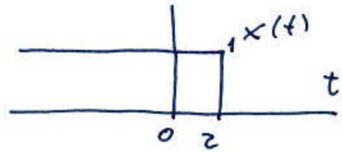
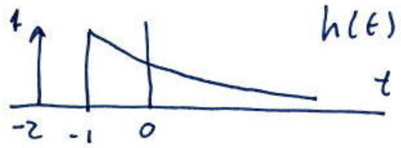
Parcial 1 — Marzo 2017

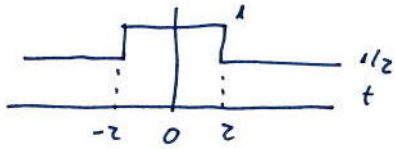
$x(t) = u(2-t)$



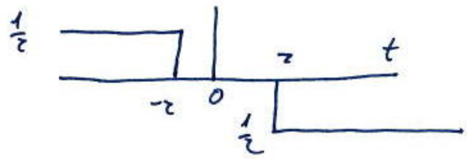
$h(t) = e^{-2t}u(t+1) + \delta(t+2)$



a) $Ev\{x(t)\} = \dfrac{x(t)+x(-t)}{2}$



$Od\{x(t)\} = \dfrac{x(t)-x(-t)}{2}$



$P_i(t) = u(2-t)$ $\qquad E = \int_{-\infty}^{\infty} u(2-t)\,dt = \int_{-\infty}^{2} 1 \cdot dt = \infty$

$$P_{AV} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{2} dt = \lim_{T\to\infty}\frac{2+T}{2T} = \frac{1}{2}$$

b) $h(t) \neq k\,\delta(t) \Rightarrow$ CON MEMORIA

$h(t) \neq 0 \begin{cases} t > 0 \\ t < 0 \end{cases} \Rightarrow$ ANTICAUSAL

$$\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{-\infty}^{\infty} \delta(t+2)\,dt + \int_{1}^{\infty} e^{-2t}\,dt = 1 + \frac{e^2}{2} < \infty \Rightarrow \text{ESTABLE}$$

c) $y(t) = x(t) * h(t) \quad \Rightarrow \quad h(t) = \delta(t+2) + \underbrace{e^{-2t}u(t+1)}_{h_1(t)}$

$= x(t) * \left(\delta(t+2) + h_1(t)\right)$

$= x(t+2) + \underbrace{x(t) * h_1(t)}_{y_1(t)} \quad \rightarrow x(t+2) = u(-t)$

$$y_1(t) = \int_{-\infty}^{2} e^{-2(t-\tau)}\,u(t+1-\tau)\,d\tau$$

$$y_1(t) = \begin{cases} t < 1 & \int_{-\infty}^{t+1} e^{-2t} e^{2\tau} d\tau = \dfrac{e^2}{2} \\ t \geq 1 & \int_{-\infty}^{2} e^{-2t} e^{2\tau} d\tau = \dfrac{e^{-2(t-2)}}{2} \end{cases}$$

$$y(t) = \begin{cases} 1 + \dfrac{e^2}{2} & t < 0 \\ \dfrac{e^2}{2} & 0 \leq t < 1 \\ \dfrac{e^{-2(t-2)}}{2} & t \geq 1 \end{cases}$$

$$y(t) = u(-t) + \frac{e^2}{2} u(1-t) + \frac{e^{-2(t-2)}}{2} u(t-1)$$

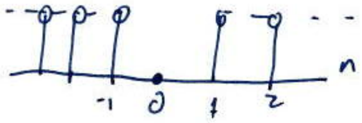
d) i) PERIÓDICA $T = \dfrac{2\pi}{\sqrt{2}}$

ii) $e^{-2\pi t} \Rightarrow$ es una exp. real, por lo tanto

NO PERIÓDICA

iii) $\Omega_0 = \dfrac{1}{2}$ $\dfrac{\Omega_0}{2\pi} \notin \mathbb{Q}$ No Periódica

iv) $\Omega_0 = 1$ $\dfrac{\Omega_0}{2\pi} \notin \mathbb{Q}$ No Periódica

v) 

$\Rightarrow$ NO PERIÓDICA

$\boxed{\text{PROBLEMA 2}}$ MAYO 2017

a) $x(t) = \text{sen}(\pi t)$

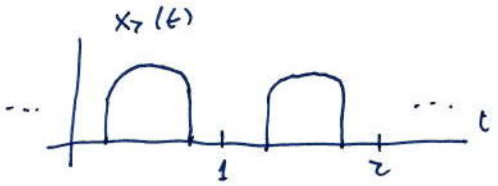
↳ La señal ya es una serie de Fourier, ya que se puede escribir como suma de exponenciales complejas

$$x(t) = \frac{1}{2j} e^{j\pi t} - \frac{1}{2j} e^{-j\pi t} = \underbrace{\frac{1}{2j} e^{j\frac{2\pi}{2}t}}_{k=1} \underbrace{-\frac{1}{2j} e^{-j\frac{2\pi}{2}t}}_{k=-1}$$

De aquí
$$C_k = \begin{cases} \frac{1}{2j} & k=1 \\ -\frac{1}{2j} & k=-1 \\ 0 & k \neq \pm 1 \end{cases} \qquad T = 2$$

Serie de Fourier: $x(t) = \sum_k C_k e^{jk\pi t}$

b) La señal $x_2(t)$ es periódica de periodo $\underline{\underline{T=1}}$



La S. Fourier: $x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk2\pi t}$

$$C_k = \int_{1/6}^{5/6} \text{sen}(\pi t) e^{-jk2\pi t} dt = \int_{1/6}^{5/6} \frac{1}{2j}\left(e^{j\pi t} - e^{-j\pi t}\right) e^{-jk2\pi t} dt$$

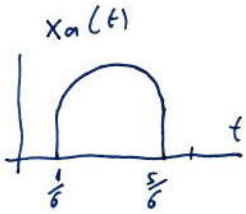
$$= \frac{1}{2\pi}\left[\frac{e^{j\frac{\pi}{6}(1-2k)} - e^{j\frac{5\pi}{6}(1-2k)}}{(1-2k)} + \frac{e^{-j\frac{\pi}{6}(1+2k)} - e^{-j\frac{5\pi}{6}(1+2k)}}{(1+2k)}\right]$$

$\forall k \in \mathbb{Z}$

Podrían simplificarse los coeficientes: $\left(e^{-j\frac{\pi}{3}k} = e^{-j\left(\frac{\pi}{3}+2\pi\right)k} = e^{j\frac{5\pi}{3}k}\right)$

$$C_k = \frac{1}{\pi(1-4k^2)}\left[\sqrt{3}\cos\left(\frac{5\pi}{3}k\right) - 2k\,\text{sen}\left(\frac{5\pi}{3}k\right)\right]$$

De manera alternativa, se puede calcular a partir de la T.F. de la señal aperiódica:



$$x_a(t) = \text{sen}(\pi t) \cdot x_1(t)$$

$$\text{con} \quad x_1(t) = \begin{cases} 1 & \frac{1}{6} < t < \frac{5}{6} \\ 0 & \text{resto} \end{cases}$$

$$X_a(\omega) = \frac{1}{2\pi} X(\omega) * \frac{\pi}{j}\left[\delta(\omega-\pi) - \delta(\omega+\pi)\right]$$

$$X_1(\omega) = \frac{2\,\text{sen}(\frac{\omega}{3})}{\omega} \cdot e^{-j\frac{\omega}{3}}$$

$$X_a(\omega) = e^{-j\frac{\omega}{2}}\left[\frac{\text{sen}\left(\frac{\omega-\pi}{3}\right)}{\omega-\pi} - \frac{\text{sen}\left(\frac{\omega+\pi}{3}\right)}{\omega+\pi}\right]$$

$$C_k = X_a(2\pi k) = \frac{(-1)^k}{\pi}\left[\frac{\text{sen}\left(\frac{2\pi}{3}k - \frac{\pi}{3}\right)}{2k-1} - \frac{\text{sen}\left(\frac{2\pi}{3}k + \frac{\pi}{3}\right)}{2k+1}\right]$$

$k \in \mathbb{Z}$

(Aunque parezca distinto a la solución anterior, se puede demostrar que es la misma)

c) Sabemos que $e^{j\omega_0 t} \rightarrow \boxed{h(t)} \rightarrow H(\omega_0)e^{j\omega_0 t}$

En el caso de una señal periódica, la SF a la entrada da una SF a la salida:

$$x(t) = \sum_k C_k e^{jk\frac{2\pi}{T}t} \longrightarrow \boxed{h(t)} \rightarrow y(t) = \sum_k \underbrace{C_k H\left(k\frac{2\pi}{T}\right)}_{d_k} e^{jk\frac{2\pi}{T}t}$$

En nuestro caso:

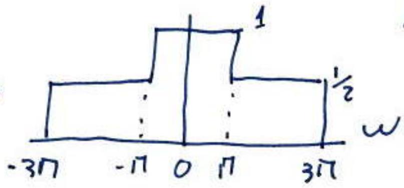
$$y(t) = \sum_k \underbrace{C_k \cdot H(k2\pi)}_{d_k} e^{jk\frac{2\pi}{T}t}$$

La S.F. a la salida tendrá el mismo periodo $T=1$ y los coef. d_k serán

$$d_k = C_k H(k2\pi)$$

Calculo la T.F. de $h(t)$

$$h(t) \xrightarrow{\mathcal{F}} H(\omega) \Rightarrow$$



Por lo tanto $H(k2\pi) = \begin{cases} 1 & k=0 \\ \frac{1}{2} & k=\pm 1 \\ 0 & |k|>1 \end{cases}$

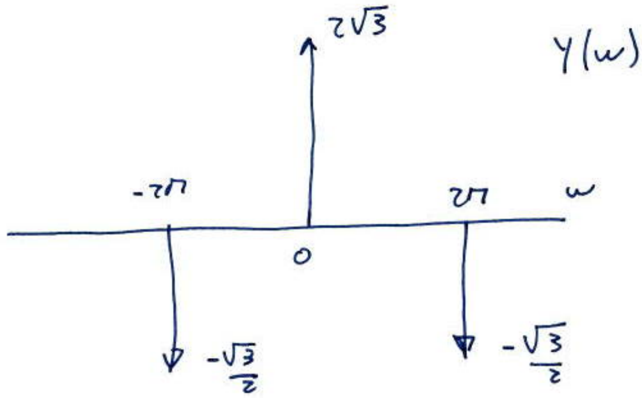
De aquí:

$$d_k = \begin{cases} C_0 & k=0 \\ \frac{C_1}{2} & k=1 \\ \frac{C_{-1}}{2} & k=-1 \\ 0 & |k|>1 \end{cases}$$

$$\boxed{y(t) = C_0 + \frac{C_1}{2} e^{j2\pi t} + \frac{C_{-1}}{2} e^{-j2\pi t}}$$

d) La T.F. de una señal periódica es de la forma

$$Y(\omega) = \sum_k 2\pi d_k \,\delta(\omega - 2\pi k) \Rightarrow \begin{cases} C_0 = \frac{\sqrt{3}}{\pi} \\ C_1 = C_{-1} = -\frac{\sqrt{3}}{2\pi} \end{cases}$$

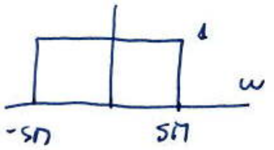


TAMBIÉN PUEDE HACERSE COMO $Y(\omega) = X(\omega) \cdot H(\omega)$

(3)

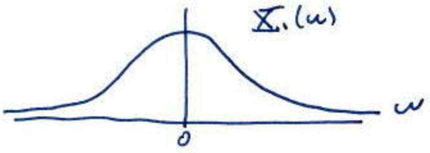
a) $x_1(t) = e^{-2|t|} \xrightarrow{\mathcal{F}} X_1(\omega) = \frac{1}{2+j\omega} + \frac{1}{2-j\omega} = \frac{4}{4+\omega^2}$

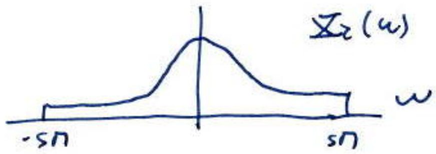
$H_c(\omega)$



$$X_2(\omega) = X_1(\omega)\cdot H_c(\omega)$$

$$= \begin{cases} \frac{4}{4+\omega^2} & |\omega| \leq 5\pi \\ 0 & |\omega| > 5\pi \end{cases}$$





$x_1(t)$ no es muestreable, ya que $X_1(\omega)$ no es de banda limitada.

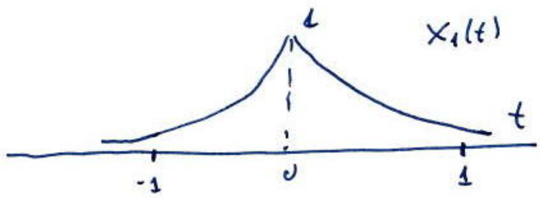
$x_2(t)$ sí es muestreable. $\omega_{MAX} = 5\pi$

Podemos muestrear a la freq. de Nyquist

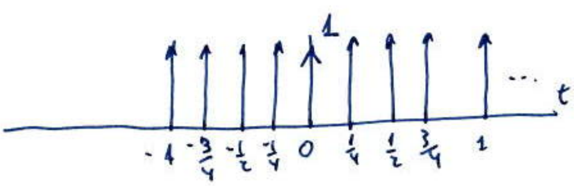
$\omega_s = 2\,\omega_{MAX} \Rightarrow \omega_s = 10\pi$

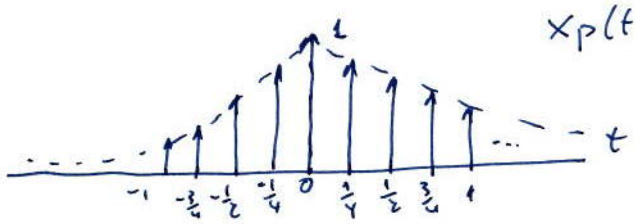
$T = \frac{1}{5}$

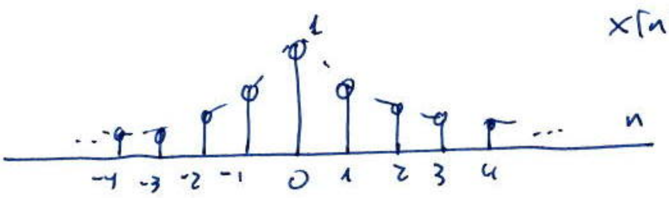
b)



$T = 1/4$

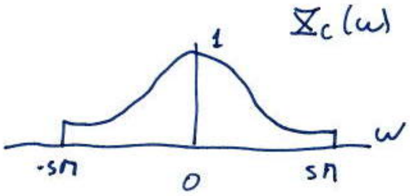


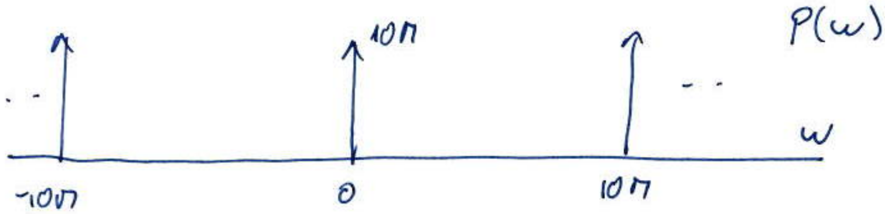


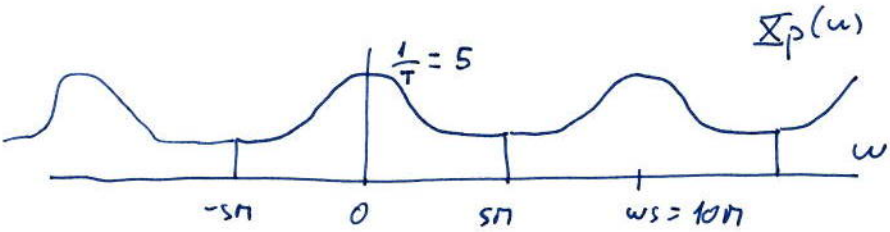


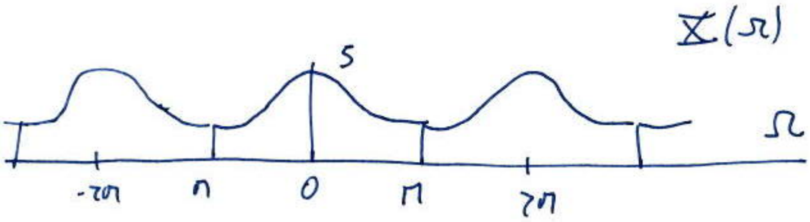
$$x[n] = x_c(n \cdot T) = x_c\left(\frac{n}{4}\right) = e^{-2\left|\frac{n}{4}\right|} = e^{-\frac{|n|}{2}}$$

c) Uso $T = 1/5$







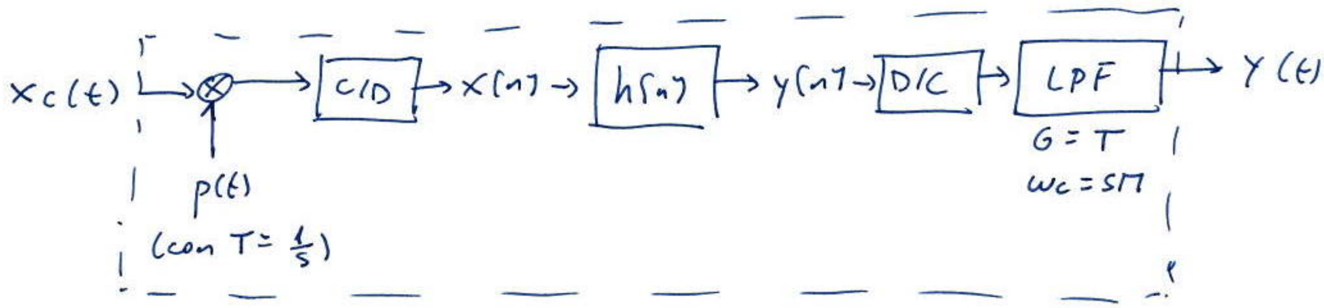


$$X(\Omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_c\left(\frac{\Omega - 2\pi k}{T}\right)$$

Por no haber aliasing: $X(\Omega) = \frac{1}{T} X_c\left(\frac{\Omega}{T}\right) \quad |\Omega| < \pi$

$$X(\Omega) = 5\, X_c(5\Omega) = \frac{20}{4 + (5\Omega)^2} \quad |\Omega| < \pi$$

d) Esquema:



$$H_c(\omega) = \frac{1}{(1+j\omega)^2} \rightarrow$$ ① Límite en Banda

$$H_c(\omega) = \begin{cases} \frac{1}{(1+j\omega)^2} & |\omega| \leq 5\pi \\ 0 & |\omega| > 5\pi \end{cases}$$

② CAMBIO VARIABLE

$$H(\Omega) = \frac{1}{(1+j5\Omega)^2} \qquad |\Omega| < \pi$$

↳ (PERIÓDICO DE PERIODO 2π)