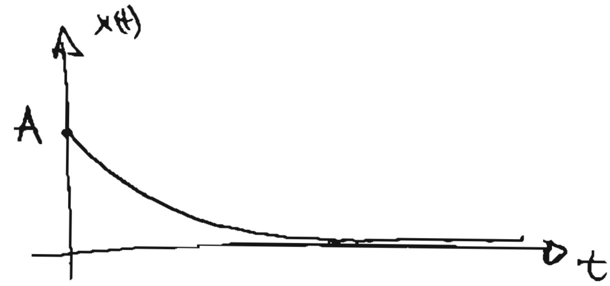
PROBLEMA 1.

(a) $x(t) = A\,e^{-t/a}\,u(t)$



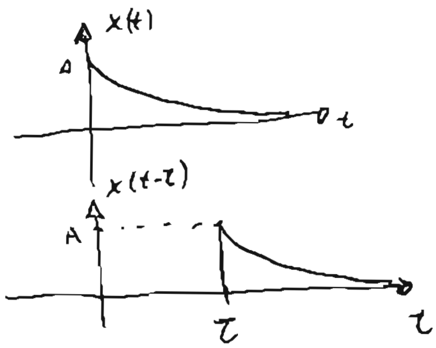
$$\boxed{E_x} = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-\infty}^{\infty} \left[A e^{-t/a} u(t)\right]^2 dt = A^2 \int_0^{\infty} e^{-2t/a}\,dt =$$

$$= A^2 \left[\frac{e^{-2t/a}}{-2/a}\right]_0^{\infty} = \boxed{\frac{A^2 a}{2}}$$

(b)

Consideremos $\tau > 0$ ya que $R_x(\tau)$ es par para el caso real.

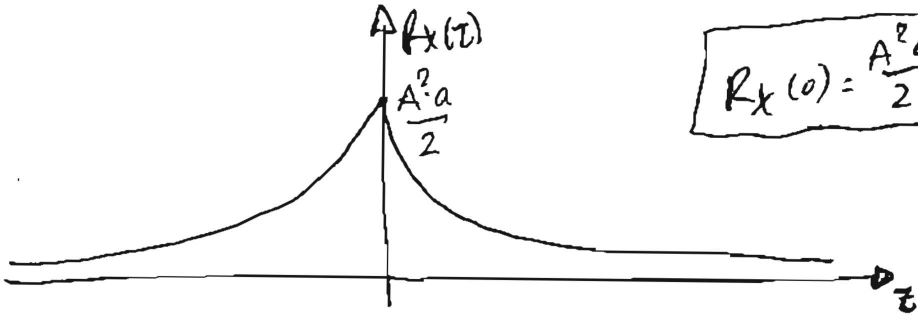
$$R_x(\tau) = \int_{-\infty}^{\infty} x(t)\,x(t-\tau)\,dt$$



$$= \int_{\tau}^{\infty} A\,e^{-t/a} \cdot A\,e^{-\left(\frac{t-\tau}{a}\right)} dt = A^2 \int_{\tau}^{\infty} e^{-2t/a}\, e^{\tau/a}\,dt = A^2 e^{\tau/a} \left[\frac{e^{-2t/a}}{-2/a}\right]_{\tau}^{\infty}$$

$$= A^2 e^{\tau/a}\, \frac{a}{2}\, e^{-2\tau/a} = \frac{A^2 a}{2}\, e^{-\tau/a} \qquad \tau > 0$$

Como tiene que ser par: $\boxed{R_x(\tau) = \frac{A^2 a}{2}\, e^{-\frac{|\tau|}{a}}}$



$$\boxed{R_x(0) = \frac{A^2 a}{2} = E_x}$$

(c) Mirando en las tablas de transformadas inmediatas:

$$X(f) = \frac{A}{1/a + j2\pi f} = \frac{Aa}{1 + j2\pi a f}$$

$X(0) = A \cdot a$. $\quad |X(BW_{3dB})|^2 = \frac{|X(0)|^2}{2}$ por definición.

$$\frac{A^2 a^2}{1 + 4\pi^2 a^2 BW_{3dB}^2} = \frac{A^2 a^2}{2} \Rightarrow 4\pi^2 a^2 BW_{3dB}^2 = 1$$

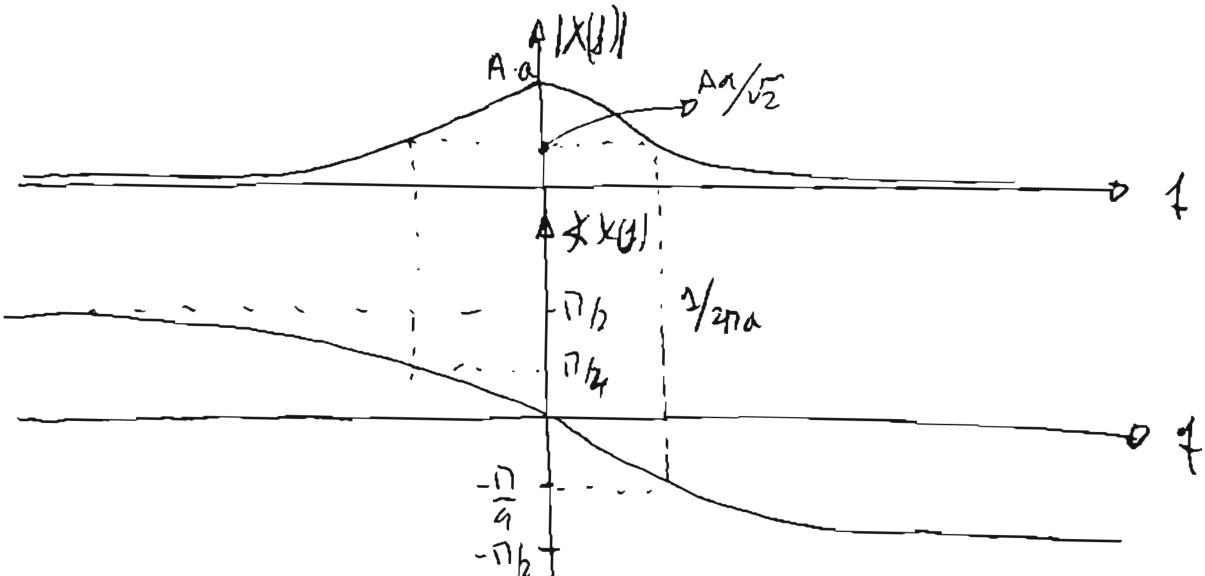
$$\boxed{BW_{3dB} = \frac{1}{2\pi a}}$$

(d) $|X(f)| = \frac{A \cdot a}{\sqrt{1 + 4\pi^2 a^2 f^2}}$ y $\measuredangle X(f) = -\operatorname{atan}(2\pi a f)$.

Para $f = 0$, $|X(f)| = A \cdot a$ $\quad \measuredangle X(f) = -\operatorname{atan}(0) = 0$

Para $f = BW_{3dB}$, $|X(f)| = \frac{Aa}{\sqrt{2}}$ $\quad \measuredangle X(f) = -\operatorname{atan}(1) = -\frac{\pi}{4}$

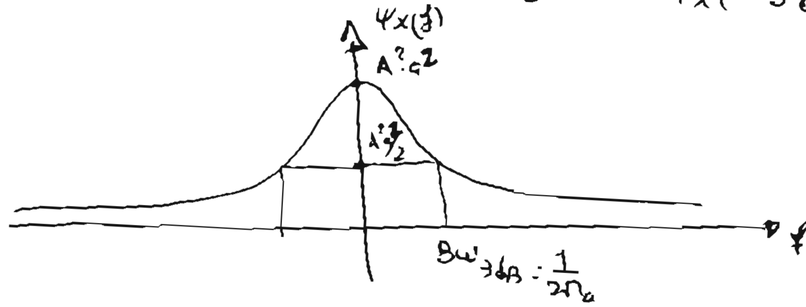
Para $f = -BW_{3dB}$ $|X(f)| = \frac{Aa}{\sqrt{2}}$ $\quad \measuredangle X(f) = -\operatorname{atan}(-1) = \frac{\pi}{4}$



3

(e) $\Psi_X(f) = TF\{R_X(\tau)\} = |X(f)|^2 = \dfrac{A^2 a^2}{1 + 4\pi^2 f^2 a^2}$

$f=0$ $\quad \Psi_X(0) = A^2 \cdot a^2$. $\qquad f = \pm BW_{3dB} \Rightarrow \Psi_X(\pm 3 B_{3dB}) = \dfrac{A^2 a^2}{2}$



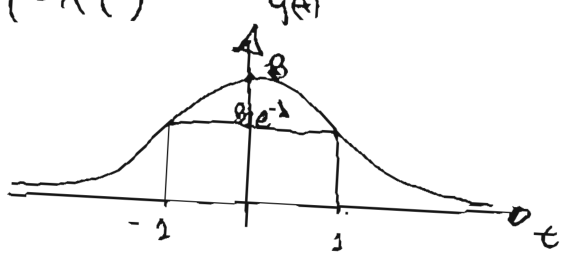
(f) $\boxed{E_{X,3dB}} = \displaystyle\int_{-BW_{3dB}}^{BW_{3dB}} \Psi_X(f)\,df = \int_{-\frac{1}{2\pi a}}^{\frac{1}{2\pi a}} \frac{A^2 a^2}{1 + 4\pi^2 a^2 f^2}\,df$ $\quad \left|\begin{array}{l} u = 2\pi a f \\ du = 2\pi a\,df \end{array}\right.$

$= A^2 a^2 \displaystyle\int_{-1}^{1} \frac{1}{1+u^2}\,\frac{du}{2\pi a} = \frac{A^2 a^2}{2\pi a}\left[\operatorname{atan}(u)\right]_{-1}^{1} = \frac{A^2 a^{\cancel{2}}}{2\cancel{\pi}\cancel{a}}\,\frac{\cancel{\pi}}{2} = \boxed{\frac{A^2 a}{4}}$

$\dfrac{E_{X,3dB}}{E_X} = 0{,}5 \Rightarrow \boxed{50\%}$

(g) $y(t) = B\exp(-\lambda t^2)$

(1)

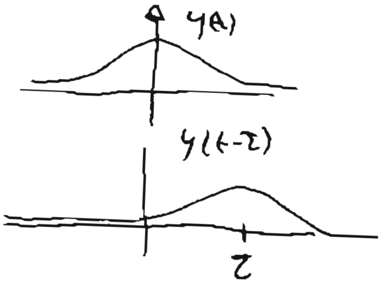


$\boxed{E_y = \displaystyle\int_{-\infty}^{\infty} y^2(t)\,dt} = \int_{-\infty}^{\infty} B^2 \exp(-2\lambda t^2)\,dt = \boxed{B^2\sqrt{\frac{\pi}{2\lambda}}}$

haciendo uso de $\displaystyle\int_{-\infty}^{\infty} \exp(-a x^2)\,dx = \sqrt{\frac{\pi}{a}} \quad a>0$.

(2)

$$R_y(\tau) = \int_{-\infty}^{\infty} y(t)\, y(t-\tau)\, dt$$



$$= B^2 \int_{-\infty}^{\infty} \exp(-\lambda t^2) \exp[-\lambda (t-\tau)^2]\, dt =$$

$$= B^2 \int_{-\infty}^{\infty} \exp[-\lambda (t^2 + (t-\tau)^2)]\, dt$$

Se puede hacer uso del hecho que

$$t^2 + (t-\tau)^2 = 2\left(t - \frac{\tau}{2}\right)^2 + \frac{\tau^2}{2}$$

entonces.

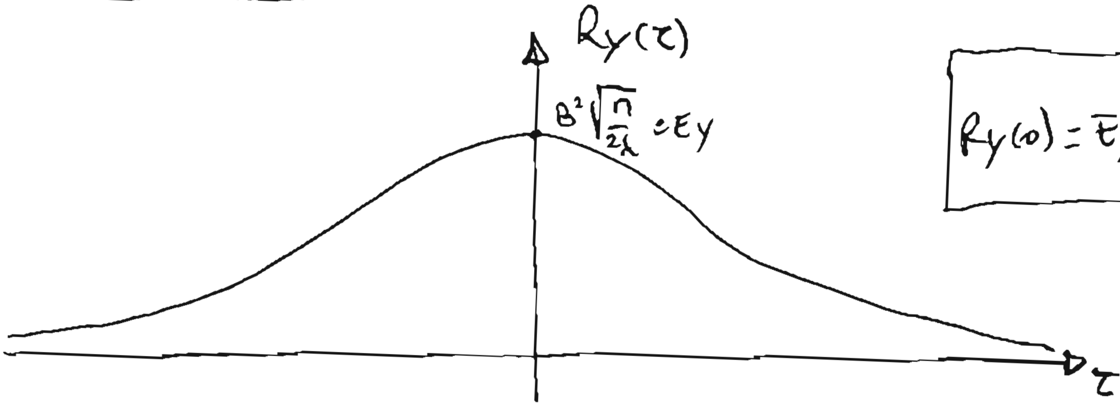
$$R_y(\tau) = B^2 \int_{-\infty}^{\infty} \exp\left[-2\lambda \left(t - \frac{\tau}{2}\right)^2\right] \exp\left(-\lambda \frac{\tau^2}{2}\right) dt$$

$$= B^2 \exp\left(-\lambda \frac{\tau^2}{2}\right) \int_{-\infty}^{\infty} \exp\left[-2\lambda \left(t - \frac{\tau}{2}\right)^2\right] dt \quad \Big| \begin{array}{l} u = t - \frac{\tau}{2} \\ du = dt \end{array}$$

$$= B^2 \exp\left(-\lambda \frac{\tau^2}{2}\right) \int_{-\infty}^{\infty} \exp\left(-2\lambda u^2\right) du$$

$$\boxed{R_y(\tau) = B^2 \sqrt{\frac{\pi}{2\lambda}} \exp\left(-\lambda \frac{\tau^2}{2}\right)}$$

haciendo uso de $\int_{-\infty}^{\infty} \exp(-ax^2)\,dx = \sqrt{\frac{\pi}{a}}$ $a>0$.



$$\boxed{R_y(0) = E_y = B^2 \sqrt{\frac{\pi}{2\lambda}}}$$

3 Usando Transformadas Inmediatas

$$\exp\left(-\pi \frac{t^2}{T^2}\right) \rightleftharpoons T\exp\left(-\pi f^2 T^2\right)$$

en nuestro caso $y(t) = B\exp(-\lambda t^2) \Rightarrow \lambda = \frac{\pi}{T^2} \Rightarrow T = \sqrt{\frac{\pi}{\lambda}}$

$$\boxed{Y(f) = B\left(\sqrt{\frac{\pi}{\lambda}}\right)\exp\left(-\pi f^2 \frac{\pi}{\lambda}\right) = B\sqrt{\frac{\pi}{\lambda}}\exp\left(-\pi^2 f^2/\lambda\right)}$$

$Y(0) = B\sqrt{\frac{\pi}{\lambda}}$ $\qquad |Y(BW_{3dB})|^2 = \frac{|Y(0)|^2}{2}$ por definición.

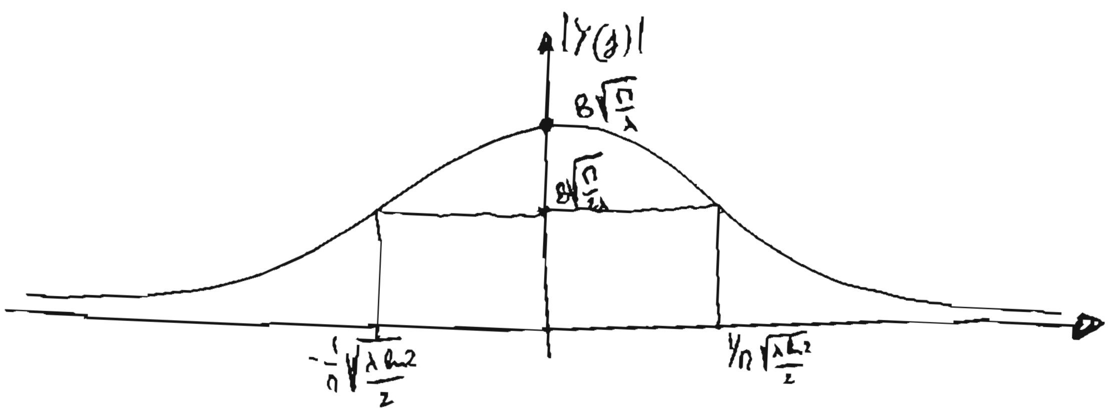
$$\left[B\sqrt{\frac{\pi}{\lambda}}\exp\left(-\pi^2 f^2/\lambda\right)\right]^2 = \frac{B^2\,\pi/\lambda}{2}$$

$$\cancel{B^2}\,\cancel{\frac{\pi}{\lambda}}\exp\left(-2\pi^2 f^2/\lambda\right) = \frac{1}{2}\cancel{B^2}\,\cancel{\frac{\pi}{\lambda}}$$

$$-\frac{2\pi^2 f^2}{\lambda} = \ln\frac{1}{2} \qquad \frac{2\pi^2 f^2}{\lambda} = \ln 2$$

$$\boxed{BW_{3dB} = \frac{1}{\pi}\sqrt{\frac{\lambda\ln 2}{2}}}$$

(4) $|Y(f)| = Y(f)$ $\qquad \measuredangle Y(f) = 0.$

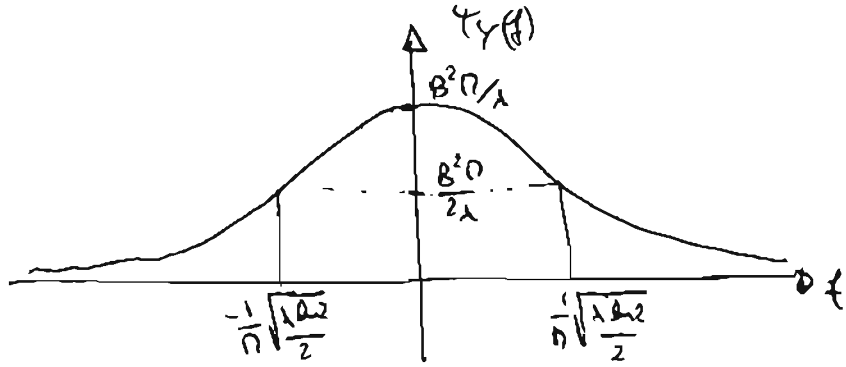


$$Y(\pm BW_{3dB}) = B\sqrt{\frac{\pi}{\lambda}}\exp\left(-\cancel{\pi^2}\,\frac{1}{\cancel{\pi^2}}\,\frac{\lambda\ln 2}{2}\,\frac{1}{\cancel{\lambda}}\right) = B\sqrt{\frac{\pi}{\lambda}}\exp\left(-\ln\sqrt{2}\right) = B\sqrt{\frac{\pi}{2\lambda}}$$

(5)
$$\Psi_Y(f) = TF\{R_Y(\tau)\} = |Y(f)|^2 = B^2 \frac{\pi}{\lambda} \exp\left(-\frac{2\pi^2 f^2}{\lambda}\right)$$

$$\Psi_Y(0) = \frac{B^2\pi}{\lambda}$$

$$\Psi_Y(\pm BW_{3dB}) = \frac{B^2\pi}{2\lambda}$$



(6)
$$E_{Y,3dB} = \int_{-BW_{3dB}}^{BW_{3dB}} \Psi_Y(f)\,df = \int_{-\frac{1}{\pi}\sqrt{\frac{\lambda\ln 2}{2}}}^{\frac{1}{\pi}\sqrt{\lambda\frac{\ln 2}{2}}} B^2 \frac{\pi}{\lambda} \exp\left(-\frac{2\pi^2 f^2}{\lambda}\right) df$$

$$= B^2\frac{\pi}{\lambda} \int_{-\frac{1}{\pi}\sqrt{\frac{\lambda\ln 2}{2}}}^{\frac{1}{\pi}\sqrt{\frac{\lambda\ln 2}{2}}} \exp\left(-\frac{2\pi^2 f^2}{\lambda}\right) df \qquad \left|\; \begin{aligned} x &= \sqrt{\frac{2}{\lambda}}\cdot\pi\cdot f \\ dx &= \sqrt{\frac{2}{\lambda}}\cdot\pi\, df \end{aligned}\right.$$

$$= B^2 \frac{\cancel{\pi}}{\lambda} \int_{-\sqrt{\ln 2}}^{\sqrt{\ln 2}} \exp(-x^2)\, \frac{dx}{\sqrt{\frac{2}{\lambda}}\cdot\cancel{\pi}} = B^2 \frac{\sqrt{\pi}}{\sqrt{2\lambda\pi}} \int_{-\sqrt{\ln 2}}^{\sqrt{\ln 2}} \exp(-x^2)\,dx$$

$$\boxed{E_{Y,3dB} = B^2 \frac{\sqrt{\pi}}{\sqrt{2\lambda}} H\left(\sqrt{\ln 2}\right) = B^2 \sqrt{\frac{\pi}{2\lambda}}\, H\left(\sqrt{\ln 2}\right)}$$

donde
$$H(u) = \frac{1}{\sqrt{\pi}} \int_{-u}^{u} \exp(-x^2)\,dx.$$

$u = \sqrt{\ln 2} = 0{,}8326$ interpolamos $H(u)$ usando la tabla

$H(0{,}80) = 0{,}742$

$H(0{,}85) = 0{,}771$

$$\boxed{H(0{,}8326)} = H(0{,}80) + \frac{H(0{,}85) - H(0{,}80)}{0'85 - 0'80} \cdot H(0'8326 - 0'80)$$

$$= 0'742 + \frac{0'029}{0'05} \cdot 0'0326 = \boxed{0'761}$$

$$\boxed{\frac{E_{y,3dB}}{E_y}} = \frac{B^2\cancel{\sqrt{\frac{\pi}{2A}}}\cdot H(\sqrt{\ln 2})}{B^2\cancel{\sqrt{\frac{\pi}{2A}}}} = H(\sqrt{\ln 2}) = H(0{,}8326) = \boxed{0'761}$$

$$\boxed{76'1\%}$$

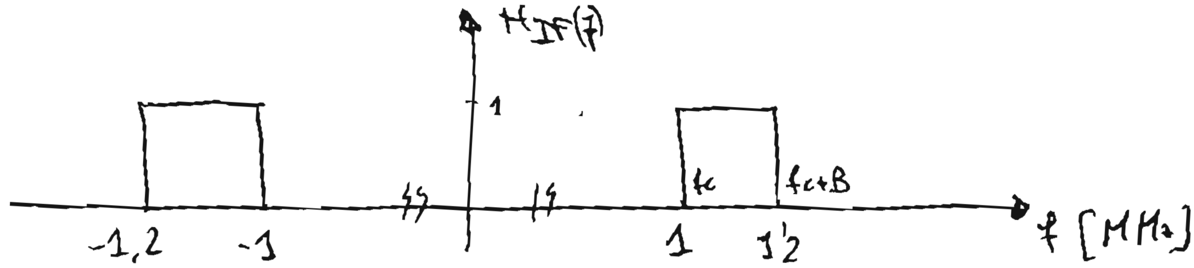
(h) La segunda señal y(t), tiene 76'1% frente al 50% de x(t). y(t) tiene la energía más concentrada entorno del origen.

PROBLEMA 2: (1)

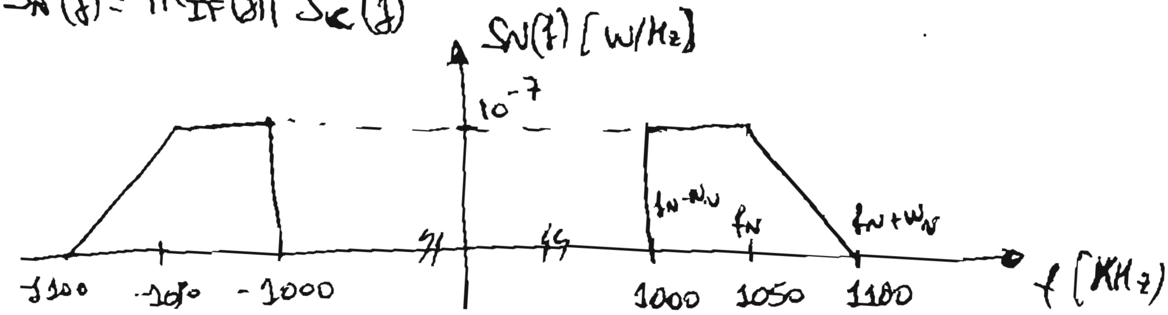
(a) El Filtro IF equivalente para la señal

$$s(t) = \frac{A_c}{2} m(t) \cos(2\pi f_c t) - \frac{A_c}{2} \hat{m}(t) \sin(2\pi f_c t)$$

es aquel que la deja pasar sin modificar. Esta señal es SSB con banda lateral superior, ocupa frecuencias $[f_c, f_c+B] = [1, 1{,}2]$ MHz



$S_N(f) = |H_{IF}(f)|^2 S_c(f)$



$f_N = 1050$ KHz $\qquad W_N = 100$ KHz

(b) $\boxed{P_{S_I}} = \frac{A_c^2 \cdot P}{4} = \frac{5^2 \cdot 30}{4} = 187500 \text{ mW} \Rightarrow \boxed{52'73 \text{ dBm}}$

$\boxed{P_{N_I}} = \int_{-\infty}^{\infty} S_N(f)df = 10^{-7}_{W/Hz} \cdot 150 \cdot 10^3 \text{ Hz} = 150 \cdot 10^{-4} \text{ W} = 15 \text{ mW} \Rightarrow \boxed{11'76 \text{ dBm}}$

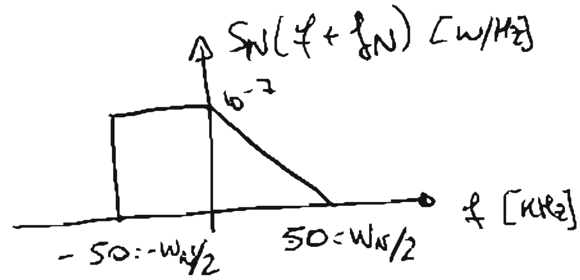
$\boxed{SNR_I = P_{S_I} - P_{N_I}} = \boxed{40'97 \text{ dB}}$

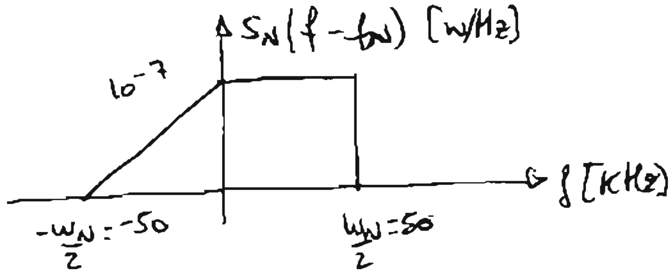
(c)

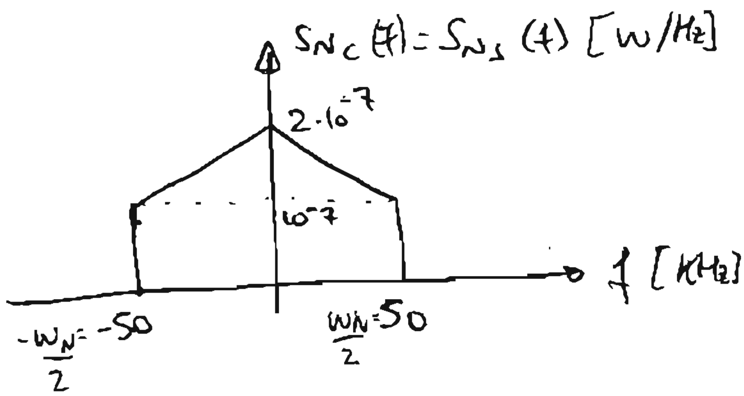
$$n(t) = n_c(t)\cos(2\pi f_N t) - n_s(t)\,\mathrm{sen}(2\pi f_N t)$$

De teoría se sabe que

$$S_{N_c}(f) = S_{N_s}(f) = \begin{cases} S_N(f-f_N) + S_N(f+f_N) & |f| < W_N/2 \\ 0 & \text{Resto} \end{cases}$$

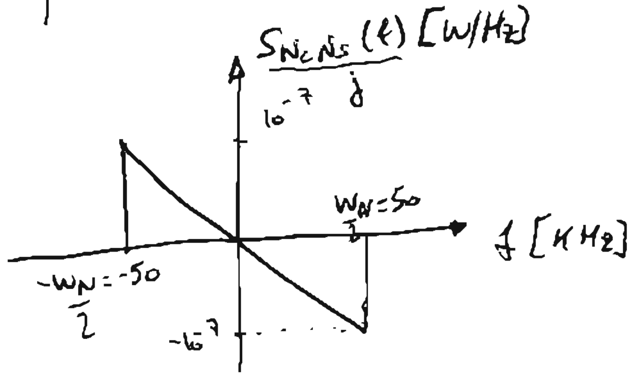






para la cruzada:

$$S_{N_cN_s}(f) = \begin{cases} j\,[S_N(f+f_N) - S_N(f-f_N)] & |f| < W_N/2 \\ 0 & \text{Resto} \end{cases}$$



(d) ③

$$x(t) = s(t) + n(t)$$

$$y_1(t) = A_c' \cos(2\pi f_c t)\,[s(t) + n(t)]$$ antes del filtro de postdetección.

$$y_1(t) = A_c' \cos(2\pi f_c t)\left[\frac{A_c}{2} m(t)\cos(2\pi f_c t) - \frac{A_c}{2}\hat{m}(t)\,\mathrm{sen}(2\pi f_c t) + n_c(t)\cos(2\pi f_N t) - n_s(t)\,\mathrm{sen}(2\pi f_N t)\right]$$

$$= \frac{A_c A_c'}{4} m(t) + \frac{A_c A_c'}{4} m(t)\cos[4\pi f_c t) - \frac{A_c A_c'}{4}\hat{m}(t)\,\mathrm{sen}(4\pi f_c t)$$

$$+ \frac{A_c'}{2} n_c(t)\cos[2\pi(f_N - f_c)t] + \frac{A_c'}{2} n_c(t)\cos[2\pi(f_N + f_c)t]$$

$$- \frac{A_c'}{2} n_s(t)\,\mathrm{sen}[2\pi(f_N - f_c)t] - \frac{A_c'}{2} n_s(t)\,\mathrm{sen}[2\pi(f_N + f_c)t]$$

Tras el filtro con frecuencia de corte B. $(f_N - f_c = W_N < B)$

$$\boxed{y(t) = \frac{A_c A_c'}{4} m(t) + \frac{A_c'}{2} n_c(t)\cos[\pi W_N t) - \frac{A_c'}{2} n_s(t)\,\mathrm{sen}(\pi W_N t)}$$

Si $y(t) = z(t) + n_o(t) \Rightarrow$ $\boxed{z(t) = \frac{A_c A_c'}{4} m(t)}$

$$\boxed{n_o(t) = \frac{A_c'}{2} n_c(t)\cos[\pi W_N t] - \frac{A_c'}{2} n_s(t)\,\mathrm{sen}[\pi W_N t]}$$

(e) $$R_{N_o}(\tau) = E[n_o(t)\,n_o(t-\tau)] = E\left[\left(\frac{A_c'}{2} n_c(t)\cos(\pi W_N t) - \frac{A_c'}{2} n_s(t)\,\mathrm{sen}(\pi W_N t)\right)\cdot\right.$$

$$\left.\cdot\left(\frac{A_c'}{2} n_c(t-\tau)\cos[\pi W_N(t-\tau)] - \frac{A_c'}{2} n_s(t-\tau)\,\mathrm{sen}[\pi W_N(t-\tau)]\right)\right] = \quad \left(R_{N_c}(\tau) = R_{N_s}(\tau)\right)$$

$$\left(R_{N_s N_c}(\tau) = -R_{N_c N_s}(\tau)\right)$$

$$= \frac{(A_c')^2}{4} R_{N_c}(\tau)\left[\cos(\pi W_N t)\cos[\pi W_N(t-\tau)] + \mathrm{sen}(\pi W_N t)\,\mathrm{sen}[\pi W_N(t-\tau)]\right]$$

$$+ \frac{(A_c')^2}{4} R_{N_c N_s}(\tau)\left[\mathrm{sen}(\pi W_N t)\cos(\pi W_N(t-\tau)) - \cos(\pi W_N t)\,\mathrm{sen}[\pi W_N(t-\tau)]\right]$$

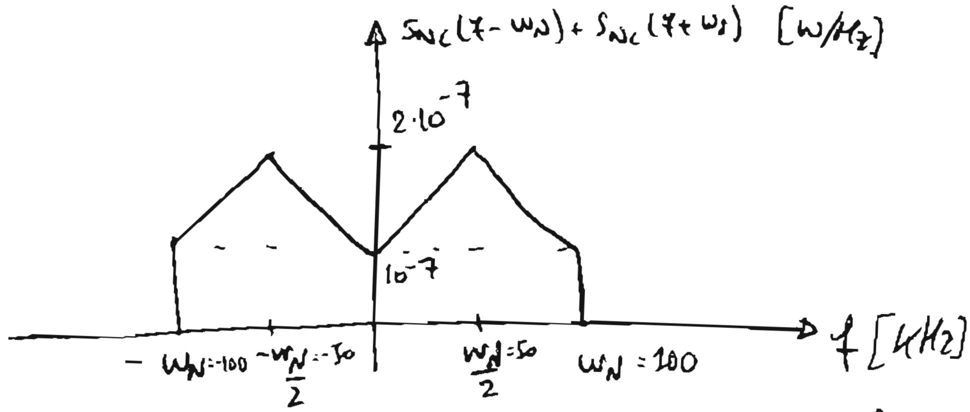
$$\boxed{R_{N_0}(\tau) = \frac{(A_c')^2}{4} R_{N_c}(\tau)\cos(\pi W_N \tau) + \frac{(A_c')^2}{4} R_{N_c N_s}(\tau)\,\text{sen}(\pi W_N \tau)}$$

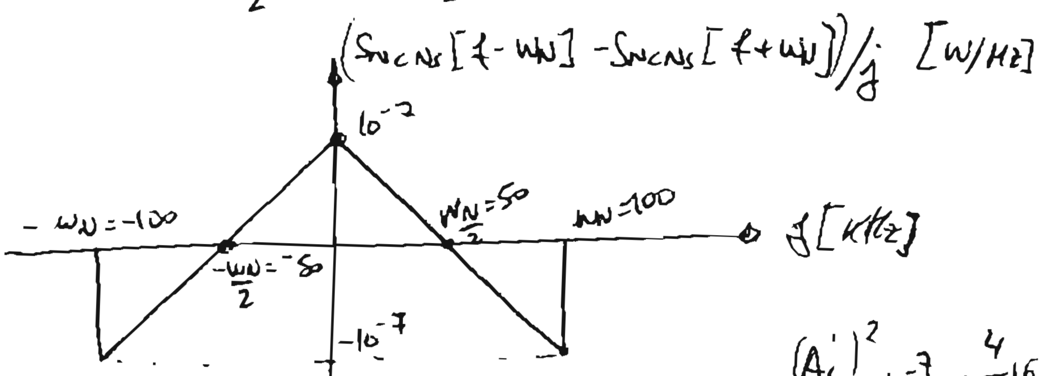
(f) Haciendo transformada de Fourier

$$S_{N_0}(f) = \frac{(A_c')^2}{8} S_{N_c}(f) * \left[\delta\left(f - \frac{W_N}{2}\right) + \delta\left(f + \frac{W_N}{2}\right)\right] +$$

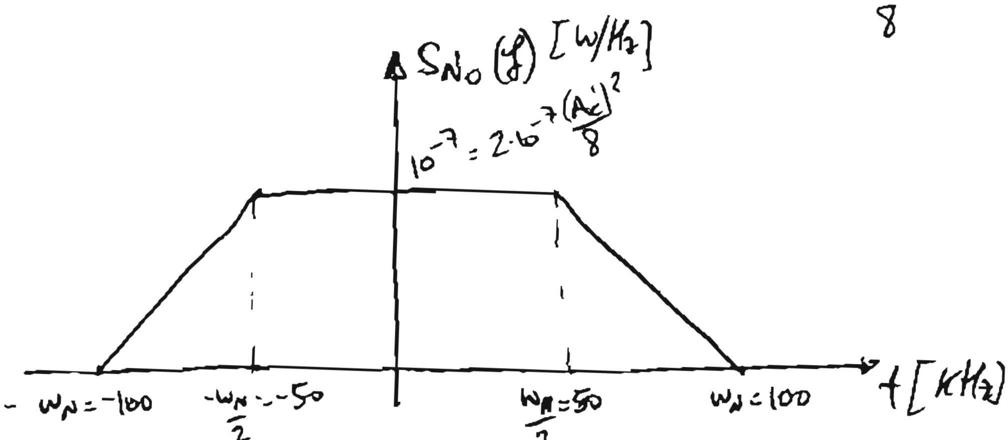
$$+ \frac{(A_c')^2}{8j} S_{N_c N_s}(f) * \left[\delta\left(f - \frac{W_N}{2}\right) - \delta\left(f + \frac{W_N}{2}\right)\right]$$

$$\boxed{S_{N_0}(f) = \frac{(A_c')^2}{8}\left[S_{N_c}\left(f - \frac{W_N}{2}\right) + S_{N_c}\left(f + \frac{W_N}{2}\right)\right] + \frac{(A_c')^2}{8j}\left[S_{N_c N_s}\left(f - \frac{W_N}{2}\right) - S_{N_c N_s}\left(f + \frac{W_N}{2}\right)\right]}$$





$$\frac{(A_c')^2}{8} \cdot 10^{-7} = \frac{4}{8} 10^{-7} = \frac{1}{2} 10^{-7}$$



(g) $\boxed{P_{S_0}} = \dfrac{A_c^2 \cdot (A_c')^2 \cdot P}{4^2} = \dfrac{5^2 \cdot 2^2 \cdot 30}{4^2} = 187500 \text{ mW} \Rightarrow \boxed{52'73 \text{dBm}}$

$\boxed{P_{N_0}} = \int_{-\infty}^{\infty} S_{N_0}(f)\, df = 10^{-7} \cancel{\text{w/Hz}}\, 150 \cdot 10^3 \cancel{\text{Hz}} = 150 \cdot 10^{-4} = 15 \text{ mW} \Rightarrow \boxed{11'76 \text{dBm}}$

$\boxed{SNR_0 = 40'97 \text{ dB}}$

Como era de esperar $SNR_0 = SNR_I$ en SSB.