PROBLEMA 1:

1)
$$g(X) \simeq g(0) + X g'(0) + \frac{X^2}{2} g''(0) + \frac{X^3}{3!} g'''(0) + \frac{X^4}{4!} g^{iv}(0) + \frac{X^5}{5!} g^{v}(0) + O(6)$$

$g(X) = 1 - \cos X \Rightarrow g(0) = 1 - \cos 0 = 1 - 1 = 0$

$g'(X) = \operatorname{sen} X \Rightarrow g'(0) = \operatorname{sen} 0 = 0$

$g''(X) = \cos X \Rightarrow g''(0) = \cos 0 = 1$

$g'''(X) = -\operatorname{sen} X \Rightarrow g'''(0) = -\operatorname{sen} 0 = 0$

$g^{iv}(X) = -\cos X \Rightarrow g^{iv}(0) = -\cos 0 = -1$

$g^{v}(X) = \operatorname{sen} X \Rightarrow g^{v}(0) = \operatorname{sen} 0 = 0$

$$\boxed{g(X) \simeq \frac{X^2}{2} - \frac{X^4}{24} + O(6)}$$

2) $m(t)$ y $c(t) = A_c \cos(2\pi f_c t)$

$x(t) = m(t) + c(t)$

$$y(t) = G(x(t)) \simeq \frac{x^2(t)}{2} - \frac{x^4(t)}{24} = \frac{[m(t) + c(t)]^2}{2} - \frac{[m(t) + c(t)]^4}{24}$$

$$y(t) \simeq \frac{m^2(t)}{2} + m(t)\,c(t) + \frac{c^2(t)}{2} - \frac{m^4(t)}{24} - \frac{m^3(t)\,c(t)}{6} - \frac{m^2(t)\,c^2(t)}{4}$$

$$- \frac{m(t)\,c^3(t)}{6} - \frac{c^4(t)}{24}$$

$$c(t) = A_c \cos(2\pi f_c t)$$

$$c^2(t) = A_c^2\left[\tfrac{1}{2} + \tfrac{1}{2}\cos[4\pi f_c t]\right] = \frac{A_c^2}{2} + \frac{A_c^2}{2}\cos(4\pi f_c t)$$

$$c^3(t) = \left[\frac{A_c^3}{2} + \frac{A_c^3}{2}\cos(4\pi f_c t)\right]\cos(2\pi f_c t) = \frac{A_c^3}{2}\cos(2\pi f_c t) + \frac{A_c^3}{2}\cos(2\pi f_c t)\cos(4\pi f_c t)$$

$$= \frac{A_c^3}{2}\cos(2\pi f_c t) + \frac{A_c^3}{4}\cos(2\pi f_c t) + \frac{A_c^3}{4}\cos(6\pi f_c t) = \frac{3A_c^3}{4}\cos(2\pi f_c t) + \frac{A_c^3}{4}\cos(6\pi f_c t)$$

$$c^4(t) = \left[\frac{3A_c^4}{4}\cos(2\pi f_c t) + \frac{A_c^4}{4}\cos(6\pi f_c t)\right]\cos(2\pi f_c t) =$$

$$= \frac{3A_c^4}{8} + \frac{3A_c^4}{8}\cos(4\pi f_c t) + \frac{A_c^4}{8}\cos(4\pi f_c t) + \frac{A_c^4}{8}\cos(8\pi f_c t)$$

$$= \frac{3A_c^4}{8} + \frac{A_c^4}{2}\cos(4\pi f_c t) + \frac{A_c^4}{8}\cos(8\pi f_c t)$$

$$y(t) \simeq \left[\frac{m^2(t)}{2} - \frac{m^4(t)}{24} + \frac{A_c^2}{4} - \frac{A_c^2\,m^2(t)}{8} - \frac{A_c^4}{64}\right] +$$

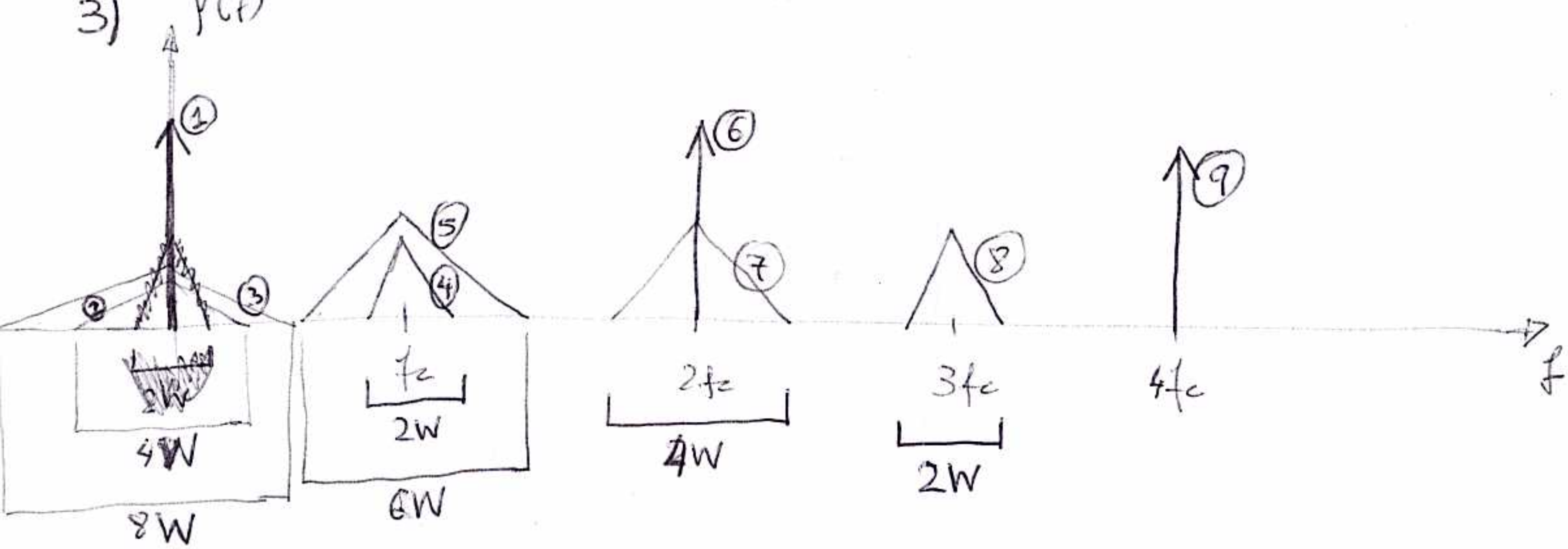
$$+ \left[A_c\, m(t) - \frac{A_c\, m^3(t)}{6} - \frac{A_c^3\, m(t)}{8}\right]\cos(2\pi f_c t) +$$

$$+ \left[\frac{A_c^2}{4} - \frac{A_c^2\, m^2(t)}{8} - \frac{A_c^4}{48}\right]\cos(4\pi f_c t) +$$

$$- \frac{A_c^3\, m(t)}{24}\cos(6\pi f_c t) - \frac{A_c^4}{192}\cos(8\pi f_c t)$$

PROBLEMA 2 (cont)

3)



FRECUENCIA 0

(1) Componente DC $\frac{A_c^2}{4} - \frac{A_c^4}{64}$

(2) Señal: $\frac{A_c^2 m^2(t)}{8} + \frac{m^2(t)}{2}$ Ancho de banda 2W

(3) Señal: $-\frac{m^4(t)}{24}$ Ancho de banda 4W

FRECUENCIA f_c

(4) Señal DSB: $\left[A_c m(t) - \frac{A_c^3 m(t)}{8}\right]\cos(2\pi f_c t)$ Ancho banda 2W

(5) Señal $-\frac{A_c m^3(t)}{6}\cos(2\pi f_c t)$ Ancho de banda 6W

FRECUENCIA $2f_c$

(6) Portadora a $2f_c$: $\left[\frac{A_c^2}{4} - \frac{A_c^4}{48}\right]\cos(4\pi f_c t)$

(7) Señal $-\frac{A_c^2 m^2(t)}{8}\cos(4\pi f_c t)$ Ancho banda 4W

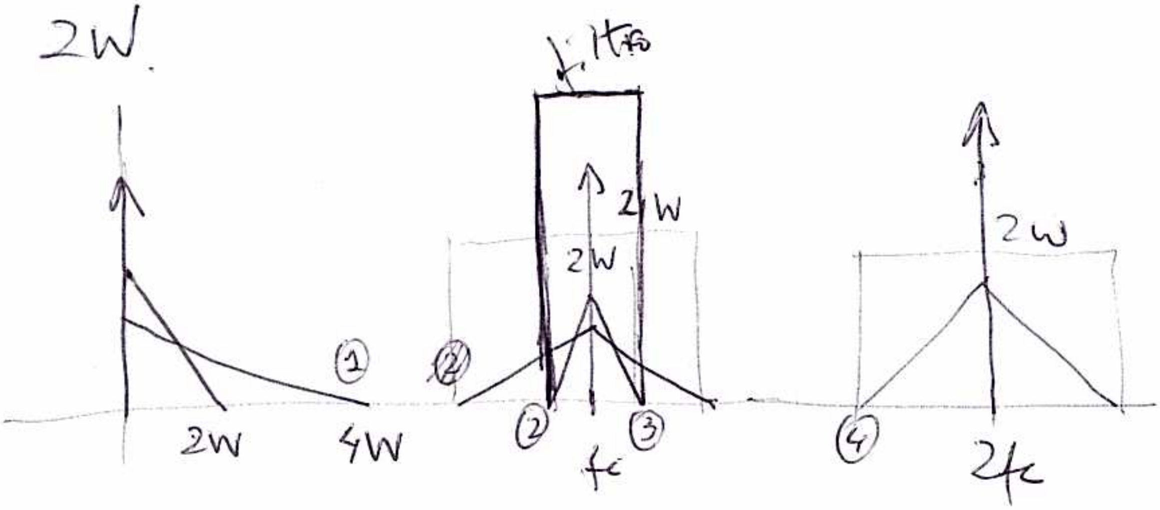
FRECUENCIA $3f_c$

(8) Señal DSB a $3f_c$ $-\frac{A_c^3 m(t)}{24}\cos(6\pi f_c t)$ Ancho banda 2W

FRECUENCIA $4f_c$

(9) Portadora a $4f_c$ $-\frac{A_c^4}{\ldots}\cos(8\pi f_c t)$

4) Filtro paso banda a f_c y ancho de banda 2W.



5) Mirando figura anterior punto 2 > punto 1 y punto 4 > punto 3. Lo que sea más restrictivo:

1 - 2 $\quad f_c - W > 4W \Rightarrow \boxed{f_c > 5W}$ MAS RESTRICTIVA

3 - 4 $\quad 2f_c - 2W > f_c + W \Rightarrow \boxed{f_c > 3W}$ ~~más restrictiva~~

$f > 5W$

6) Dentro del ancho de banda del filtro cae:

$$\left[A_c\, m(t) - \frac{A_c^3\, m(t)}{8}\right] \cos(2\pi f_c t)$$

que es la señal DSB deseada que pasa sin modificarse por el filtro.

PROBLEMA 1 (cont)

pero también pasa la señal

$$-\frac{A_c\, m^3(t)}{6}\cos(2\pi f_c t)$$

a la salida:

$$\left[-\frac{A_c\, m^3(t)}{6}\cos(2\pi f_c t)\right] * h(t)$$

donde $h(t)$ es la respuesta al impulso del filtro. No es posible eliminarlo. La relación amplitudes:

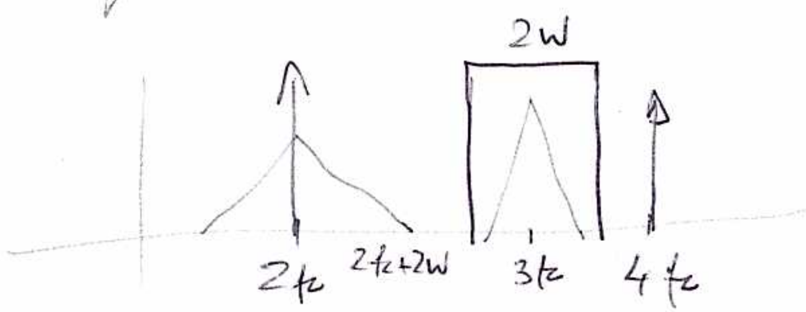
$$\text{DISTORSION} \cong \frac{A_c/6\; m^3(t)}{\left|A_c\, m(t) - \frac{A_c^3\, m(t)}{8}\right|} = \frac{m^2(t)}{\left|6 - \frac{3}{4}A_c^2\right|}$$

para reducir la distorsión: aumentar A_c

disminuir $m(t)$.

7) Si, se puede elegir la señal 8 a $3f_c$ $\Rightarrow$

filtro paso banda $3f_c$ y ancho de banda $2W$.



$$4f_c > 3f_c + W \Rightarrow f_c > W$$

$$3f_c - W > 2f_c + 2W \Rightarrow \boxed{f_c > 3W}$$ más restrictiva.

$$\boxed{s(t) = -\frac{A_c^3\, m(t)}{24} \cos(6\pi f_c t)}$$

PROBLEMA 2:

(a) $s_1(t) = 4\cos(2\pi f_c t)$

$s_2(t) = 10\sin(2\pi f_c t)$ independiente de la anterior

ya que, aunque tienen la misma frecuencia, están en cuadratura y por tanto son ortogonales.

$$s_3(t) = 9\cos\left(2\pi f_c t + \frac{\pi}{4}\right) = 9\cos(2\pi f_c t)\cos\left(\frac{\pi}{4}\right) -$$

$$9\,\mathrm{sen}(2\pi f_c t)\,\mathrm{sen}\left(\frac{\pi}{4}\right) = \frac{9\sqrt{2}}{2}\cos(2\pi f_c t) - \frac{9\sqrt{2}}{2}\mathrm{sen}(2\pi f_c t)$$

se puede poner como combinación lineal de $s_1(t)$ y $s_2(t)$.

$s_4(t) = 5\,\mathrm{sen}(4\pi f_c t)$

independiente de las anteriores ya que tiene frecuencia $2f_c$.

$$s_5(t) = 3\,\mathrm{sen}(2\pi f_c t)\cos(2\pi f_c t) = \frac{3}{2}\mathrm{sen}(4\pi f_c t)$$

es proporcional a $s_4(t)$

$$s_6(t) = 2\,\mathrm{se}(4\pi f_c t + \pi) = 2\,\mathrm{sen}(4\pi f_c t)\cos(\pi) + 2\cos(4\pi f_c)\mathrm{sen}(\pi)$$

$$= -2\,\mathrm{se}(4\pi f_c t)$$

es proporcional a $s_4(t)$

$$s_7(t) = 2\cos(2\pi f_c t)\left[1 + 4\,\mathrm{se}(2\pi f_c t)\right] =$$

$$= 2\cos(2\pi f_c t) + 8\,\mathrm{sen}(2\pi f_c t)\cos(2\pi f_c t) = 2\cos(2\pi f_c t) + 4\,\mathrm{se}(4\pi f_c t)$$

es combinación lineal de $s_1(t)$ y $s_4(t)$.

$N=3$ ~~dep~~ dado por $s_1(t)$, $s_2(t)$ y $s_4(t)$ que además son ortogonales.

(b) Como el conjunto $s_1(t)$, $s_2(t)$ y $s_4(t)$ además de ser linealmente independiente son ortogonales, solo tenemos que garantizar que tengan energía unidad

$$E_1=\int_0^T s_1^2(t)\,dt \Rightarrow \phi_1(t)=\frac{s_1(t)}{\sqrt{E_1}}$$

$$E_2=\int_0^T s_2^2(t)\,dt \Rightarrow \phi_2(t)=\frac{s_2(t)}{\sqrt{E_2}}$$

$$E_4=\int_0^T s_4^2(t)\,dt \Rightarrow \phi_3(t)=\frac{s_4(t)}{\sqrt{E_4}}$$

$$E_1=\int_0^T (4\cos(2\pi f_c t))^2 dt = 16\int_0^T \cos^2(2\pi f_c t)\,dt = 16\int_0^T\left[\frac{1}{2}+\frac{1}{2}\cos(4\pi f_c t)\right]$$

$$=16\int_0^T\frac{dt}{2}+16\int_0^T\frac{\cos(4\pi f_c t)}{2}\,dt = 8T+8\cancel{\left[\frac{\operatorname{sen}(4\pi f_c t)}{4\pi f_c}\right]_0^T}^{\,0}$$

$$E_2=10^2\cdot\frac{T}{2}=50T \qquad E_4=\frac{25T}{2}$$

$$\phi_1(t)=\frac{4\cos(2\pi f_c t)}{\sqrt{8T}}=\sqrt{\frac{2}{T}}\cos(2\pi f_c t)$$

PROBLEMA 2 (cont)

$$\phi_2(t) = \frac{10 \text{ sen}(2\pi f_c t)}{\sqrt{50T}} = \frac{10 \text{ se}(2\pi f_c t)}{10\sqrt{T/2}} = \sqrt{\frac{2}{T}} \text{ se}(2\pi f_c t)$$

$$\phi_3(t) = \frac{5 \text{ se}(4\pi f_c t)}{\sqrt{\frac{25T}{2}}} = \sqrt{\frac{2}{T}} \text{ sen}(4\pi f_c t)$$

(c) $s_1(t) = 4\cos(2\pi f_c t)$ $\quad \bar{s}_1 = [\cancel{4}\, 2\sqrt{2T},\ 0,\ 0]$

$s_2(t) = 10 \text{ sen}(2\pi f_c t)$ $\quad \bar{s}_2 = [\cancel{5}\ 0,\ 5\sqrt{2T},\ 0]$

$$s_3(t) = \frac{9\sqrt{2}}{2}\cos(2\pi f_c t) - \frac{9\sqrt{2}}{2} \text{ se}(2\pi f_c t)$$

$$\bar{s}_3 = \left[\frac{9}{2}\sqrt{T},\ -\frac{9}{2}\sqrt{T},\ 0\right] = \left[\frac{9\sqrt{2}}{4}\sqrt{2T}\ -\frac{9\sqrt{2}}{4}\sqrt{2T},\ 0\right]$$

$s_4(t) = 5 \text{ se}(4\pi f_c t)$, $\quad \bar{s}_4 = \left[0,\ 0,\ 5\sqrt{\frac{T}{2}}\right] = \left[0,\ 0,\ \frac{5\sqrt{2T}}{2}\right]$

$s_5(t) = 3/2 \text{ se}(4\pi f_c t)$ $\quad \bar{s}_5 = \left[0,\ 0,\ \frac{3}{2}\sqrt{\frac{T}{2}}\right] = \left[0,\ 0,\ \frac{3\sqrt{2T}}{4}\right]$

$s_6(t) = -2 \text{ sen}(4\pi f_c t)$ $\quad \bar{s}_6 = [0,\ 0,\ -\sqrt{2T}]$

$$s_7(t) = 2\cos(2\pi f_c t) + 4 \text{ sen}(4\pi f_c t)$$

$$\bar{s}_7 = [\sqrt{2T},\ 0,\ 2\sqrt{2T}]$$

(d) $$E_i = \sum_{j=1}^{N} s_{ij}^2 = \sum_{j=1}^{3} s_{ij}^2$$

$$E_1 = \left(2\sqrt{2T}\right)^2 = 4\cdot 2T = 8T$$

$$E_2 = \left(5\sqrt{2T}\right)^2 = 25\cdot 2T = 50T$$

$$E_3 = \left(\frac{9}{2}\sqrt{T}\right)^2 + \left(-\frac{9}{2}\sqrt{T}\right)^2 = \frac{81}{4}T + \frac{81}{4}T = \frac{81}{2}T$$

$$E_4 = \left(\frac{5\sqrt{2T}}{2}\right)^2 = \frac{25\cdot 2T}{4} = \frac{25}{2}T$$

$$E_5 = \left(\frac{3\sqrt{2T}}{4}\right)^2 = \frac{9\cdot 2T}{16} = \frac{9}{8}T$$

$$E_6 = \left(-\sqrt{2T}\right)^2 = 2T$$

$$E_7 = \left(\sqrt{2T}\right)^2 + \left(2\sqrt{2T}\right)^2 = 2T + 4\cdot 2T = 2T + 8T = 10T$$

(e)