Sistemas Lineales

Tema 6. Soluciones de la Hoja de Problemas

1. (a) $X(s) = \frac{1}{s+a}$, $\Re\{s\} > -a$
   (b) $X(s) = \frac{1}{s-a}$, $\Re\{s\} < a$
   (c) $X(s) = \frac{1}{s-a}$, $\Re\{s\} > a$
   (d) $X(s) = \frac{2a}{a^2-s^2}$, $-a < \Re\{s\} < a$
   (e) $X(s) = \frac{1}{s}$, $\Re\{s\} > 0$
   (f) $X(s) = e^{-st_0}$, para todo s
   (g) $X(s) = \sum_{k=0}^{\infty} a^k e^{-skT}$, para todo s
   (h) $X(s) = -\frac{d}{ds}\left(\frac{1}{s+a}\right)$, $\Re\{s\} > -a$
   (i) $X(s) = \frac{s\cos\phi - \omega_0 \sin\phi}{s^2+\omega_0^2}$, $\Re\{s\} > 0$

2. (a) $x(t) = e^{-t}u(t)$
   (b) $x(t) = -e^{-t}u(-t)$
   (c) $x(t) = \cos(2t)u(t)$
   (d) $x(t) = 2e^{-3t}u(t) - e^{-2t}u(t)$
   (e) $x(t) = -2e^{-3t}u(-t) + e^{-2t}u(-t)$
   (f) $x(t) = -tu(t) - e^{t}u(-t)$
   (g) $x(t) = -tu(t) + e^{-t}u(t)$
   (h) $x(t) = e^{-t}\cos(2t)u(t)$

3. (a) $X(s) = \frac{1}{s+1}$, $\Re\{s\} > -1$
   $H(s) = \frac{1}{s+2}$, $\Re\{s\} > -2$
   (b) $Y(s) = \frac{1}{(s+1)(s+2)}$, $\Re\{s\} > -1$
   (c) $y(t) = e^{-t}u(t) - e^{-2t}u(t)$

4. (a) $H(s) = \frac{1}{(s-2)(s+1)}$. Polos en $s = 2$ y $s = -1$. Cero en ∞.
   (b) i. $-1 < \Re\{s\} < 2$, $h(t) = \frac{1}{3}\left(-e^{2t}u(-t) - e^{-t}u(t)\right)$
   ii. $\Re\{s\} > 2$, $h(t) = \frac{1}{3}\left(e^{2t}u(t) - e^{-t}u(t)\right)$
   iii. $\Re\{s\} < -1$, $h(t) = \frac{1}{3}\left(-e^{2t}u(-t) + e^{-t}u(-t)\right)$

5. $H(s) = \frac{2}{s(s+4)}$, $\Re\{s\} > 0$ $(b = 1)$.

6. (a) $\frac{dy(t)}{dt} + 2y(t) = 2\frac{dx(t)}{dt} + 5x(t)$.
   (b) $h(t) = 2\delta(t) + e^{-2t}u(t)$
   (c) $y(t) = \left(-e^{-2t} + 3e^{-t}\right)u(t)$
   (d) No se puede calcular $y(t)$ porque no existe ROC para $Y(s)$, debido a que la intersección de la ROC de $H(s)$ con la ROC de $X(s)$ es $\emptyset$.

7. (a) $X(s) = -e^{-2s}\left(\frac{1+2s}{s^2}\right) + e^{s}\left(\frac{1-s}{s^2}\right)$, $\forall s$

   (b) $Y(s) = -4\pi^2 e^{-2s}\left(\frac{1+2s}{s(s+4\pi)}\right) + 4\pi^2 e^{s}\left(\frac{1-s}{s(s+4\pi)}\right)$, $\Re\{s\} > 0$

   $y(t) = \pi\left[1 - (4\pi+1)e^{-4\pi(t+1)}\right]u(t+1) - \pi\left[1 - (1-8\pi)e^{-4\pi(t-2)}\right]u(t-2)$

8. Soluciones en el libro.