PROBLEMA 1.

(a) $\boxed{X(f)} = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt = \int_a^b A e^{-ct} e^{-j2\pi f t} dt = A \int_a^b e^{-(c+j2\pi f)t} dt$

$$= A \left[\frac{e^{-(c+j2\pi f)t}}{-c-j2\pi f} \right]_a^b = \boxed{A \frac{e^{-(c+j2\pi f)a} - e^{-(c+j2\pi f)b}}{c+j2\pi f}}$$

(b) Hay una propiedad de la transformada de Fourier que nos dice que el área temporal es igual al valor de la transformada de Fourier en el origen ($f=0$):

$$\int_{-\infty}^{\infty} x(t) dt = X(0)$$

A partir de $X(f)$ fijando $f=0$

$$\boxed{A_x} = X(0) = X(f)\Big|_{f=0} = \boxed{A \frac{e^{-ca} - e^{-cb}}{c}}$$

(c) $\boxed{\Psi_x(f)} = |X(f)|^2 = A^2 \frac{\left| e^{-(c+j2\pi f)a} - e^{-(c+j2\pi f)b} \right|^2}{c^2 + 4\pi^2 f^2} =$

$$= \frac{A^2}{c^2+4\pi^2 f^2} \left| \left[\cos(2\pi f a) - j\,\mathrm{sen}(2\pi f a) \right] e^{-ca} - \left[\cos(2\pi f b) - j\,\mathrm{sen}(2\pi f b) \right] e^{-cb} \right|^2$$

$$= \frac{A^2}{c^2+4\pi^2 f^2} \left| \left[e^{-ca} \cos(2\pi f a) - e^{-cb} \cos(2\pi f b) \right] + j \left[e^{-cb} \mathrm{sen}(2\pi f b) - e^{-ca} \mathrm{sen}(2\pi f a) \right] \right|^2$$

$$= \frac{A^2}{c^2+4\pi^2 f^2} \left\{ \left[e^{-ca} \cos(2\pi f a) - e^{-cb} \cos(2\pi f b) \right]^2 + \left[e^{-cb} \mathrm{sen}(2\pi f b) - e^{-ca} \mathrm{sen}(2\pi f a) \right]^2 \right\}$$

$$= \frac{A^2}{c^2+4\pi^2 f^2} \Big[e^{-2ca} \cos^2(2\pi f a) + e^{-2cb} \cos^2(2\pi f b) - 2 e^{-ca} e^{-cb} \cos(2\pi f a) \cos(2\pi f b) +$$

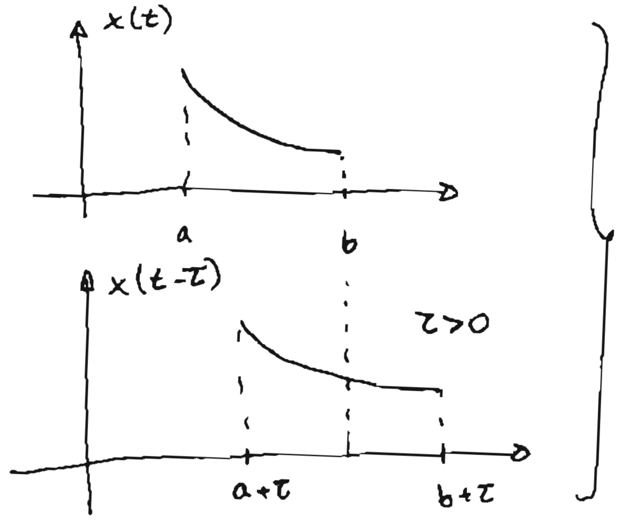
$$+ e^{-2cb} \mathrm{sen}^2(2\pi f b) + e^{-2ca} \mathrm{sen}^2(2\pi f a) - 2 e^{-cb} e^{-ca} \mathrm{sen}(2\pi f b)\,\mathrm{sen}(2\pi f a) \Big]$$

$$\boxed{= \frac{A^2}{c^2+4\pi^2 f^2} \left[e^{-2ca} + e^{-2cb} - 2 e^{-c(a+b)} \cos[2\pi f (b-a)] \right]}$$

(d) $R_x(\tau) = \langle x(t), x(t-\tau)\rangle = \int_{-\infty}^{\infty} x(t)\,x(t-\tau)\,dt$

2

Sabemos que $R_x(\tau) = R_x(-\tau)$ es par para $x(t)$ real. Vamos a calcular $R_x(\tau)$ para $\tau > 0$.



Estas funciones se solapan para $\tau>0$ en el intervalo

$\underline{0 < \tau < b-a}$

ya que

$b = a+\tau \Rightarrow \tau = b-a$

Para $\tau > b-a \Rightarrow R_x(\tau) = 0$.

En el intervalo $0 < \tau < b-a$:

$$R_x(\tau) = \int_{a+\tau}^{b} A\cdot e^{-ct}\cdot A\cdot e^{-c(t-\tau)}\,dt = \int_{a+\tau}^{b} A^2 e^{-2ct} e^{c\tau}\,dt = A^2 e^{c\tau}\int_{a+\tau}^{b} e^{-2ct}\,dt =$$

$$= A^2 e^{c\tau}\left[\frac{e^{-2ct}}{-2c}\right]_{a+\tau}^{b} = A^2\,\frac{e^{-2c(a+\tau)} - e^{-2cb}}{2c}\cdot e^{c\tau} = \frac{A^2}{2c}\left[e^{-c(2a+\tau)} - e^{-c(2b-\tau)}\right]$$

$0 < \tau < b-a$

Para que la función sea par, sustituyo τ por $|\tau|$, además será cero fuera del intervalo $|\tau| > b-a$:

$$\boxed{R_x(\tau) = \frac{A^2}{2c}\left[e^{-c(2a+|\tau|)} - e^{-c(2b-|\tau|)}\right]\prod\left(\frac{\tau}{2(b-a)}\right)}$$

(e) $$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_a^b A^2 e^{-2ct}\,dt = A^2\left[\frac{e^{-2ct}}{-2c}\right]_a^b = \boxed{\frac{A^2}{2c}\left[e^{-2ca} - e^{-2cb}\right]}$$

(f) Sabemos que $E_x = R_x(0) \Rightarrow$

$$\boxed{E_x = R_x(0) = R_x(\tau)\Big|_{\tau=0} = \frac{A^2}{2c}\left[e^{-2ca} - e^{-2cb}\right]}$$ coincide

(g) Sabemos que por las propiedades de la transformada de Fourier, el área en un dominio es el valor en el origen del otro dominio, además sabemos que (3)

$$R_X(\tau) \rightleftharpoons \Psi_X(f)$$

Por lo que:

$$A_R = \int_{-\infty}^{\infty} R_X(\tau)\, d\tau = \Psi_X(f)\Big|_{f=0} = \Psi_X(0)$$

por lo que

$$A_R = \Psi_X(0) = \Psi_X(f)\Big|_{f=0} = \frac{A^2}{c^2}\left[e^{-2ca} + e^{-2cb} - 2e^{-c(a+b)}\right]$$

$$= \frac{A^2}{c^2}\left[e^{-ca} - e^{-cb}\right]^2 = \boxed{\left[\frac{A\left(e^{-ca} - e^{-cb}\right)}{c}\right]^2 = A_X^2}$$

(h) Observando el resultado del apartado (c) uno de los términos es un coseno dividido por una función cuadrática en f, por lo que se parece bastante al integrando que nos piden.

Además sabemos que:

$$E_X = \int_{-\infty}^{\infty} \Psi_X(f)\, df$$

y por el apartado (e): $E_X = \frac{A^2}{2c}\left[e^{-2ca} - e^{-2cb}\right]$

$$\int_{-\infty}^{\infty} \Psi_X(f)\, df = \frac{A^2}{2c}\left[e^{-2ca} - e^{-2cb}\right]$$

sustituyendo $\Psi_X(f)$ por el valor determinado en el apartado (c)

$$\int_{-\infty}^{\infty} \cancel{A^2}\, \frac{e^{-2ca} + e^{-2cb} - 2e^{-c(a+b)}\cos[2\pi f(b-a)]}{c^2 + 4\pi^2 f^2}\, df = \frac{\cancel{A^2}}{2c}\left[e^{-2ca} - e^{-2cb}\right]$$

4

$$\frac{1}{c^2}\int_{-\infty}^{\infty} \frac{e^{-2ca}+e^{-2cb}-2e^{-c(a+b)}\cos(2\pi f(b-a))}{1+\left(\frac{2\pi f}{c}\right)^2}\,df = \frac{1}{2c}\left[e^{-2ca}-e^{-2cb}\right]$$

Hacemos el cambio de variable $u=\frac{2\pi f}{c}$, $du=\frac{2\pi}{c}df$, $df=\frac{c\cdot du}{2\pi}$, $2\pi f=u\cdot c$

$$\cancel{\frac{1}{c}}\int_{-\infty}^{\infty} \frac{e^{-2ca}+e^{-2cb}-2e^{-c(a+b)}\cos[u\cdot c(b-a)]}{1+u^2}\,\frac{\cancel{c}\cdot du}{\cancel{2}\pi} = \cancel{\frac{1}{2}}\left[e^{-2ca}-e^{-2cb}\right]$$

$$\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{e^{-2ca}+e^{-2cb}}{1+u^2}\,du + \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{-2e^{-c(a+b)}\cos[u\cdot c(b-a)]}{1+u^2}\,du = e^{-2ca}-e^{-2cb}$$

$$\frac{1}{\pi}(e^{-2ca}+e^{-2cb})\int_{-\infty}^{\infty}\frac{1}{1+u^2}\,du - \frac{2}{\pi}e^{-c(a+b)}\int_{-\infty}^{\infty}\frac{\cos[c(b-a)u]}{1+u^2}\,du = e^{-2ca}-e^{-2cb}$$

$$\frac{1}{\pi}(e^{-2ca}+e^{-2cb})\left[\operatorname{atan}(u)\right]_{-\infty}^{\infty} - \frac{2}{\pi}e^{-c(a+b)}\int_{-\infty}^{\infty}\frac{\cos[c(b-a)u]}{1+u^2}\,du = e^{-2ca}-e^{-2cb}$$

$$\cancel{\frac{1}{\pi}}(e^{-2ca}+e^{-2cb})\cancel{\left[\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right]} - \frac{2}{\pi}e^{-c(a+b)}\int_{-\infty}^{\infty}\frac{\cos[c(b-a)u]}{1+u^2}\,du = e^{-2ca}-e^{-2cb}$$

$$\cancel{e^{-2ca}}+e^{-2cb} - \frac{2}{\pi}e^{-c(a+b)}\int_{-\infty}^{\infty}\frac{\cos[c(b-a)u]}{1+u^2}\,du = \cancel{e^{-2ca}}-e^{-2cb}$$

$$\cancel{2}e^{-2cb} = \frac{\cancel{2}}{\pi}e^{-c(a+b)}\int_{-\infty}^{\infty}\frac{\cos[c(b-a)u]}{1+u^2}\,du$$

$$\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\cos[c(b-a)u]}{1+u^2}\,du = \frac{e^{-2cb}}{e^{-ca}e^{-cb}} = e^{-2cb}e^{ca}e^{cb} = e^{-cb}e^{ca} = e^{-c(b-a)}$$

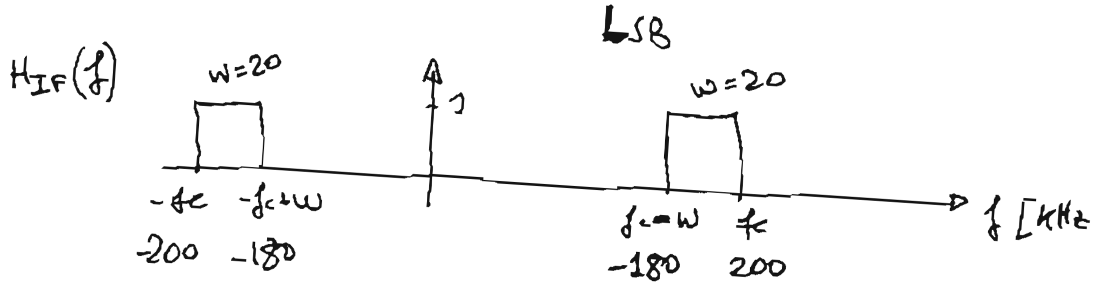
Haciendo $\lambda = c(b-a)$

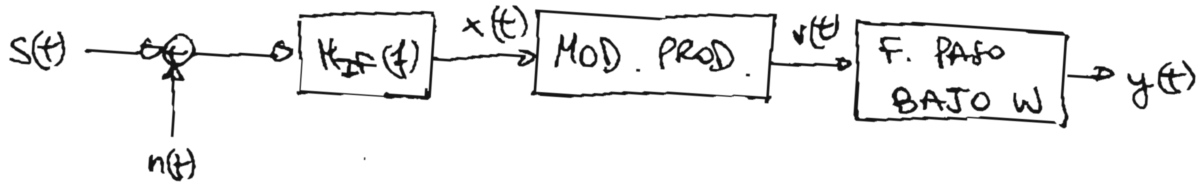
$$\int_{-\infty}^{\infty}\frac{\cos(\lambda u)}{1+u^2}\,du = \pi e^{-\lambda}$$, ahora como $\cos(\cdot)$ es par y $1+u^2$

también $\Rightarrow$

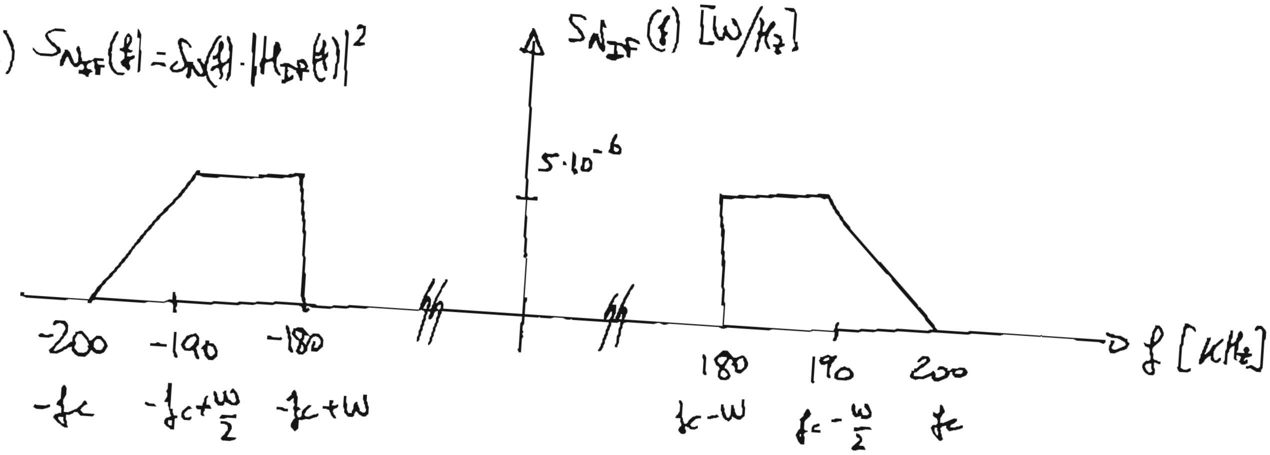
$$\int_{-\infty}^{\infty}\frac{\cos(\lambda u)}{1+u^2}\,du = 2\int_{0}^{\infty}\frac{\cos(\lambda u)}{1+u^2}\,du = \pi e^{-\lambda} \Rightarrow \boxed{\int_{0}^{\infty}\frac{\cos(\lambda u)}{1+u^2}\,du = \frac{\pi}{2}e^{-\lambda}}$$

PROBLEMA 2





(a) $S_{N_{IF}}(f) = S_N(f) \cdot |H_{IF}(f)|^2$



(b) $x(t) = s(t) + n_{IF}(t)$

LSB

$$s(t) = \frac{A_c}{2} m(t) \cos(2\pi f_c t) + \frac{A_c}{2} \hat{m}(t) \operatorname{sen}(2\pi f_c t)$$

$$n_{IF}(t) = n_c(t) \cos\left[2\pi\left(f_c - \frac{W}{2}\right)t\right] - n_s(t) \operatorname{sen}\left[2\pi\left(f_c - \frac{W}{2}\right)t\right]$$

$v(t) = x(t) \cdot \cos(2\pi f_c t) =$

$$= \frac{A_c}{2} m(t) \frac{1 + \cos(4\pi f_c t)}{2} + \frac{A_c}{2} \hat{m}(t) \frac{\operatorname{sen}(4\pi f_c t)}{2}$$

$$+ n_c(t) \frac{\cos(\pi W t) + \cos\left[2\pi\left(2f_c - \frac{W}{2}\right)t\right]}{2} + n_s(t) \frac{\operatorname{sen}(\pi W t) - \operatorname{sen}\left[2\pi\left(2f_c - \frac{W}{2}\right)t\right]}{2}$$

Tras el filtro paso bajo los términos en torno a $2f_c$ se eliminan:

$$\boxed{y(t) = \frac{A_c}{4} m(t) + \frac{n_c(t)}{2} \cos(\pi W t) + \frac{n_s(t)}{2} \operatorname{sen}(\pi W t)} \Rightarrow \begin{cases} \boxed{y_s(t) = \frac{A_c\, m(t)}{4}} \\ \boxed{y_n(t) = \frac{n_c(t)}{2} \cos(\pi W t) + \frac{n_s(t)}{2} \operatorname{sen}(\pi W t)} \end{cases}$$

(c) $P_{Y_S} = \dfrac{A_c^2 P}{16}$ con P la potencia de $m(t)$ (2)

$P_{S_I} = \dfrac{A_c^2 P}{4}$ potencia de la señal a la entrada del receptor

$P_{Y_S} = \dfrac{P_{S_I}}{4}$ tomando logaritmos:

$10\log_{10} P_{Y_S} = 10\log_{10} P_{S_I} - 10\log_{10} 4$

$$\boxed{P_{Y_S}(dBm) = P_{S_I}(dBm) - 10\log_{10} 4 = 59\,dBm - 6\,dB = 53\,dBm}$$

(d) $y_n(t) = \dfrac{n_c(t)}{2}\cos(\pi W t) + \dfrac{n_s(t)}{2}\mathrm{sen}(\pi W t)$

$$R_{Y_n}(\tau) = E\left\{ y_n(t)\, y_n(t-\tau)\right\} = E\left\{\left[\frac{n_c(t)}{2}\cos(\pi W t) + \frac{n_s(t)}{2}\mathrm{sen}(\pi W t)\right]\cdot\right.$$

$$\left.\cdot\left[\frac{n_c(t-\tau)}{2}\cos[\pi W(t-\tau)] + \frac{n_s(t-\tau)}{2}\mathrm{sen}[\pi W(t-\tau)]\right]\right\}$$

$$= \frac{1}{4}E\{n_c(t)n_c(t-\tau)\}\cos(\pi W t)\cos[\pi W(t-\tau)] +$$

$$+\frac{1}{4}E\{n_s(t)n_s(t-\tau)\}\mathrm{sen}(\pi W t)\,\mathrm{sen}[\pi W(t-\tau)] +$$

$$+\frac{1}{4}E\{n_c(t)n_s(t-\tau)\}\cos(\pi W t)\,\mathrm{sen}[\pi W(t-\tau)] +$$

$$+\frac{1}{4}E[n_s(t)n_c(t-\tau)]\,\mathrm{sen}(\pi W t)\cos[\pi W(t-\tau)]$$

Sabemos que $R_{N_c}(\tau) = E\{n_c(t)n_c(t-\tau)\} = R_{N_s}(\tau) = E\{n_s(t)n_s(t-\tau)\}$

$$R_{N_c N_s}(\tau) = E[n_c(t)n_s(t-\tau)\} = -R_{N_s N_c}(\tau) = -E[n_s(t)n_c(t-\tau)\}$$

Entonces:

$$\boxed{R_{Y_n}(\tau) = \frac{1}{4}R_{N_c}(\tau)\cos(\pi W \tau) - \frac{1}{4}R_{N_c N_s}(\tau)\,\mathrm{sen}(\pi W \tau)}$$

(e) $R_{y_n}(\tau) \rightleftharpoons S_{y_n}(f)$ $\quad S_{N_c}(f) \rightleftharpoons R_{N_c}(\tau)$ y $S_{N_cN_s}(f) \rightleftharpoons R_{N_cN_s}(\tau)$

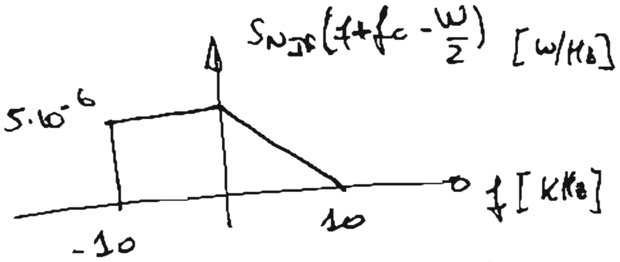
$$\boxed{S_{y_n}(f)} = \frac{1}{4} S_{N_c}(f) * \left[\frac{\delta(f-W/2)}{2} + \frac{\delta(f+W/2)}{2} \right]$$

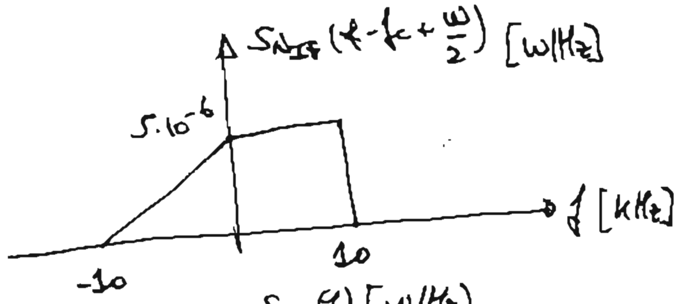
$$- \frac{1}{4} S_{N_cN_s}(f) * \left[\frac{\delta(f-W/2)}{2j} - \frac{\delta(f+W/2)}{2j} \right]$$

$$\boxed{= \frac{1}{8}\left[S_{N_c}\left(f-\frac{W}{2}\right) + S_{N_c}\left(f+\frac{W}{2}\right)\right] + \frac{1}{8j}\left[S_{N_cN_s}\left(f+\frac{W}{2}\right) - S_{N_cN_s}\left(f-\frac{W}{2}\right)\right]}$$

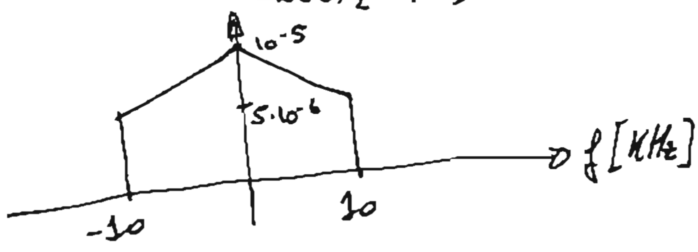
(f)

$$S_{N_c}(f) = \begin{cases} S_{N_{IF}}\left(f-f_c+\frac{W}{2}\right) + S_{N_{IF}}\left(f+f_c-\frac{W}{2}\right) & |f| < \frac{W}{2} \\ 0 & \text{Resto} \end{cases}$$

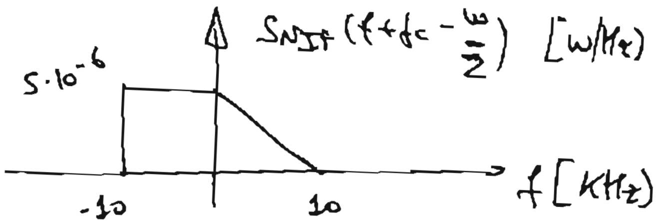


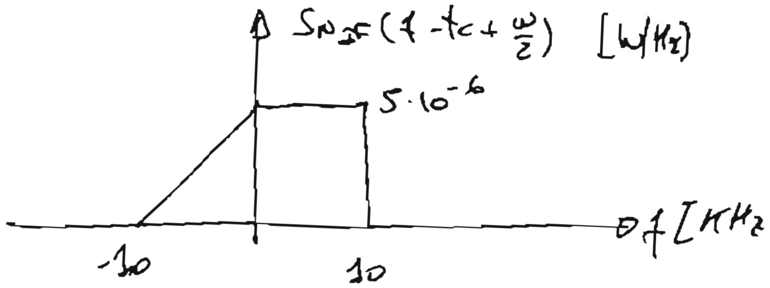


Sumando

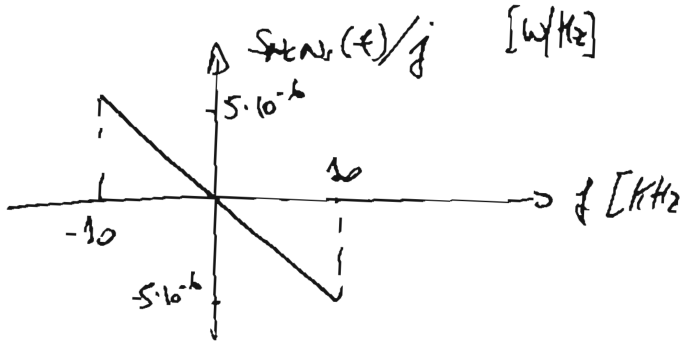


$$S_{N_cN_s}(f) = \begin{cases} j\left[S_{N_{IF}}\left(f+f_c-\frac{W}{2}\right) - S_{N_{IF}}\left(f-f_c+\frac{W}{2}\right)\right] & |f| < \frac{W}{2} \\ 0 & \text{Resto} \end{cases}$$





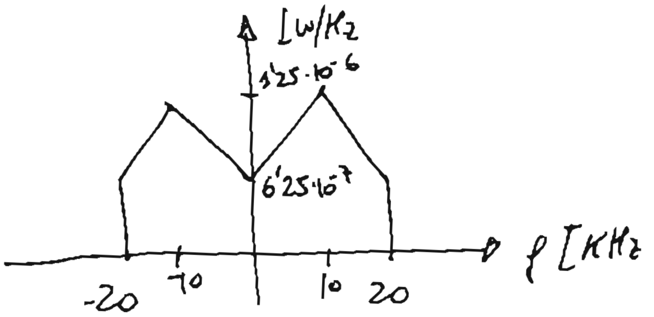
Restando



(g) $S_{y_u}(f) = \frac{1}{8}\left[S_{N_c}\left(f-\frac{W}{2}\right) + S_{N_c}\left(f+\frac{W}{2}\right)\right]$ (1ª parte)

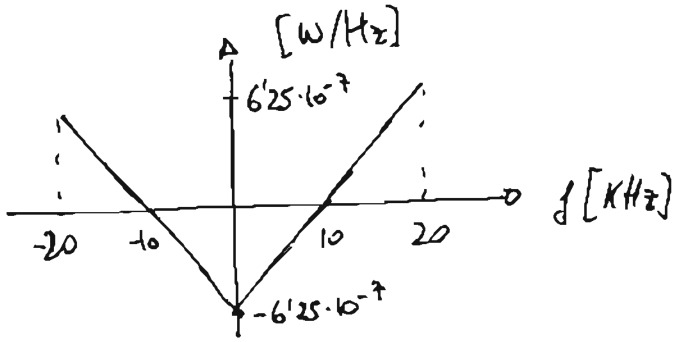
$+\frac{1}{8j}\left[S_{N_cN_s}\left(f+\frac{W}{2}\right) - S_{N_cN_s}\left(f-\frac{W}{2}\right)\right]$ (2ª parte)

1ª parte, cómo los componentes no se solapan se tiene

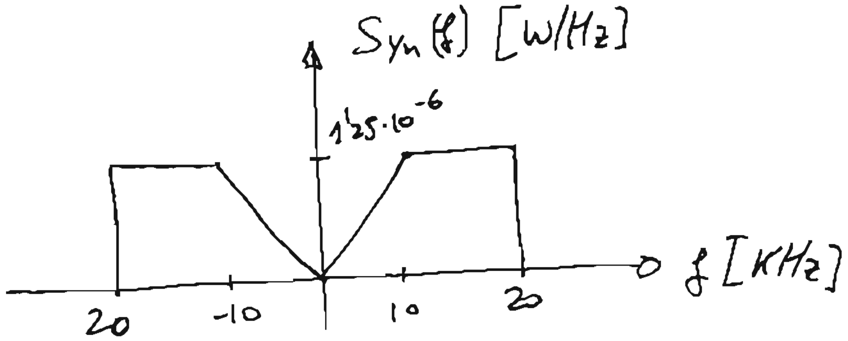


(5)

2ª parte



Sumando:



(h) $P_{y_n} = \int_{-\infty}^{\infty} S_{y_n}(f)\,df = 1'25\cdot10^{-6}\left[10^4 + 10^4 + 2\cdot5\cdot10^3\right]$

$= 1'25\cdot10^{-6}\cdot3\cdot10^4 = 3'75\cdot10^{-2}\ W = 37'5\ mW$

$\boxed{P_{y_n}\,(dBm) = 10\log_{10} 37'5\,mW = 15'74\ dBm}$

(i) $\boxed{SNR} = P_{y_s}\,(dBm) - P_{y_n}\,(dBm) = 53\ dBm - 15'74\ dBm = \boxed{37'26\ dB}$