PARCIAL 2 MAYO 2016

a) $z(t) = e^{-5|t-1|}\cos(2\pi t)$ [0'9 PUNTOS]

1 $z_1(t) = e^{-5|t|} = \underbrace{e^{-5t}u(t)}_{z_{11}(t)} + \underbrace{e^{5t}u(-t)}_{z_{12}(t)}$

$$e^{-5t}u(t) \xrightarrow{\mathcal{F}} \frac{1}{5+j\omega} = Z_{11}(\omega)$$

$$z_{12}(t) = z_{11}(-t) \xrightarrow{\mathcal{F}} Z_{12}(\omega) = Z_{11}(-\omega) = \frac{1}{5-j\omega}$$

$$Z_1(\omega) = Z_{11}(\omega) + Z_{12}(\omega) = \frac{1}{5+j\omega} + \frac{1}{5-j\omega} = \frac{10}{25+\omega^2}$$

2 $z_2(t) = e^{-5|t-1|} = z_1(t-1) \xrightarrow{\mathcal{F}} Z_2(\omega) = Z_1(\omega)\cdot e^{-j\omega}$

$$= \frac{10\cdot e^{-j\omega}}{25+\omega^2}$$

3 $z(t) = z_2(t)\cdot\cos(2\pi t)$

$$\xrightarrow{\mathcal{F}} Z(\omega) = \frac{1}{2\pi} Z_2(\omega) * \pi\left[\delta(\omega-2\pi)+\delta(\omega+2\pi)\right]$$

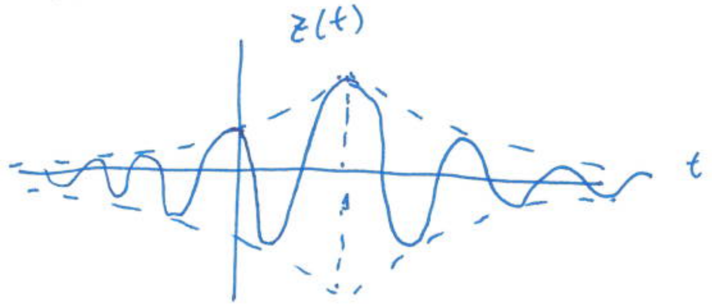
$$= \frac{1}{2}\left[Z_2(\omega-2\pi) + Z_2(\omega+2\pi)\right]$$

$$Z(\omega) = \frac{5\, e^{-j(\omega+2\pi)}}{25+(\omega+2\pi)^2} + \frac{5\, e^{-j(\omega-2\pi)}}{25+(\omega-2\pi)^2}$$

$(e^{-j\pi\cdot 2} = 1)$

$$\boxed{Z(\omega) = 5\, e^{-j\omega}\left[\frac{1}{25+(\omega+2\pi)^2} + \frac{1}{25+(\omega-2\pi)^2}\right]}$$

DIBUJO DE $z(t)$



b)

La serie de Fourier será de la forma [0.8 PUNTOS]

$$x(t) = \sum_{k=-\infty}^{\infty} C_k \cdot e^{jk\frac{2\pi}{3}t}$$

con $T=3$ y C_k:

$$C_k = \frac{1}{3}\int_{-1}^{2} x(t)\, e^{-jk\frac{2\pi}{3}t}\, dt$$

$$= \frac{1}{3}\int_{-1}^{0} e^{3t} e^{-jk\frac{2\pi}{3}t}\, dt + \frac{1}{3}\int_{0}^{2} e^{-2t} e^{-jk\frac{2\pi}{3}t}\, dt$$

$$= \frac{1}{3}\left[\frac{1-e^{-3}e^{jk\frac{2\pi}{3}}}{3-jk\frac{2\pi}{3}} + \frac{1-e^{-4}e^{-jk\frac{4\pi}{3}}}{2+jk\frac{2\pi}{3}}\right] \quad k \in \mathbb{Z}$$

(No es necesario calular otros valores de k, como C_0)

c)

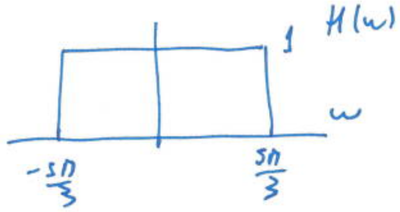
$$x(t) \longrightarrow \boxed{LTI} \longrightarrow y(t)$$ [0.8 PUNTOS]

Por ser periódica:

$$x(t) = \sum_k C_k e^{jk\frac{2\pi}{3}t} \longrightarrow \boxed{LTI} \longrightarrow y(t) = \sum_k \underbrace{C_k \cdot H\left(k\frac{2\pi}{3}\right)}_{d_k} e^{jk\frac{2\pi}{3}t}$$

$$d_k = C_k \cdot H\left(k\frac{2\pi}{3}\right)$$

$H(\omega) \longrightarrow$

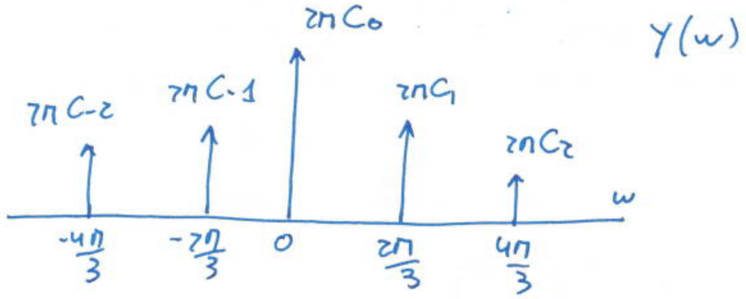


La T.F. de $y(t)$ será de la forma:

$$Y(\omega) = \sum_{k=-\infty}^{\infty} 2\pi \cdot C_k \cdot H\left(k\frac{2\pi}{3}\right) \delta\left(\omega - k\frac{2\pi}{3}\right)$$

(TREN DE DELTAS)

Dado que $H(\omega) = 0$ $|\omega| > \frac{5\pi}{3}$, sólo quedan 5 deltas:



Los coeficientes $C_0, C_1, C_2, C_{-1}, C_{-2}$ se calculan dando valores a los C_k del apartado anterior:

$$C_0 = \frac{1}{3}\left[\frac{1-e^{-3}}{3} + \frac{1-e^{-4}}{2}\right]$$

$$C_1 = C_{-1}^* = \ldots$$

$$C_2 = C_{-2}^* = \ldots$$