Sistemas Lineales

Tema 7. Filtrado. Soluciones Numéricas de la Hoja de Problemas

1. (a) $H_{\mathrm{LP}}(\omega) = \begin{cases} 1, & |\omega| < 2\pi 25\ 10^3 \\ 0, & |\omega| > 2\pi 25\ 10^3 \end{cases}$

   $h_{\mathrm{LP}}(t) = \frac{\mathrm{sen}(2\pi 25\ 10^3 t)}{\pi t} = 50\ 10^3 \,\mathrm{sinc}(50\ 10^3 t)$

   (b) $H_{\mathrm{LP}}(\Omega) = \begin{cases} 1, & 0 < |\Omega| < \pi/6 \\ 0, & \pi/6 < |\Omega| < \pi \end{cases}$, periódico de periodo 2π.

   $h_{\mathrm{LP}}[n] = \frac{\mathrm{sen}(\pi n/6)}{\pi n} = \frac{1}{6}\,\mathrm{sinc}\left(\frac{n}{6}\right)$

   (c) $H_{\mathrm{HP}}(\omega) = \begin{cases} 0, & |\omega| < 2\pi 50\ 10^6 \\ 1, & |\omega| > 2\pi 50\ 10^6 \end{cases}$

   $h_{\mathrm{HP}}(t) = \delta(t) - \frac{\mathrm{sen}(2\pi 50\ 10^6 t)}{\pi t} = \delta(t) - 10^8\,\mathrm{sinc}(10^8 t)$

   (d) $H_{\mathrm{HP}}(\Omega) = \begin{cases} 0, & 0 < |\Omega| < 6\pi/7 \\ 1, & 6\pi/7 < |\Omega| < \pi \end{cases}$, periódico de periodo 2π.

   $h_{\mathrm{HP}}[n] = \delta[n] - \frac{\mathrm{sen}(6\pi n/7)}{\pi n} = \delta[n] - \frac{6}{7}\,\mathrm{sinc}\left(\frac{6}{7}n\right)$, o bien
   $h_{\mathrm{HP}}[n] = \frac{(-1)^n\,\mathrm{sen}(\pi n/7)}{\pi n} = \frac{(-1)^n}{7}\,\mathrm{sinc}\left(\frac{n}{7}\right)$

   (e) $H_{\mathrm{BP}}(\omega) = \begin{cases} 1, & 2\pi 5\ 10^6 < |\omega| < 2\pi 6.3\ 10^6 \\ 0, & resto \end{cases}$

   $h_{\mathrm{BP}}(t) = \frac{\mathrm{sen}(2\pi 6.3\ 10^6 t) - \mathrm{sen}(2\pi 5\ 10^6 t)}{\pi t} = 1.26\ 10^7\,\mathrm{sinc}(1.26\ 10^7 t) - 10^7\,\mathrm{sinc}(10^7 t)$,
   o bien
   $h_{\mathrm{BP}}(t) = \frac{2\cos(2\pi 5.65\ 10^6 t)\,\mathrm{sen}(2\pi 0.65\ 10^6 t)}{\pi t} = 2.6\ 10^6 \cos(2\pi 5.65\ 10^6 t)\,\mathrm{sinc}(1.3\ 10^6 t)$

   (f) $H_{\mathrm{BP}}(\Omega) = \begin{cases} 0, & 0 < |\Omega| < 0.75\pi \\ 1, & 0.75\pi < |\Omega| < 0.85\pi \\ 0, & 0.85\pi < |\Omega| < \pi \end{cases}$, periódico de periodo 2π.

   $h_{\mathrm{BP}}[n] = \frac{\mathrm{sen}(0.85\pi n) - \mathrm{sen}(0.75\pi n)}{\pi n} = 0.85\,\mathrm{sinc}(0.85n) - 0.75\,\mathrm{sinc}(0.75n)$, o bien
   $h_{\mathrm{BP}}[n] = \frac{2\cos(0.8\pi n)\,\mathrm{sen}(0.05\pi n)}{\pi n} = 0.1\cos(0.8\pi n)\,\mathrm{sinc}(0.05n)$

2. $H(\omega) = \begin{cases} 1, & |\omega| < 2\pi 10^6 \\ 0, & |\omega| > 2\pi 10^6 \end{cases}$

   $h(t) = \frac{\mathrm{sen}(2\pi 10^6 t)}{\pi t} = 2\ 10^6\,\mathrm{sinc}(2\ 10^6 t)$

   (a) $y_1(t) = \cos(2\pi 1500 t)$
   (b) $y_2(t) = 0$
   (c) $y_3(t) = \frac{1}{2}\cos\left(2\pi(10^6 - 1500)t\right)$

3. Los resultados se muestran en la figura 1.

4. $H(\omega) = \begin{cases} e^{-j\omega}, & |\omega| < 2\pi 40 \\ 0, & |\omega| > 2\pi 40 \end{cases}$

   (a) $y(t) = \cos(2\pi 30(t-1)) = \cos(2\pi 30 t)$
   (b) $y(t) = \frac{1}{2\pi}\int_{-2\pi 40}^{2\pi 40} \frac{e^{j\omega(t-1)}}{1+j\omega}\mathrm{d}\omega$
   (c) $y(t) = 0$
   (d) $y(t) = \frac{\mathrm{sen}(2\pi 40(t-1))}{\pi(t-1)} = \frac{\mathrm{sen}(2\pi 40 t)}{\pi(t-1)} = 80\,\mathrm{sinc}\left(80(t-1)\right)$
   (e) $y(t) = e^{j2\pi 30(t-1)} = e^{j2\pi 30 t}$
   (f) $y(t) = 0$

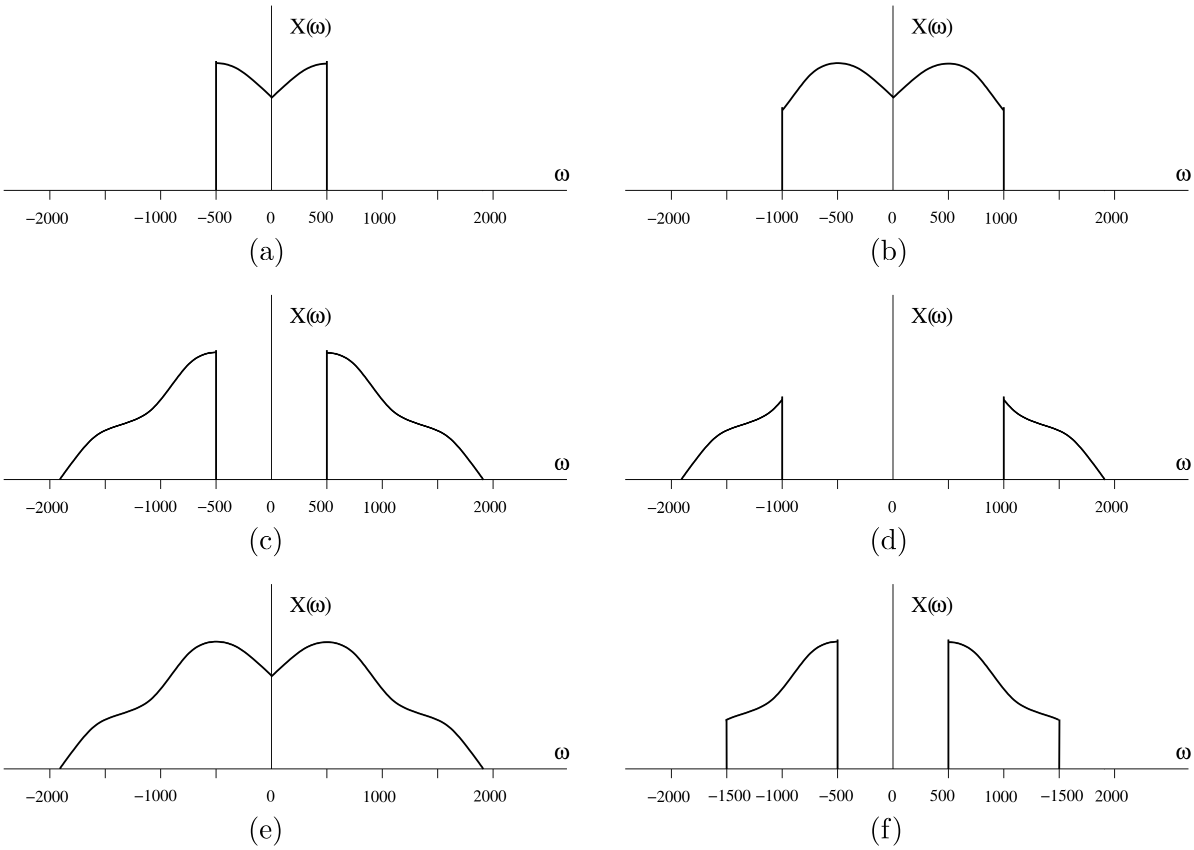


Figure 1: Resultados del problema 3.

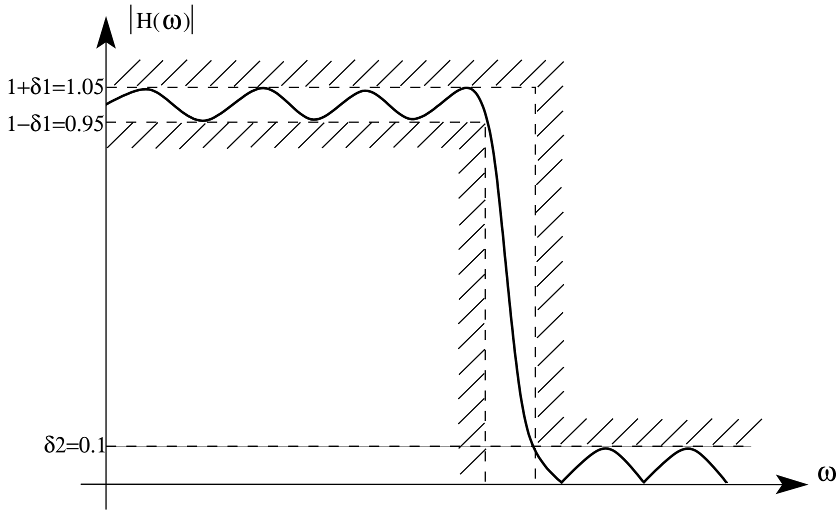


Figure 2: Resultado del problema 5.

5. El resultado se muestra en la figura 2

6. (a) $X_2(\omega)$ como se muestra en la figura 3(a).

   (b) $x(t)$ se puede recuperar a partir de $x_2(t)$ mediante un filtro paso banda ideal, como el que se muestra en la figura 3(b), con $4\pi \leq \omega_{c1} \leq 8\pi$, y $12\pi \leq \omega_{c1} \leq 16\pi$. El valor óptimo es $\omega_{c1} = 6\pi$ y $\omega_{c2} = 14\pi$. La ganancia del filtro, $G = 1$.

   (c) $\omega_1 \geq \frac{\omega_2}{2}$

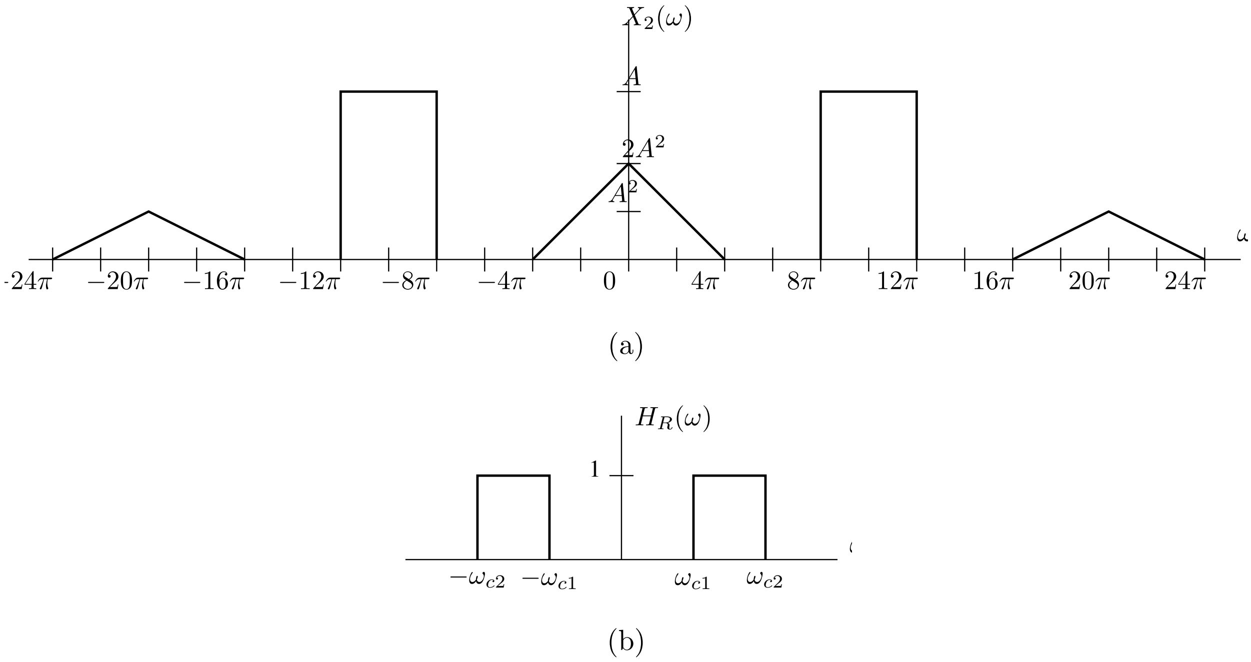


Figure 3: Soluciones al problema 6

7. (a) Los coeficientes a_k para todo k, valen:

$$a_k = \frac{-2}{\pi(4k^2-1)}.$$

   (b)

$$X(\omega) = -4 \sum_{k=-\infty}^{\infty} \frac{\delta(\omega - 2\pi k)}{4k^2-1}.$$

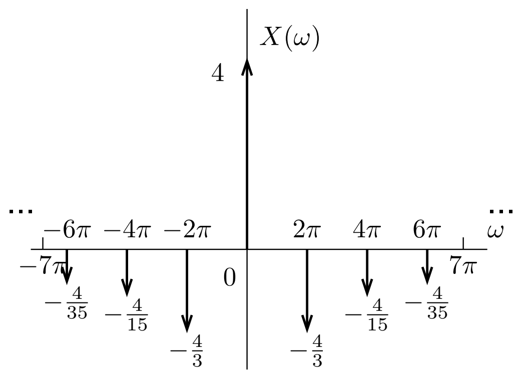


Figure 4: Resultado del problema 7b para $-7\pi \leq \omega \leq 7\pi$.

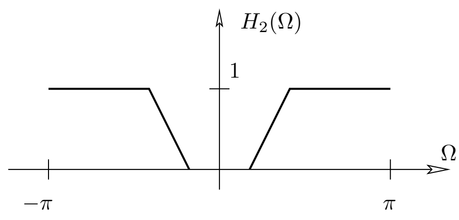
8. (a) a_k periódico de periodo $N = 4$. Dentro de un periodo $0 \leq k \leq 3$: $a_0 = \frac{1+\sqrt{2}}{4}$; $a_1 = -\frac{1}{4}$; $a_2 = \frac{1-\sqrt{2}}{4}$; $a_3 = -\frac{1}{4}$.

   (b) $X(\Omega)$ como se muestra a continuación:

$$
\begin{aligned}
X(\Omega) &= \tfrac{\pi}{2}\sum_{l=-\infty}^{\infty}\left[\left(1+\sqrt{2}\right)\delta(\Omega-2\pi l)-\delta\left(\Omega-\tfrac{\pi}{2}-2\pi l\right)+\right.\\
&\quad \left.+\left(1-\sqrt{2}\right)\delta(\Omega-\pi-2\pi l)-\delta\left(\Omega-\tfrac{3\pi}{2}-2\pi l\right)\right]=\\
&= \tfrac{\pi}{2}\left(1+\sqrt{2}\right)\delta_p(\Omega-2\pi l)-\tfrac{\pi}{2}\delta_p\left(\Omega-\tfrac{\pi}{2}-2\pi l\right)+\\
&\quad +\tfrac{\pi}{2}\left(1-\sqrt{2}\right)\delta_p(\Omega-\pi-2\pi l)-\tfrac{\pi}{2}\delta_p\left(\Omega-\tfrac{3\pi}{2}-2\pi l\right).
\end{aligned}
$$

(c) $y[n]=-\frac{1}{2}\cos\left(\frac{\pi}{2}n\right)$.

9. (Examen Ene. 2004, ejercicio 4)
$H_2(\Omega)=H_1(\Omega-\pi)$.
Es un filtro paso-alto tal y como está representado en la figura, en el intervalo $[-\pi,\pi]$.

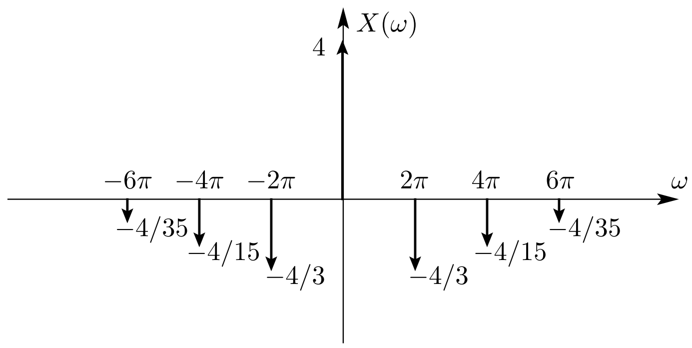


10. (Examen Sep. 2007, ejercicio 2)

(a)

$$X(\omega)=\sum_{k=-\infty}^{\infty}\frac{4}{1-4k^2}\delta(\omega-2\pi k).$$

La transformada de Fourier en el intervalo $|\omega|\leq 7\pi$ es:



(b) $z(t)=\frac{2}{\pi}-\frac{4}{15\pi}\cos(4\pi t)$.