Sistemas Lineales

Tema 2. Soluciones Numéricas de la Hoja de Problemas (V 2.0)

1. (a) $y_1[n] = x[n] * h[n] = 2\delta[n+1] + 4\delta[n] + 2\delta[n-1] + 2\delta[n-2] - 2\delta[n-4]$

   (b) $y_2[n] = x[n+2]*h[n] = y_1[n+2] = 2\delta[n+3]+4\delta[n+2]+2\delta[n+1]+2\delta[n]-2\delta[n-2]$

   (c) $y_3[n] = x[n]*h[n+2] = y_1[n+2] = 2\delta[n+3]+4\delta[n+2]+2\delta[n+1]+2\delta[n]-2\delta[n-2]$

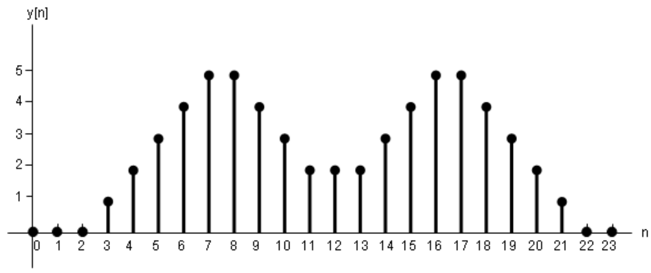
2. (a) $x[n] = \alpha^n u[n]$,
   $h[n] = \beta^n u[n],\ \alpha \neq \beta$
   $y[n] = \frac{\beta^{n+1}-\alpha^{n+1}}{\beta-\alpha} u[n]$

   (b) $x[n] = h[n] = \alpha^n u[n]$
   $y[n] = \alpha^n (n+1) u[n]$

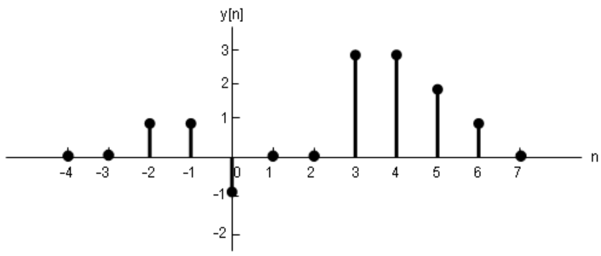
   (c) $x[n] = 2^n u[-n]$
   $h[n] = u[n]$
   $y[n] = \begin{cases} 2^{n+1}, & n < 0 \\ 2, & n \geq 0 \end{cases}$

   (d) $x[n] = (-1)^n \left(u[-n] - u[-n-8]\right)$
   $h[n] = u[n] - u[n-8]$
   $$y[n] = \begin{cases} 0, & n < -7 \\ \left\{ \begin{array}{ll} -1, & n \text{ impar} \\ 0, & n \text{ par} \end{array} \right\}, & -7 \leq n < 0 \\ \left\{ \begin{array}{ll} 1, & n \text{ impar} \\ 0, & n \text{ par} \end{array} \right\}, & 0 \leq n \leq 7 \\ 0, & n > 7 \end{cases}$$

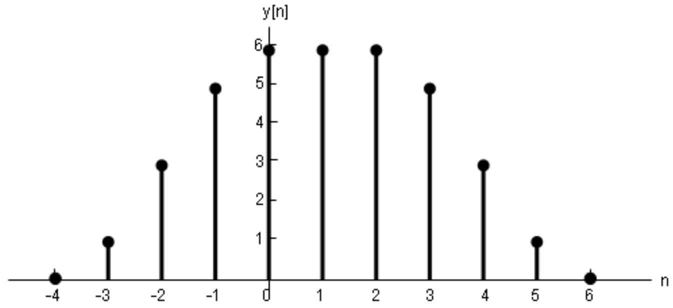
   (e) y[n]



   (f) y[n]



(g) y[n]



(h) $x[n] = 1$ para todo n, $h[n] = \begin{cases} \left(\frac{1}{2}\right)^n, & n \geq 0 \\ 4^n, & n < 0 \end{cases}$

$y[n] = \frac{7}{3}$ para todo n.

(i) $x[n] = u[n] - u[-n]$ para todo n, $h[n] = \begin{cases} \left(\frac{1}{2}\right)^n, & n \geq 0 \\ 4^n, & n < 0 \end{cases}$

$y[n] = \left[\frac{7}{3} - 3\left(\frac{1}{2}\right)^n\right] u[n] + \left[\frac{5}{3}4^n - \frac{7}{3}\right] u[-n-1]$

(j) $x[n] = \left(\frac{1}{2}\right)^n u[n]$

$h[n] = 4^n u[2-n]$

$y[n] = \begin{cases} \frac{1}{7}2^{2n+3}, & n < 2 \\ \frac{1}{7}2^{9-n}, & n \geq 0 \end{cases}$

(k) $x[n] = \alpha^n(u[n] - u[n-10]), \quad 0 < \alpha < 1$

$y[n] = \beta^n u[n+5], \quad 0 < \beta < 1$

$y[n] = \begin{cases} 0, & n < -5 \\ \frac{\beta^{n+6} - \alpha^{n+6}}{\beta^5(\beta - \alpha)}, & -5 \leq n \leq 4 \\ \frac{\beta^{n+1} - \beta^{n-9}\alpha^{10}}{\beta - \alpha}, & n > 4 \end{cases}$

3. (a) $x(t) = e^{-\alpha t}u(t)$

$h(t) = e^{-\beta t}u(t)$ (Haga este ejercicio para $\alpha \neq \beta$ y para $\alpha = \beta$).

$y(t) = \frac{e^{-\alpha t} - e^{-\beta t}}{\beta - \alpha}u(t)$, si $\alpha \neq \beta$.

$y(t) = te^{-\alpha t}u(t)$, si $\alpha = \beta$.

(b) $x(t) = u(t) - 2u(t-2) + u(t-5)$

$h(t) = e^{2t}u(1-t)$

$y(t) = \begin{cases} \frac{1}{2}\left(e^{2t} - 2e^{2(t-2)} + e^{2(t-5)}\right), & t \leq 1 \\ \frac{1}{2}\left(e^{2} - 2e^{2(t-2)} + e^{2(t-5)}\right), & 1 \leq t \leq 3 \\ \frac{1}{2}\left(e^{2(t-5)} - e^{2}\right), & 3 \leq t \leq 6 \\ 0, & t \geq 6 \end{cases}$

o bien:

$$\begin{aligned} y(t) = \; & \tfrac{1}{2}\left[e^{2t} - e^{2}\right]u(1-t) + \left[e^{2} - e^{2(t-2)}\right]u(3-t) + \\ & \tfrac{1}{2}\left[-e^{2} + e^{2(t-5)}\right]u(6-t) \end{aligned}$$

(c) $x(t) = e^{-3t}u(t)$

$h(t) = u(t-1)$

$y(t) = \frac{1}{3}\left(1 - e^{-3(t-1)}\right)u(t-1)$

(d) $x(t) = e^{-2t}u(t+2) + e^{3t}u(-t+2)$

$h(t) = e^{t}u(t-1)$

$$y(t) \begin{cases} \frac{1}{2}e^{(3t-2)}, & t \leq -1 \\ \frac{1}{2}e^{(3t-2)} - \frac{1}{3}e^{(-2t+3)} + \frac{1}{3}e^{(t+6)}, & -1 \leq t \leq 3 \\ \frac{1}{2}e^{(t+4)} - \frac{1}{3}e^{(-2t+3)} + \frac{1}{3}e^{(t+6)}, & t \geq 3 \end{cases}$$

o bien:

$$\begin{aligned} y(t) = \; & \left[\tfrac{1}{2}e^{(3t-2)} - \tfrac{1}{3}e^{(-2t+3)} + \tfrac{1}{3}e^{(t+6)}\right] + \\ & \tfrac{1}{3}\left[e^{(-2t+3)} - e^{(t+6)}\right] u(-1-t) + \\ & \tfrac{1}{2}\left[e^{(t+4)} - e^{(3t-2)}\right] u(t-3) \end{aligned}$$

(e) $x(t) = \begin{cases} e^t, & t < 0 \\ e^{5t} - 2e^{-t}, & t > 0 \end{cases}$

$h(t)$ como se muestra en la figura.

$$y(t) = \begin{cases} e^t - e^{(t-1)}, & t < 0 \\ \frac{1}{5}e^{5t} - e^{(t-1)} + 2e^{-t} - \frac{6}{5}, & 0 \leq t < 1 \\ \frac{1}{5}\left[e^{5t} - e^{5(t-1)}\right] + 2\left[e^{-t} - e^{-(t-1)}\right], & t \geq 1 \end{cases}$$

o bien:

$$\begin{aligned} y(t) = \; & \left[e^t - e^{(t-1)}\right] + \left[\tfrac{1}{5}e^{5t} - e^t + 2e^{-t} - \tfrac{6}{5}\right] u(t) + \\ & \left[-\tfrac{1}{5}e^{5(t-1)} + e^{(t-1)} - 2e^{-(t-1)} + \tfrac{6}{5}\right] u(t-1) \end{aligned}$$

(f) $x(t)$ y $h(t)$ como se muestran en la figura.

$$y(t) = \begin{cases} 0, & t \leq 1 \\ \frac{1}{\pi}\left[1 + cos(\pi t)\right], & 1 < t \leq 3 \\ -\frac{1}{\pi}\left[1 + cos(\pi t)\right], & 3 < t \leq 5 \\ 0, & t > 5 \end{cases}$$

o bien:

$$\begin{aligned} y(t) = \; & \tfrac{1}{\pi}\left[1 + cos(\pi t)\right] u(t-1) - \tfrac{2}{\pi}\left[1 + cos(\pi t)\right] u(t-3) + \\ & \tfrac{1}{\pi}\left[1 + cos(\pi t)\right] u(t-5) \end{aligned}$$

(g) $x(t)$ como se muestra en la figura, y $h(t) = u(-2-t)$.

$$y(t) = \begin{cases} 7, & t \leq -1 \\ 4 - 3t, & -1 < t \leq 0 \\ 4 - t, & 0 < t \leq 4 \\ 0, & t > 4 \end{cases}$$

o bien:

$$y(t) = 7 - 3(t+1)u(t+1) + 2tu(t) + (t-4)u(t-4)$$

(h) $x(t) = \delta(t) - 2\delta(t-1) + \delta(t-2)$, y $h(t)$ como se muestra en la figura.

$$y(t) = \begin{cases} 0, & t < 0 \\ 1, & 0 < t < 1 \\ 0, & 1 < t < 2 \\ -3, & 2 < t < 3 \\ 2, & 3 < t < 4 \\ 0, & t > 4 \end{cases}$$

(i) $x(t)$ y $h(t)$ como se muestran en la figura.

$y(t) = at + b$

(j) $x(t)$ y $h(t)$ como se muestran en la figura.

$$y(t) = \frac{dx(t)}{dt} = \begin{cases} 0, & t < -1 \\ 1, & -1 < t < 0 \\ -1, & 0 < t < 1 \\ 0, & t > 1 \end{cases}$$

(k) $x(t)$ y $h(t)$ como se muestran en la figura.

y(t) es periódico con periodo $T_0 = 2$, por serlo x(t), siendo en un periodo:

$$y(t) = \begin{cases} -t^2 + t + \frac{1}{4}, & -\frac{1}{2} < t \leq \frac{1}{2} \\ t^2 - 3t + \frac{7}{4}, & \frac{1}{2} < t \leq \frac{3}{2} \end{cases}$$

(l) $x(t)$ como se muestra en la figura, y $h(t) = e^{-t}\left[u(t-1) - u(t-2)\right]$.

$$y(t) = \begin{cases} 0, & t < 1 \\ (t-2)e^{-1} + e^{-t}, & 1 \leq t < 2 \\ (3-t)e^{-2} + e^{-1} - e^{(1-t)}, & 2 \leq t < 3 \\ e^{(2-t)} - e^{-2}, & 3 \leq t < 4 \\ 0, & t \geq 4 \end{cases}$$

o bien:

$$\begin{aligned} y(t) = \;& \left[(t-2)e^{-1} + e^{-t}\right] u(t-1) + \left[(3-t)(e^{-1} + e^{-2}) - (1+e)e^{-t}\right] u(t-2) + \\ & \left[-e^{-1} + (t-4)e^{-2} + (e+e^2)e^{-t}\right] u(t-3) + \left[e^{-2} - e^{(2-t)}\right] u(t-4) \end{aligned}$$

(m) $x(t)$ y $h(t)$ como se muestran en la figura.

$$y(t) = \begin{cases} 0, & t < -1 \\ \frac{1}{2}t^2 - \frac{1}{2}, & -1 \leq t < 0 \\ -\frac{1}{6}t^3 + t^2 - \frac{1}{2}, & 0 \leq t < 1 \\ -\frac{1}{2}t^2 + \frac{5}{2}t - \frac{5}{3}, & 1 \leq t < 2 \\ \frac{1}{6}t^3 - \frac{3}{2}t^2 + \frac{7}{2}t - 1, & 2 \leq t < 3 \\ \frac{1}{2}t^2 - 4t + 8, & 3 \leq t < 4 \\ 0, & t \geq 4 \end{cases}$$

o bien:

$$\begin{aligned} y(t) = \;& \left[\tfrac{1}{2}t^2 - \tfrac{1}{2}\right] u(t+1) + \left[-\tfrac{1}{6}t^3 + \tfrac{1}{2}t^2\right] u(t) + \left[\tfrac{1}{6}t^3 - \tfrac{3}{2}t^2 + \tfrac{5}{2}t - \tfrac{7}{6}\right] u(t-1) + \\ & \left[\tfrac{1}{6}t^3 - t^2 + t + \tfrac{2}{3}\right] u(t-2) + \left[-\tfrac{1}{6}t^3 + 2t^2 - \tfrac{15}{2}t + 9\right] u(t-3) + \\ & \left[-\tfrac{1}{2}t^2 + 4t - 8\right] u(t-4) \end{aligned}$$

(n) $x(t) = e^{-2t}[u(t-1) - u(t-4)]$,

$h(t) = e^{2t}[u(1-t) - u(-1-t)]$.

$$y(t) = \begin{cases} 0, & t < 0 \\ \frac{1}{4}\left(e^{(2t-4)} - e^{-(2t+4)}\right), & 0 \leq t < 2 \\ \frac{1}{4}\left(e^{(-2t+4)} - e^{-(2t+4)}\right), & 2 \leq t < 3 \\ \frac{1}{4}\left(e^{(-2t+4)} - e^{(2t-16)}\right), & 3 \leq t < 5 \\ 0, & t \geq 5. \end{cases}$$

(o) $h(t)$ como en la figura, y $x(t) = \sin(\pi t)[u(t+1) - u(t-1)]$.

$y(t) = 0$.

4. (a) Por ser un sistema LPI razonamos a partir de su respuesta al impulso $h(t)$:

Causalidad: El sistema es no causal, ya que no se cumple que $h(t) = 0$ para $t > 0$ (causal) o para $t \leq 0$ (anticausal).

Memoria: se trata de un sistema con memoria, ya que $h(t) \neq K\delta(t)$.

Estabilidad: Para que un sistema LTI sea estable, su repuesta al impulso ha de ser absolutamente integrable:

$$\int_{-\infty}^{\infty} |h(t)| dt = \int_{-\infty}^{\infty} |e^{j2\pi t}| dt = \int_{-\infty}^{\infty} 1 dt = \infty.$$

Por lo tanto, el sistema no es estable.

(b) Lo resolvemos a partir de la integral de convolución:

$$\begin{aligned} y(t) &= x(t) * h(t) \\ &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\ &= \int_{-1}^{1} 2e^{-3\tau}e^{j2\pi(t-\tau)}d\tau + \int_{1}^{\infty} e^{-3\tau}e^{j2\pi(t-\tau)}d\tau \\ &\vdots \\ &= \frac{1}{3+j2\pi}\left(2e^{3}-e^{-3}\right)\, e^{j2\pi t}. \end{aligned}$$

5. Definiendo:
$a(t) = \begin{cases} 1, & 0 \le t < 1/2 \\ 0, & \text{resto,} \end{cases}$
y
$b(t) = a(t) * a(t) = \begin{cases} t, & 0 \le t < 1/2 \\ 1-t, & 1/2 \le t \le 1 \\ 0, & \text{resto.} \end{cases}$

   (a) $z_0(t) = b(t+4) + b(t+3) + b(t+2) - b(t) - b(t-1) - b(t-2)$.

   (b) $z_1(t) = z_0(t+1) = b(t+5) + b(t+4) + b(t+3) - b(t+1) - b(t) - b(t-1)$.

   (c) $z_2(t) = -b(t+7/2) - b(t+5/2) - b(t+3/2) + b(t-1/2) + b(t-3/2) + b(t-5/2)$.

   (d) $z_3(t) = \frac{dz_0(t)}{dt} = a(t+4) - a(t+7/2) + a(t+3) - a(t+5/2) + a(t+2) - a(t+3/2) - a(t) + a(t-1/2) - a(t-1) + a(t-3/2) - a(t-2) + a(t-5/2)$.

6. (a) $h(t) = e^{-(t-2)}u(t-2)$

   (b) $y(t) = \begin{cases} 0, & t < 1 \\ 1-e^{-(t-1)}, & 1 \le t < 4 \\ e^{-(t-4)} - e^{-(t-1)} = (1-e^{-3})e^{-(t-4)}, & t \ge 4 \end{cases}$
   o bien:
   $y(t) = \left[1-e^{-(t-1)}\right]u(t-1) + \left[e^{-(t-4)}-1\right]u(t-4)$

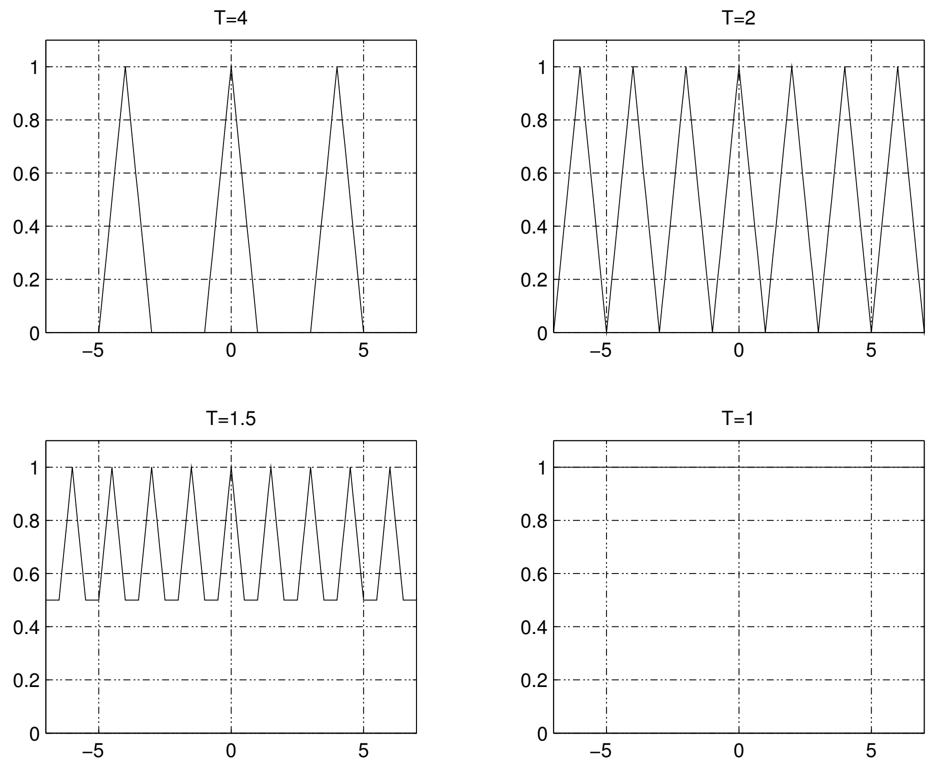
   (c) $h_2(t) = e^{-(t-2)}u(t-2) - e^{-(t-3)}u(t-3)$
   $$\begin{aligned} y_2(t) = y(t) - y(t-1) = \; & \left[1-e^{-(t-1)}\right]u(t-1) - \left[1-e^{-(t-2)}\right]u(t-2) + \\ & \left[e^{-(t-4)}-1\right]u(t-4) - \left[e^{-(t-5)}-1\right]u(t-5) \end{aligned}$$

7. Si se convoluciona una señal con un tren de impulsos, a la salida tenemos un tren de versiones desplazadas de la señal.

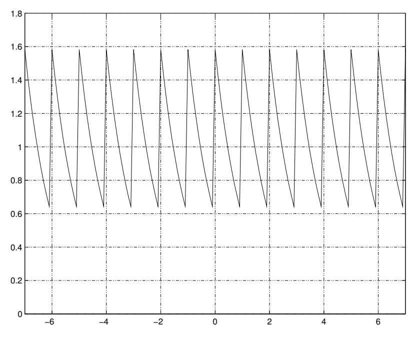
   (a) $y(t) = \sum_{k=-\infty}^{\infty} h(t-kT)$

   (b) $y(t) = \sum_{k=-\infty}^{\infty} e^{-(t-k)}u(t-k)$
   $y(t)$ es una señal periódica de periodo $T_0 = 1$, siendo en el intervalo $0 < t < 1$:
   $y(t) = \frac{e^{(1-t)}}{e-1}, \quad 0 < t < 1.$

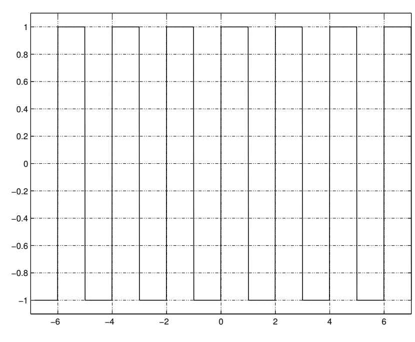
   (c)
   $$\begin{aligned} y(t) = \; & \sum_{k=-\infty}^{\infty} (-1)^k h(t-k) = \\ & \sum_{k=-\infty}^{\infty} (-1)^k \left[u(t-k) - u(t-k-1)\right] = \\ & \cdots + 2u(t+2) - 2u(t+1) + 2u(t) - 2u(t-1) + 2u(t-2) + \ldots \end{aligned}$$



(a)



(b)



(c)

Figure 1: Resultados gráficos del problema 6.

8. Calcule y dibuje $y[n] = x[n] * h[n]$.

$$y[n] = \begin{cases} 0, & n < 7 \\ n - 6, & 7 \leq n \leq 11 \\ 6, & 12 \leq n \leq 18 \\ 24 - n, & 19 \leq n \leq 23 \\ 0, n \geq 24 \end{cases}$$

9. (a) $y(t) = \frac{1}{3}\left[1 - e^{-3(t-3)}\right]u(t-3) + \frac{1}{3}\left[e^{-3(t-5)} - 1\right]u(t-5)$

o bien:

$$y(t)=\begin{cases} 0, & t<3 \\ \frac{1}{3}\left[1-e^{-3(t-3)}\right], & 3\leq t<5 \\ \frac{1}{3}e^{-3(t-5)}\left[1-e^{-6}\right], & t\geq 5 \end{cases}$$

(b) $g(t)=e^{-3(t-3)}u(t-3)-e^{-3(t-5)}u(t-5)$

o bien:

$$g(t)=\begin{cases} 0, & t<3 \\ e^{-3(t-3)} & 3\leq t<5 \\ e^{-3(t-5)}\left[e^{-6}-1\right], & t\geq 5 \end{cases}$$

(c) $g(t)=\frac{dy(t)}{dt}$

10. (a) Estable.

    (b) Estable.

11. $y[n]=\left(\frac{1}{4}\right)^{(n-1)}u[n-1]$

12. (a) $y(t)=\frac{\sin(3\pi t)}{2(1+9\pi^2)}-3\pi\frac{\cos(3\pi t)}{2(1+9\pi^2)}-\frac{\sin(\pi t)}{2(1+\pi^2)}+\pi\frac{\cos(\pi t)}{2(1+\pi^2)}$.

    (b) Con memoria, no causal, no invertible y no estable.

13. (a) Con memoria, causal y no estable.

    (b) (...).

14. Sitema no estable.

15. Falso.