Soluciones Problemas SAR

1.-

a) $P(1) = 0.5833$

b) El C, con $P(C/1) = 3/7$.

2.- a) $P(\text{destrucción}) = 0.025$

b) $P(B/\text{dest.}) = 0.6$

3.- a) 0.5

b) 0.35

c) 0.5

d) $0.\widehat{33}$

e) 0.412

f) 0.125

g) $0'06 \neq 0'05$
No son independientes.

4.- a) 0.9942

b) 0.9770

5.- a) $P(R_1) = q_0 q + p_1 (1-q)$

$P(R_0) = p_0 q + q_1 p$

b) $P(T_0/R_0) = \dfrac{p_0 q}{p_0 q + q_1 p}$

6.- a) $\binom{n}{k} p^k (q+r)^{n-k}$

b) Todos a "2"

c) Todos a "1"

d) $\binom{n}{k_1} \binom{n-k_1}{k_2} p^{k_1} q^{n-(k_1+k_2)} r^{k_2}$

7.- a) No

b) 0.766

c) $1 - G(1.18) \cong 0.119$

8.- 0.8010

9.- $F_{X}(x) = \sum_{k=0}^{n} P(X=k)\, u(n-k)$

$P(X=k) = \binom{n}{k} p^k q^{n-k}$, $q = e^{-ct_0}$, $p = 1-q$.

10.- a) $f_y(y) = \begin{cases} \frac{1}{\sqrt{y}} & 0 \le y \le 1/4 \\ 0 & y > 1/4 \end{cases}$

b) $f_y(y) = \dfrac{e^{-\frac{(\ln y - \eta)^2}{2\sigma^2}}}{(\sigma\sqrt{2\pi})\, y} \qquad y \geq 0$

c) $f_y(y) = \dfrac{1}{\pi\sqrt{A^2 - y^2}} \qquad -a \le y \le a$

11.-

a) e^{-1}

b) e^{-1}

c) $0.4016 \longrightarrow P(y \geq 1) = 0.5984$

d) 0.11

12.-

a) $P(y=n) = \dfrac{\tau^{n-1}}{(n-1)!}\left(1 - \dfrac{\tau}{n}\right)$ con $\tau = \dfrac{t}{k}$.

b) $e^{t/k}$

c) $k > \dfrac{t}{\ln n_0}$

13.-

a) 2.44

b) 0.051

c) 0.9185

14.-

a) $F_\eta(y) = \frac{1}{8} u(y+b_2) + \frac{3}{8} u(y+b_1) + \frac{3}{8} u(y-b_1) + \frac{1}{8} u(y-b_2)$

b) $b_1 = \frac{2}{9}$; $b_2 = \frac{2}{3}$

15.-

a) $P(\eta=n) = q^{n-1} p$

b) $E\{\eta\} = \frac{1}{p}$

c) $n_\eta > 490.3 \Rightarrow n_\eta = 491$

16.-

a) $\mu = 0$

b) $w = \frac{6.215}{a}$

c) $P(\eta \geq M) = \sum_{k=M}^{N} \binom{N}{k} p^k q^{N-k}$ $\quad p = 1 - P_1$

$P(\eta \geq 4) = 0.999990$

17.-

a) $f_\eta(y) = \begin{cases} 1 & 0 \leq y \leq 1 \\ 0 & \text{resto.} \end{cases}$

b) $f_\xi(x) = e^{-x}$ $\quad 0 \leq x < \infty$

c) Mínimo $\Rightarrow \mu = 0$ ó $\mu = \infty$

Máximo $\Rightarrow \mu = \ln e$

18.-

a) $a = 6/7, \quad b = 2/7$

b) $$\hat{q}_2 = \frac{x-1}{\ln x}$$

c) $r_{x\hat{q}_1} = 1$

$C_{x\hat{q}_2} = \frac{1}{24}$

19.-

a) $$f_{xz}(x,z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(z-x)^2}{2}} \qquad \begin{array}{l} -\infty < z < \infty \\ 0 < x < 1 \end{array}$$

b) $f_z(z) = G(1-z) - G(-z)$

c) $$E\{x|z\} = \frac{e^{-z^2/2} - e^{-(1-z)^2/2}}{\sqrt{2\pi}\left[G(1-z) - G(-z)\right]} + z.$$

d) 0.5401

20.-

a) $a = 0.675\,\sigma_0$

b) $E\{z\} = 0.48$

$\sigma_z^2 = 0.7696$

c) $b = 23.3$

d) 0.9976

21.-

a) 0.3560

b) 3.279

c)

i	P_i
0	0.21
1	0.2980
2	0.2770
3	0.2150

j	P_j
1	0.2670
2	0.3970
3	0.3020
4	0.034

$\eta_x = 1.4970 \qquad \eta_y = 2.1030$

$\sigma_x^2 = 1.1 \qquad \sigma_y^2 = 0.6944$

d) 0.688

e) Estan correladas. No son independientes

$C_{\xi\eta} = 0.1308 \qquad r = 0.1497.$

22:-

a) $C_{\xi\eta} = R_{\xi\eta} - \eta_\xi \eta_\eta$

$\eta_\xi = 0 \;;\quad \eta_\eta = 0 \;;\quad R_{\xi\eta} = 0 \;\Rightarrow\; C_{\xi\eta} = 0.$

b) $f_{\xi\eta}(0,0) = 0 \neq f_\xi(x)\big|_{x=0}\, f_\eta(y)\big|_{y=0}$

No factorizable $\Rightarrow$ no independientes.

c) Si

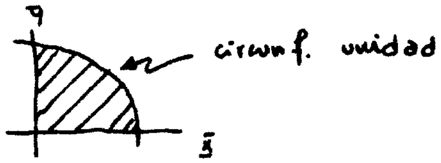
$a < b \Rightarrow \theta = -\dfrac{\pi}{4}$

$a > b \Rightarrow \theta = \dfrac{\pi}{4}$

d) Si $a = b \Rightarrow E\{ZW\} = 0$

23:-

a) $k = \dfrac{4}{\pi}$



b) $f_\xi(x) = \dfrac{4}{\pi}\sqrt{1-x^2} \qquad 0 \le x \le 1$

$f_\eta(y)\big|_x = \dfrac{1}{\sqrt{1-x^2}} \quad$ con $\quad 0 < y < \sqrt{1-x^2}$

$E\{\eta|_x\} = \dfrac{\sqrt{1-x^2}}{2} \qquad\qquad E\{\xi\eta\} = \dfrac{1}{2\pi}$

c) $f_{\rho\theta}(\rho,\theta) = \dfrac{4}{\pi}\rho \qquad \left.\begin{array}{l} 0 \le \rho \le 1 \\ 0 \le \theta \le \pi/2 \end{array}\right\}$ indep.

d) $P(\rho\cos\theta > 0.5) = 0.3910$

24.-

a) $f_z(z) = q\lambda e^{-\lambda z} u(z) + p\lambda e^{-\lambda(z-\alpha)} u(z-\alpha)$

b) $k_1 > \frac{1}{\lambda\alpha} \ln\left[\frac{q + p e^{\lambda\alpha}}{p_0} \right]$

c) $k_2 = \frac{1}{\lambda\alpha} \ln \frac{1}{p_0}$

25.-

a)

$a = 1.65\sigma$

b)

$\sigma^2 \left[2\,G\left(\frac{a}{\sigma}\right) - 1 \right] \longrightarrow 0.9\sigma^2$

c) $\gamma = \left[2\,G\left(\frac{a}{\sigma}\right) - 1 \right] \longrightarrow 0.9$

26.-

a)

$\eta_z = 7$

$\sigma_z^2 = 13/3$

b)

$r_{yz} = \frac{2\sqrt{3}}{\sqrt{13}}$ $\qquad r_{xz} = \frac{1}{\sqrt{13}}$

c)

$\hat{z} = y + \eta_x$

$\hat{z} = x + \eta_y$

27.- a) $\eta_y = 0$

$\sigma_y^2 = \frac{1}{n}\sigma^2$

b) $G\left(\frac{1-z}{\sigma_y}\right) - G\left(\frac{-z}{\sigma_y}\right) = f_z(z) \quad \forall z.$

$f_{xz}(x,z) = \frac{1}{\sigma_y\sqrt{2\pi}} e^{-\frac{(x-z)^2}{2\sigma_y^2}}$ $\qquad \forall z$, $0 \le x \le 1$

c) $$E\{z|x\} = x + \frac{\sigma_y}{\sqrt{2\pi}} \frac{e^{-\frac{x^2}{2\sigma_y^2}} - e^{-\frac{(x-1)^2}{2\sigma_y^2}}}{G\left(\frac{1-x}{\sigma_y}\right) - G\left(-\frac{x}{\sigma_y}\right)}$$

28.-

a) $\eta_x = 90$ usg.

$\sigma_x^2 = 58.3$ usg.

$r_{yx} = 0.65$

b) $$f_x(x) = \frac{1}{20}\left[G\left(\frac{x}{5} - 16\right) - G\left(\frac{x}{5} - 20\right)\right] \quad \forall x.$$

c) $$E\{z|_y\} = Ay + B = \frac{y}{2} + 40.$$

29.-

a) $$E\{y\} = \int_a^b g(x)\,dx.$$

b) $$E\{y\} = \int_a^b g(x)\,dx$$

c)

i) $\sigma_y^2 = \frac{4}{45n}$ ii) $\frac{1}{72n}$

$$\frac{1}{72n} < \frac{4}{45n}$$

30.-

a)

$x \backslash y$	0	1	2
0	q^2	qp	0
1	0	qp	p^2

$P(x=0) = q$

$P(x=1) = p$

$P(y=0) = q^2$

$P(y=1) = 2pq$

$P(y=2) = p^2$

No independientes.

b) $r = \dfrac{1}{\sqrt{2}}$ $\hat{Y} = X + p.$

c) $\hat{Y} = X + p.$

31.-

a) $r = \dfrac{1}{\sqrt{n}}$

$\eta_y = 0$

b) $Y \sim N(\eta_Y, \sigma_Y)$ $\sigma_Y^2 = \left[1 - \dfrac{1}{n}\right] \sigma_X^2$

c) $E\{Y|_{X_1}\} = \left[1 - \dfrac{1}{n}\right] (X_1 - \eta_X)$

32.-

a) $S_X(\omega) = k \quad \forall\omega$

b) $H(\omega) = \dfrac{\alpha}{\alpha + j\omega} \quad \alpha > 0$

c) $S_Y(\omega) = \dfrac{\alpha^2}{\alpha^2 + \omega^2} K$

d) $R_Y(\tau) = \dfrac{\alpha K}{2} e^{-\alpha|\tau|}$

33.-

a) $\hat{X}[n] = \dfrac{R_X[k]}{R_X[0]} X[n-k]$

$\hat{X}[n-k] = \dfrac{R_X[-k]}{R_X[0]} X[n]$

b) $R_{EX}[p] = R_X[p] - \dfrac{R_X[k]}{R_X[0]} R_X[p-k] \qquad p = n_1 - n_2$

$R_{EX}[k] = 0$

34.-

a) $V = R_N(\tau_R - \tau_T)$

b) $\tau_T = \tau_R$

c) - Considerar un $R_{máx}$ a rastrear $\Rightarrow \tau_{T_{máx}}$

- Probar con varios $\tau_T \in (0, \tau_{T_{máx}})$

- Considerar $\tau_T = \tau_R$ en el máximo absoluto de $R_N(\tau)$

35.-

a) $R_{ZW}(0) = 1 - e^{-at_0}$

$R_{ZW}(0) = 1 - e^{-at_0} + \alpha$

$R_{WW}(0) = 1 - e^{-at_0} + \frac{a}{2}$

b) $S_W(\omega) = \frac{\alpha^2}{\alpha^2 + \omega^2}\left[1 + \frac{\operatorname{sen} \omega t_0}{\omega/2}\right]$

c) $a = \frac{\ln 2t_0}{t_0}$

36.-

a) $E\{X(t)\} = \eta_Y$

$R_X(\tau) = \frac{A^2}{2} + R_Y(\tau)$

b) $M_T = A\cos\phi + \eta_Y \mp \eta_Y$

$A_T = A^2\cos^2\phi + 2A\cos\phi\,\eta_Y + R_Y(\tau) \mp R_X(\tau)$

c) $E\{Z(t)\} = M_T[\{Z(t)]] = 0$

37.- Sea $Y(t) = A_c(1 + m\,X(t))\cos(\omega_c t + \theta)$

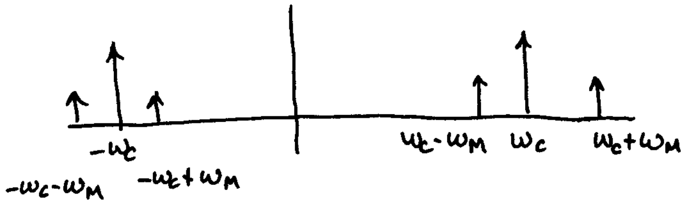
a) $E\{Y(t)\} = 0 \quad (\forall t)$

$R_Y(t_1, t_2) = \frac{A_c^2}{2}\left(1 + m^2 R_X(\tau)\right)\cos\omega_c\tau. = R_Y(\tau)$

b) $R_Y(\tau) = \frac{A_c^2}{2}\left(1 + m^2\cos\omega_M\tau\right)\cos\omega_c\tau$

c) $P_Y = R_Y(\tau = 0) = \frac{A_c^2}{2}(1 + m^2)$

$$S_q(\omega) = \frac{A_c^2}{2}\pi\,\delta(\omega \pm \omega_c) + \frac{w^2}{4}\pi\left[\delta(\omega \pm (\omega_c - \omega_M)) + \delta(\omega \pm (\omega_c + \omega_M))\right]$$



d) $$|H(\omega)|^2 = \frac{\omega^2}{\alpha^2+\omega^2}$$

$$S_z(\omega) = S_y(\omega)\,|H(\omega)|^2$$

38.-

a)
$$E\{z(t)\} = 0$$
$$E\{z(t_1)z(t_2)\} = \frac{1}{2}\sum_{i=1}^{m}\cos\omega_i\tau + R_N(\tau) \qquad \tau = t_1 - t_2$$

b) $$S_z(\omega) = \frac{\pi}{2}\left[\delta(\omega-\omega_1) + \delta(\omega+\omega_1)\right] + 1$$

c)

$$\left.\begin{array}{l} P_{Ruido} = \dfrac{\omega_c}{\pi} \\ P_s = \dfrac{1}{4} \end{array}\right\} \quad \omega_c = \frac{\pi}{4}$$

39.-

a)
$$\left.\begin{array}{l} R_{zz}[n_1,n_2] = \text{mínimo}(n_1,n_2) \\ E\{z[n]\} = 0 \end{array}\right\} \quad \text{No es WSS}$$

b)
$$\left.\begin{array}{l} R_y[n_1,n_2] = \begin{cases} N - |n_1-n_2| & \text{si } |n_1-n_2| \le N \\ 0 & \text{otro.} \end{cases} \\ \\ E\{y[n]\} = 0 \end{array}\right\} \quad \text{WSS}$$

c) $$N \gg 1 \Rightarrow y[n] \sim N(\eta_y, \sigma_y^2)$$
$$\eta_y = 0$$
$$\sigma_y^2 = R_y[0] = N.$$

40.-

a) $E\{A\} = E\{B\} = 0$

$E\{AB\} = 0$

$E\{A^2\} = E\{B^2\}$

b) Igual que a)

c) $E\{x(t_1)\,y(t_2)\} = E\{A^2\} \operatorname{sen} \omega_0 \tau$

41.-

a) $\eta_x = 0$

$R_x(\tau) = \frac{A^2}{2}\cos(\omega\tau)$

b) WSS.

c) $M_T = 0$

$A_T = \frac{A^2}{2}\cos \omega\tau$.

d) Ergódico respecto media y correlación.

e) $E\{y(t)\} = \frac{A^2}{2}$

$R_y(\tau) = \frac{A^4}{4}\left(1 + \frac{1}{2}\cos 2\omega\tau\right)$

f) $M_T[y(t)] = \frac{A^2}{2}$

$A_T[y(t)] = \frac{A^4}{4}\left(1 + \frac{1}{2}\cos 2\omega\tau\right)$

g) Ergódico respecto media y correlación

h) $H(\omega) = \frac{\operatorname{sen}\omega T}{\omega T}$

i) $C_z(\tau) = R_z(\tau) - \eta_z^2$

$\eta_z = \eta_y$

$R_z(\tau) = h(\tau) * h^*(-\tau) * R_y(\tau)$

$S_z(\omega) = |H(\omega)|^2\, S_y(\omega)$

42.-

a) $E\{\eta(t)\} = 0$

$R_\eta(t_1,t_2) = R_x(\tau)\,\frac{1}{2}\cos\omega_0\tau = R_\eta(\tau) \qquad \tau = t_1 - t_2$

b) $M_T(\eta(t)) \xrightarrow[T\to\infty]{} 0$

$\Delta_T(\eta(t)) \xrightarrow[T\to\infty]{} \frac{A^2}{2}\cos\omega_0\tau \rightarrow$ es una v. aleatoria

No es ergódico respecto cor

43.-

a) $E\{\eta[n]\} = 0$

$\sigma_\eta^2 = \sigma_x^2 \frac{1}{1-a^2}$

b) $R_\eta[m] = \frac{a^{|m|}}{1-a^2}\sigma_x^2$

$S_\eta(\omega) = S_x(\omega)\,|H(\omega)|^2 = \frac{\sigma_x^2}{1-2a\cos\omega + a^2}$

c) $E\{\eta[n+1]\,|\,\eta[n]\} = a\,\eta[n]$

d) $E\{|\epsilon|^2\} = \sigma_x^2$

44.-

a) $R_x[n] = \delta[n]$

$S_x(\omega) = 1 \quad \forall\omega$

b) $E\{x[n_1]\,\eta[n_2]\} = a_0\,\delta[m] + a_1\,\delta[m-1] = R_{x\eta}[m]$

$m = n_1 - n_2$

c) $R_\eta[m] = (a_0^2 + a_1^2)\,\delta[m] + a_0a_1\,\delta[m+1] + a_1a_0\,\delta[m-1]$

$S_\eta(\omega) = a_0^2 + a_1^2 + 2a_0a_1\cos\omega$

45.-

a) $E\{\xi(t)\} = \dfrac{1}{at}(1 - e^{-at})$

$$E\{\xi(t_1)\,\xi(t_2)\} = \frac{1}{a(t_1+t_2)}\left[1 - e^{-a(t_1+t_2)}\right]$$

b)
$$\eta(t) = \xi(t) * h(t)$$
$$= \int_0^{t_0} \xi(t-\tau)\, h(\tau)\, d\tau.$$

$$E\{\eta(t)\}\Big|_{t=0} = \int_0^{t_0} E\{\xi(t-\tau)\big|_{t=0}\}\, h(\tau)\, d\tau$$
$$= \int_0^{t_0} \frac{\beta}{a(-\tau)}\,\tau\,(1 - e^{a\tau})\,d\tau = \frac{\beta}{a}\left[\frac{1}{a}\left(e^{at_0} - 1\right) - t_0\right]$$

PROBLEMAS COMPLEMENTARIOS.

C.1

$P(S_A|C) = 0.0085$

$P(S_B|C) = 0.0057$

$P(S_C|C) = 0.0064$

C.2

a) $P(M) = \alpha^2 \left(\frac{1-\alpha}{2}\right)^2 P_1 + \alpha\left(\frac{1-\alpha}{2}\right)^3 P_2 + \alpha\left(\frac{1-\alpha}{2}\right)^3 P_3$

b) $P(M_1|M) = \dfrac{P(M|M_1)\,P(M_1)}{P(M)}$

C.3

a) $P(M) = \binom{a+b}{a}(1-p)^a p^b$

b) $P(N+M) = \binom{a+b+1}{a}(1-p)^a p^{b+1} + q\,P(M)$

C.4.

$$\eta_I - \epsilon < I < \eta_I + \epsilon \qquad \eta_I + \epsilon = 1.9402\, i_0$$

Práctico

$$0 < I < 1.9402\, i_0$$

C5.-

a) $E\{\mathbf{x}\} = N(3p-1)$

$\sigma_{\mathbf{x}}^2 = 9Npq$

b) $p > 1 - 2^{-1/3}$

c) $1 - G(0) = 1/2$

C6.-

a) $f_\varphi(\varphi) = \frac{1}{a}\left[1 + tg^2\varphi\right] \qquad 0 \le \varphi \le tg^{-1} a$

b) $f_y(y) = \dfrac{y}{a\sqrt{(y^2-1)}} \qquad 1 \le y \le \sqrt{1+a^2}$

C7.-

a) $\mu = \eta$

$\alpha^2 = \dfrac{2}{\sigma^2}$

b) 0.0185

c) 0.7252

C8.-

a) $f_{\mathbf{x}}(x) = \dfrac{c}{x}(x-27) \qquad 27 \le x \le 33$

$f_{\mathbf{y}}(y) = c \ln \dfrac{33}{y} \qquad 27 \le y \le 33$

$c = \dfrac{1}{6 - 27 \ln \frac{33}{27}}$

b) $f_X(x|y) = \dfrac{1}{x \ln \frac{33}{y}}$ $\qquad 27 \le y \le x \le 33$

$f_Y(y|x) = \dfrac{1}{x - 27}$ $\qquad 27 \le y \le x \le 33$

c) $P\{\ \} = c\left[1 - 27 \ln \dfrac{28}{27} + \ln \dfrac{30}{28} \right]$

d) $P\left(X > 32 \,|\, Y = 30 \right) = \dfrac{\ln 33/32}{\ln 33/30}$

e) $\hat{X} = \dfrac{33 - y}{\ln 33 - \ln y}$

$\hat{Y} = \dfrac{1}{2}(x + 27)$

C9.-

a) $T = 174.4$

b) $P_{FA} = 0.0902$

c) i) $P_D = 0.6772$

ii) $P_D = 0.1219$

C10.-

a) $P_{00} = q^2 + q(p^2 + q^2)$

$P_{01} = pq + 2pq^2$

$P_{10} = p^2 + p(p^2 + q^2)$

$P_{11} = pq + 2p^2 q$

b) $C_{XZ} = pq(1 - 2p)$

c) $C_{XZ} = 0 \quad \left\{ \begin{array}{l} p = 0,\ q = 1 \\ q = 0,\ p = 1 \\ p = 1/2,\ q = 1/2 \end{array} \right\}$ $\underline{\underline{\text{indep}}}$

C11.-

a) $f_{XY} = f_X(x)\, f_Y(y|_x)$

b) $C_{XY} \neq 0 \quad \left(\frac{T^2}{12} \cdot \frac{T^2}{8} \right)$

c) $P(Y < X) = \ln 2$

C12.-

a)

$Z \backslash W$	0	1
0	q^2	$2pq$
1	0	p^2

$P(Z=0) = q^2 + 2pq = q(p+1)$

$P(Z=1) = p^2$

$P(W=0) = q^2$

$P(W=1) = p(1+q)$

b)

$$\hat{Z} = \frac{p}{1+q} W$$

$$\hat{W} = \frac{qZ + 2p}{1+p}$$

c) Igual a b)

C13.-

a) $q_{opt} = \frac{p}{p + qe^{-c}}$

b) $P(X=1 | W=a) = \frac{p}{p+qe^{-c}}$

$P(X=0 | W=0) = 1$

C14.-

a) $f_U(u) = u\, e^{-u^2/2} \qquad u \geq 0$

$f_{UY}(u,y) = u\, e^{-u^2/2} \qquad u \geq 0 \quad 0 \leq y \leq 1$

b) $$f_{ZW} = \frac{1}{2\pi} e^{-\frac{z^2+w^2}{2}} = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \frac{1}{\sqrt{2\pi}} e^{-w^2/2}$$

$$\left.\begin{array}{l} Z \sim N(0,1) \\ W \sim N(0,1) \end{array}\right\} \text{ independientes.}$$