Blind Estimation of Spatially Variant Noise in GRAPPA MRI

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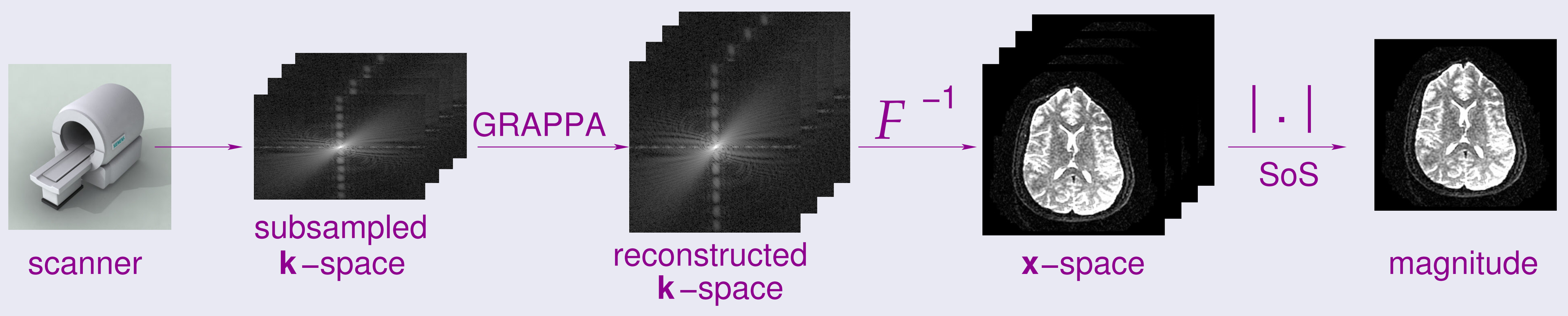
The reconstruction process in multiple coil MRI scanners makes the noise features in the final magnitude image become non-stationary, i.e. the variance of noise becomes position-dependent. Therefore, most noise estimators proposed in the literature cannot be used in multiple-coil acquisitions. This effect is augmented when parallel imaging methods, such as GRAPPA, are used to increase the acquisition rate. We propose a new technique that allows the estimation of the spatially variant maps of noise from the GRAPPA reconstructed signal when only one single image is available and no additional information is provided. Other estimators in the literature need extra information that is not always available, which has supposed an important limitation in the usage of noise models for GRAPPA. The proposed approach uses a homomorphic separation of the spatially variant noise in two terms: a stationary noise term and one low frequency signal that correspond to the x-dependent variance of noise. The non-stationary variance of noise is estimated by a low pass filtering. The noise term is obtained via prior wavelet decomposition. Results in real and synthetic experiments evidence the suitability of the simplification used and the good performance of the proposed methodology.

Simplified noise model in GRAPPA



GRAPPA reconstructs the full **k**-space from a sub–sampled **k**-space acquisition. The reconstructed lines are estimated through a linear combination of the existing samples. The composite magnitude signal (CMS) after sum-of-squares (SoS) can be approximated by a non-stationary nc-χ distribution with a (reduced) *effective number of coils* L_{eff} and an (increased) *effective variance of noise* σ^2_{eff} [Aja10]:

$$L_{\text{eff}}(\mathbf{x}) = \frac{|\mathbf{A}_{\mathscr{R}}|^2\,\text{tr}(\mathbf{C}_X) + (\text{tr}(\mathbf{C}_X))^2}{\mathbf{A}^*_{\mathscr{R}}\mathbf{C}_X\mathbf{A}_{\mathscr{R}} + ||\mathbf{C}_X||_F^2}; \quad (1)$$

$$\sigma^2_{\text{eff}}(\mathbf{x}) = \frac{\text{tr}(\mathbf{C}_X)}{L_{\text{eff}}}. \quad (2)$$

If high SNR is assumed, $A_l^{\mathscr{R}}(\mathbf{x}) >> \sigma_l^R(\mathbf{x})$, then the SoS can be approximated as:

$$M_L(\mathbf{x}) \approx A_T(\mathbf{x}) + N^{\mathscr{R}}(\mathbf{x}; 0, \sigma_T^2(\mathbf{x})), \quad (3)$$

where $A_T^2(\mathbf{x}) = \mathbf{A}^*_{\mathscr{R}}\mathbf{A}_{\mathscr{R}}$ and $N^{\mathscr{R}}(\mathbf{x}; 0, \sigma_T^2(\mathbf{x}))$ is a non-stationary Gaussian noise with zero mean and variance:

$$\sigma_T^2(\mathbf{x}) = |\Omega|^2 \frac{\mathbf{A}^*_{\mathscr{R}}\mathbf{C}_X\mathbf{A}_{\mathscr{R}}}{\mathbf{A}^*_{\mathscr{R}}\mathbf{A}_{\mathscr{R}}} = |\Omega|^2 \frac{\mathbf{A}^*_{\mathscr{S}}\mathbf{W}^*\mathbf{W}\Sigma\mathbf{W}^*\mathbf{W}\mathbf{A}_{\mathscr{S}}}{\mathbf{A}^*_{\mathscr{S}}\mathbf{W}^*\mathbf{W}\mathbf{A}_{\mathscr{S}}}. \quad (4)$$

Matrix $\mathbf{A}_{\mathscr{S}} = [A_1^{\mathscr{S}}, \cdots, A_L^{\mathscr{S}}]^T$ is the original sampled signal (without noise), and $\mathbf{A}_{\mathscr{R}} = \mathbf{W}\cdot\mathbf{A}_{\mathscr{S}}$. The variance of noise depends on the position, the GRAPPA reconstruction coefficients, the original covariance matrix and the noise-free signals.

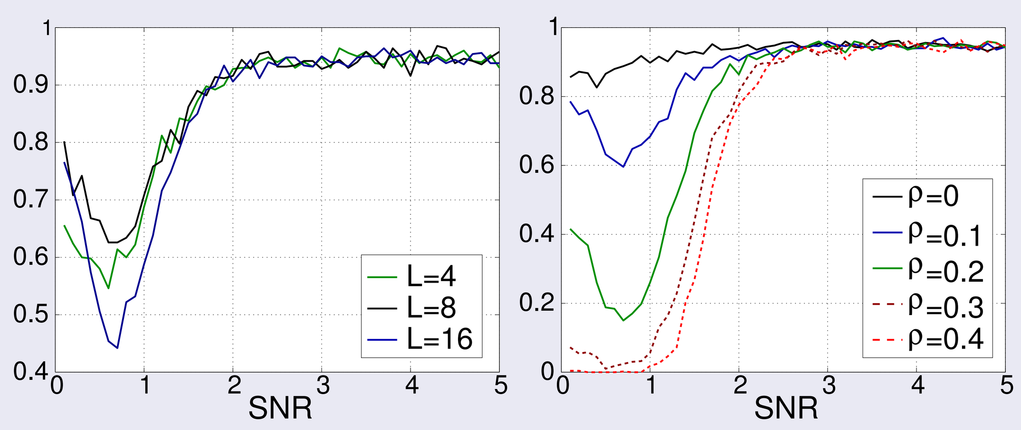


Figure: Ratio of experiments in which the Gaussian distribution hypothesis is accepted, using a Pearson goodness-of-fit test. Left: $\rho = 0.1$. Right: $L = 8$.

Homomorphic noise estimation

Homorphic estimation of non-stationary noise is originally proposed in [Aja15]. A non-stationary Gaussian noise $N(\mathbf{x}; 0, \sigma^2(\mathbf{x}))$ can be seen as

$$N(\mathbf{x}; 0, \sigma^2(\mathbf{x})) = \sigma(\mathbf{x}) \cdot N(\mathbf{x}; 0, 1)$$

$$\log|N(\mathbf{x}; 0, \sigma^2(\mathbf{x}))| = \underbrace{\log\sigma(\mathbf{x})}_{\text{low frequency}} + \underbrace{\log|N(\mathbf{x})|}_{\text{higher frequency}}.$$

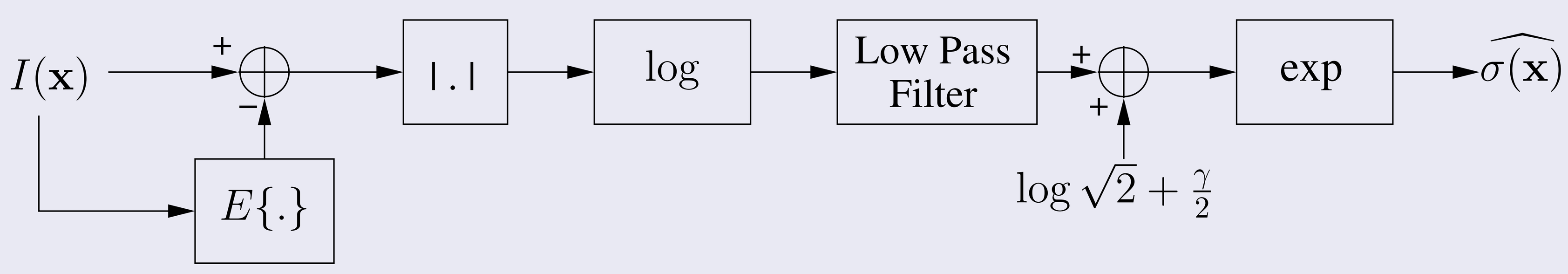
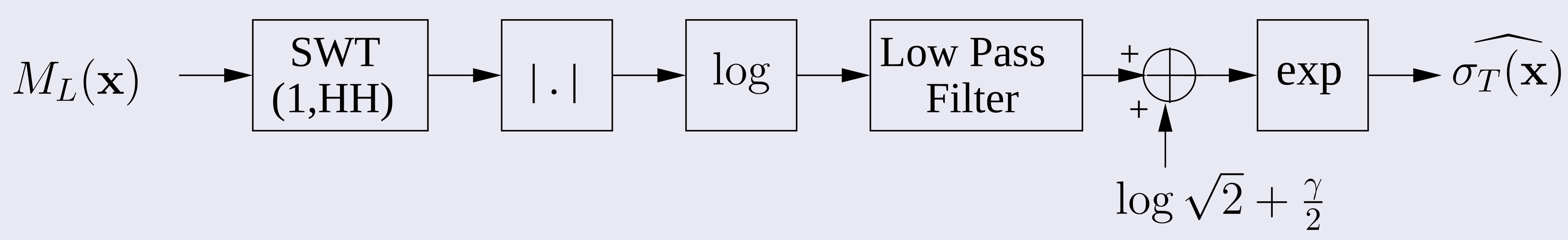


Figure: Pipeline of $\sigma(\mathbf{x})$ estimation assuming Gaussian noise. $I(\mathbf{x})$ is the original image, $E\{.\}$ is the local expected value of the signal and $|.|$ is the absolute value.

Blind Noise estimation for GRAPPA



Simplified non-stationary Gaussian model:

$$M_L(\mathbf{x}) = A_T(\mathbf{x}) + \sigma_T(\mathbf{x}) \cdot N(\mathbf{x}; 0, 1)$$

To remove the contribution of the signal $A_T(\mathbf{x})$, we will use the stationary wavelet transform (SWT):

$$I^{(1,\text{HH})}(\mathbf{x}) \approx \sigma_T(\mathbf{x}) \cdot N(\mathbf{x}; 0, 1).$$

with $I^{(1,\text{HH})}(\mathbf{x})$ the high-high subband coefficients of the SWT of the image $M_L(\mathbf{x})$ at the scale $s = 1$. After homomorphic processing:

$$\widehat{\sigma_T(\mathbf{x})} = \sqrt{2}e^{\text{LPF}\{\log|I^{(1,\text{HH})}(\mathbf{x})|\} + \gamma/2}. \quad (5)$$

Results

Synthetic experiment: from Brainweb, 8-coil systems where simulated using a realistic sensitivity map, with the image in range [0-255]. Each coil is corrupted with Gaussian noise with variance $\sigma^2 = 100$ and correlation between coils $\rho = 0.1$. The **k**–space is uniformly subsampled by a factor of 2, keeping 32 ACS lines and reconstructed using GRAPPA and SoS. 100 repetitions of the experiment are considered. The Daubechies (db7) wavelet is used. A Pearson goodness-of-fit test is carried out for the Gaussian assumption: in the 91.85% of the area of interest the null hypothesis is accepted.

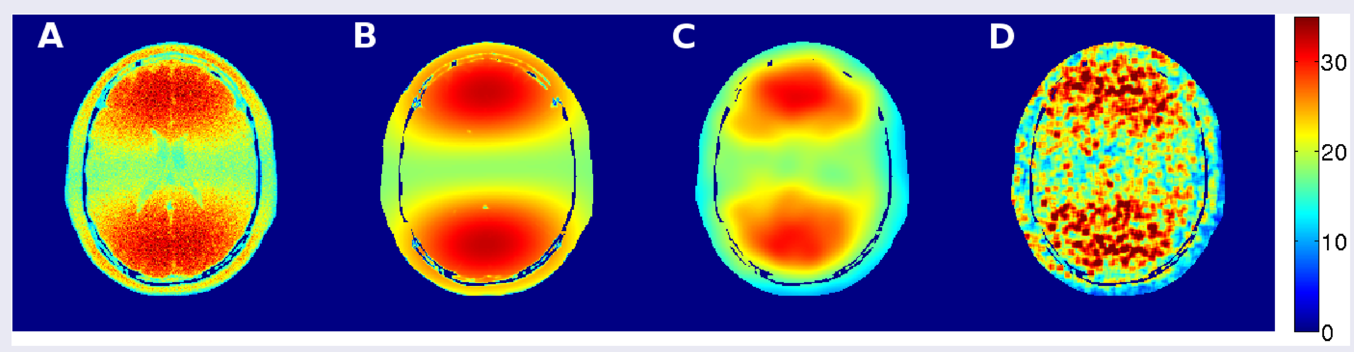


Figure: Estimation of $\sigma_T(\mathbf{x})$ (synthetic). a) Std. of 100 samples; b) Theoretical value; c) Homomorphic (1 sample); d) Goossens (1 sample).

Real data: 100 repetitions of phantom, 8-channel GE Signa 1.5T EXCITE 12m4 scanner with FGRE Pulse Sequence. Matrix size 128×128, TR/TE 8.6/3.38 ms, FOV 21×21cm, slice thickness 1mm. All the 100 samples are 2× subsampled, then GRAPPA reconstructed using 16 ACS lines (same coefficients for the 100 slices). For the 94.02% of points inside the signal area the null hypothesis (Gaussianity) is accepted, 91.1% if the whole image is considered.

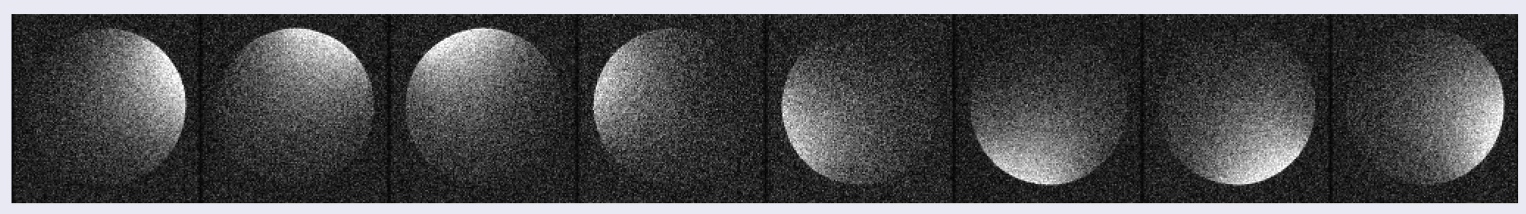


Figure: Slice of an 8-coil acquisition of a doped ball phantom.

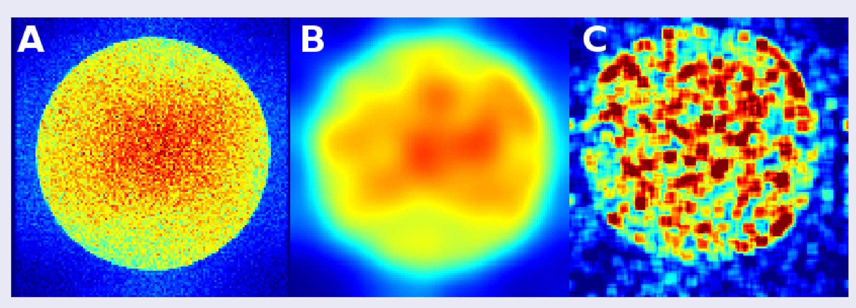


Figure: Noise estimation over the phantom. a) Standard deviation of 100 samples; b) Homomorphic estimation of one sample; c) Estimation using Goossens' method.

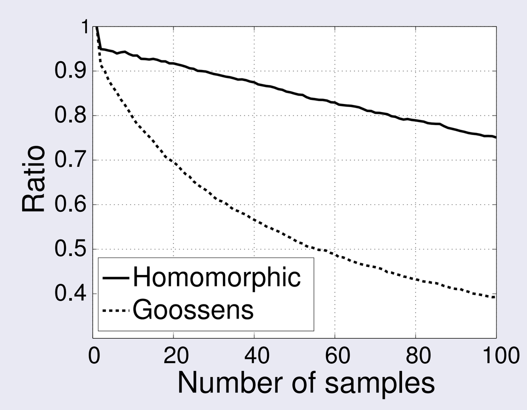


Figure: **Capability to estimate the variance of noise with one single image:** Ratio of pixels whose variance estimates calculated with one image lay within the 95% confidence interval of the non-stationary variance of noise for an increasing number of acquisitions. The proposed method provides with just one acquisition similar results to the ones obtained with a higher number of acquisitions.

Acknowledgements

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