PROBLEMA 1.

(a)
$$\boxed{a_0} = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g(t)\,dt \underset{\substack{\uparrow \\ \text{por ser} \\ \text{simétrica}}}{=} \frac{2}{T_0}\int_0^T A e^{-at}\,dt = \frac{2A}{T_0}\left[\frac{e^{-at}}{-a}\right]_0^T = \boxed{\frac{2A(1-e^{-aT})}{a T_0}}$$

(b) puesto que la señal es par, el coeficiente $b_n = 0$. $f_0 = \frac{1}{T_0}$

$$a_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g(t)\cos(2\pi n f_0 t)\,dt \underset{\substack{\uparrow \\ \text{por ser} \\ \text{el integrando par}}}{=} \frac{2}{T_0}\int_0^T A e^{-at}\cos(2\pi n f_0 t)\,dt$$

$$= \frac{\cancel{2}A}{T_0}\int_0^T e^{-at}\,\frac{e^{j2\pi n f_0 t} + e^{-j2\pi n f_0 t}}{\cancel{2}}\,dt = \frac{A}{T_0}\int_0^T\left[e^{(-a+j2\pi n f_0)t} + e^{-(a+j2\pi n f_0)t}\right]dt$$

$$= \frac{A}{T_0}\left[\frac{e^{(-a+j2\pi n f_0)t}}{-a+j2\pi n f_0} + \frac{e^{-(a+j2\pi n f_0)t}}{-a-j2\pi n f_0}\right]_0^T = \frac{A}{T_0}\left[\frac{1-e^{-(a-j2\pi n f_0)T}}{a-j2\pi n f_0} + \frac{1-e^{-(a+j2\pi n f_0)T}}{a+j2\pi n f_0}\right]$$

$$= \frac{A}{T_0}\left[\frac{a+\cancel{j2\pi n f_0} - (a+j2\pi n f_0)e^{-aT}e^{j2\pi n f_0 T} + a - \cancel{j2\pi n f_0} - (a-j2\pi n f_0)e^{-aT}e^{-j2\pi n f_0 T}}{a^2+4\pi^2 n^2 f_0^2}\right]$$

$$= \frac{A}{T_0}\left[\frac{2a - 2ae^{-aT}\cos(2\pi n f_0 T) + 4\pi n f_0 e^{-aT}\operatorname{sen}(2\pi n f_0 T)}{a^2+4\pi^2 n^2 f_0^2}\right]$$

$$= \frac{2A}{T_0}\left[\frac{a - ae^{-aT}\cos(2\pi n f_0 T) + 2\pi n f_0 e^{-aT}\operatorname{sen}(2\pi n f_0 T)}{a^2+4\pi^2 n^2 f_0^2}\right]$$

(c)
$$g_p(t) = a_0 + 2\sum_{n=1}^{\infty} a_n\cos(2\pi n f_0 t) + b_n \operatorname{sen}(2\pi n f_0 t)$$

$$= \frac{2A(1-e^{-aT})}{aT_0} + \frac{4A}{T_0}\sum_{n=1}^{\infty}\left(\frac{a - ae^{-aT}\cos(2\pi n f_0 T) + 2\pi n f_0 e^{-aT}\operatorname{sen}(2\pi n f_0 T)}{a^2+4\pi^2 n^2 f_0^2}\right)\cos(2\pi n f_0 t)$$

(d)
$$G_p(f) = \frac{2A(1-e^{-aT})}{aT_0}\delta(f) + \frac{2A}{T_0}\sum_{n=1}^{\infty}\frac{a - ae^{-aT}\cos(2\pi n f_0 T) + 2\pi n f_0 e^{-aT}\operatorname{sen}(2\pi n f_0 T)}{a^2+4\pi^2 n^2 f_0^2}\left[\delta(f-nf_0)+\delta(f+nf_0)\right]$$

(e) $$\boxed{P} = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_p^2(t)\,dt \underset{\substack{\uparrow \\ \text{por ser} \\ \text{par}}}{=} \frac{2}{T_0}\int_0^T A^2 e^{-2at}\,dt = \frac{2A^2}{T_0}\left[\frac{e^{-2at}}{-2a}\right]_0^T$$

$$= \boxed{\frac{A^2\left(1-e^{-2aT}\right)}{a\,T_0}}$$

(f) $$S_{gp}(f) = \frac{4A^2(1-e^{-aT})^2}{a^2T_0^2}\delta(f) + \frac{4A^2}{T_0^2}\sum_{n=1}^{\infty}\left(\frac{a-ae^{-aT}\cos(2\pi nf_0T)+2\pi nf_0e^{-aT}\operatorname{sen}(2\pi nf_0T)}{a^2+4\pi^2n^2f_0^2}\right)^2\left[\delta(f-nf_0)+\delta(f+nf_0)\right]$$

(g) El segundo armónico tiene coeficiente $2a_2$ con

$$a_2 = \frac{2A}{T_0}\left[\frac{a-ae^{-aT}\cos(4\pi f_0T)+4\pi f_0e^{-aT}\operatorname{sen}(4\pi f_0T)}{a^2+16\pi^2f_0^2}\right]$$

el término es:

$$2a_2\cos(4\pi f_0 t)$$

tiene una potencia

$$P_2 = (2a_2)^2\cdot\frac{1}{2} = 4a_2^2\cdot\frac{1}{2} = 2a_2^2$$

$$P_2 = \frac{8A^2}{T_0^2}\left[\frac{a-ae^{-aT}\cos(4\pi f_0T)+4\pi f_0e^{-aT}\operatorname{sen}(4\pi f_0T)}{a^2+16\pi^2f_0^2}\right]^2$$

$$\%\,pot = \frac{P_2}{P}\cdot 100 = \frac{\frac{8\cancel{A^2}}{T_0^{\cancel{2}}}\left[\frac{a-ae^{-aT}\cos(4\pi f_0T)+4\pi f_0e^{-aT}\operatorname{sen}(4\pi f_0T)}{a^2+16\pi^2f_0^2}\right]^2}{\frac{\cancel{A^2}(1-e^{-2aT})}{T_0\cdot a}}\,100$$

$$=\left(\frac{800\,a}{T_0}\right)\left(\frac{\left[a-ae^{-aT}\cos(4\pi f_0T)+4\pi f_0e^{-aT}\operatorname{sen}(4\pi f_0T)\right]^2}{(a^2+16\pi^2f_0^2)^2\,(1-e^{-2aT})}\right)$$

PROBLEMA 2

(a) $$\boxed{P_X = \int_{-\infty}^{\infty} S_X(f)\,df = \frac{2 \cdot 3000 \cdot 10^{-4}}{2} = 300\,mW \equiv 24{,}77\,dBm}$$

$$K_a\,|m(t)|_{max} = \mu \qquad \boxed{K_a = \frac{\mu}{|m(t)|_{max}} = \frac{0.45}{3} = 0{,}15}$$

(b) El filtro IF no modifica la componente de señal modulada, puesto que tiene un ancho de banda de 6KHz y entonces pasa por la zona plana del filtro IF.

$$P_R = \frac{P_T}{Att} \qquad Att = 10^{\frac{75}{10}} = 3'1623 \cdot 10^7$$

$$\boxed{P_R = \frac{500\,W}{3{,}1623 \cdot 10^7} = 0{,}0158\,mW \equiv -18{,}01\,dBm}$$

$$S_N(f) = S_W(f)\,|H_{IF}(f)|^2 = \frac{N_0}{2}\,|H_{IF}(f)|^2$$

$$\boxed{P_N = N_0 \cdot \left(\frac{4000}{2} + 6000\right) = 8000\,N_0 = 8000 \cdot 2 \cdot 10^{-14} = 1{,}6 \cdot 10^{-7}\,mW = -67{,}96\,dBm}$$

$$\boxed{SNR_I = P_R\,(dBm) - P_N\,(dBm) = -18{,}01 + 67{,}96 = 49{,}95\,dB}$$

(c) A la entrada del modulador producto:

$$s(t) = A_c\left[1 + K_a\,\cancel{x}(t)\right]\cos(2\pi f_c t) + n_c(t)\cos(2\pi f_c t) - n_s(t)\,sen(2\pi f_c t)$$

A la salida del modulador producto:

$$v(t) = 2\cos(2\pi f_c t)\cdot s(t) = 2A_c\left[1 + K_a\,\cancel{x}(t)\right]\cos^2(2\pi f_c t) + 2n_c(t)\cos^2(2\pi f_c t)\,-$$

$$-2n_s(t)\cos(2\pi f_c t)\,sen(2\pi f_c t)$$

$$= A_c\left[1+K_a x(t)\right]\left[1+\cos(4\pi f_c t)\right] + n_c(t)\left[1+\cos(4\pi f_c t)\right] - n_s(t)\,\mathrm{sen}(4\pi f_c t)$$

tras el filtro paso bajo a 4KHz:

$$y(t) = A_c + A_c K_a x(t) + n_o(t)$$

siendo $n_o(t)$ la salida del filtro tras meter a la entrada $n_c(t)$

$$n_c(t) \longrightarrow \underset{\text{PASO BAJO}}{\boxed{4\text{KHz}}} \longrightarrow n_o(t)$$

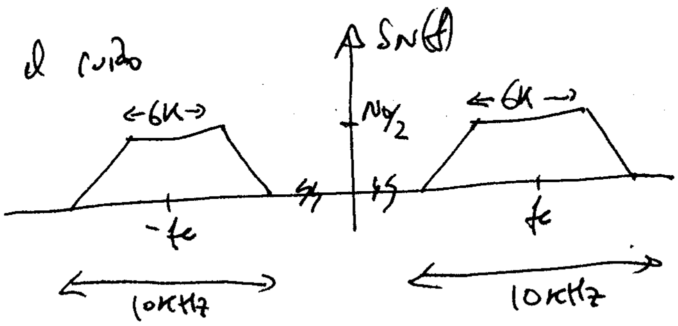
$P_{so} = A_c^2 K_a^2 \cdot P_x$ Nos falta saber A_c, pero sabemos que a la entrada:

$$P_R = \frac{A_c^2}{2}(1+K_a^2 P_x) \Rightarrow A_c^2 = \frac{2P_R}{1+K_a^2 P_x}$$

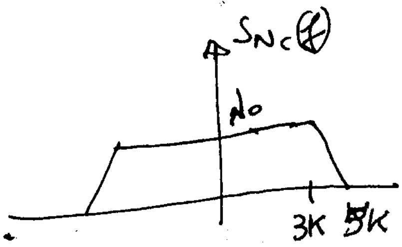
$$\boxed{P_{so}} = \frac{2P_R K_a^2 P_x}{1+K_a^2 P_x} = \frac{2\cdot 0{,}0158\,\text{mW}\cdot 0{,}15^2\cdot 0{,}3}{1+0{,}15^2\cdot 0{,}3} = 2{,}118\cdot 10^{-4}\,\text{mW}$$

$$= \boxed{-36{,}74\ \text{dBm}}$$

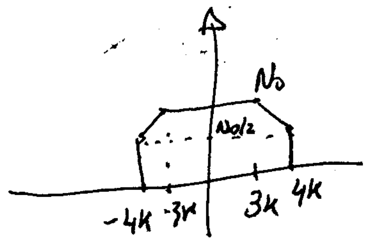
Para el ruido



$$S_{Nc}(f) = \begin{cases} S_N(f-f_c)+S_N(f+f_c) & |f|\le 5000 \\ 0 & \text{resto}\end{cases}$$



tras filtro paso bajo



$$P_{No} = \int_{-4000}^{4000} S_{No}(f)\,df = \frac{N_o}{2}\cdot 8000 + \frac{N_o}{2}(6000+1000) = 7500\,N_o = 1{,}5\cdot 10^{-7} = -68{,}24\ \text{dBm}$$

PROBLEMA 3 (CONT)

$$SNR_o = P_{So}(dBm) - P_{No}(dBm) = -36,74\,dBm + 68,24\,dBm = 31,5\ dB$$

(d) Supuesto banda lateral superior

ocupación de la señal f_c a $f_c + 3000$, por lo que el filtro IF no modifica la señal

$\boxed{P_R = -18,01\,dBm}$ $\boxed{P_N = -67,96\,dBm}$ (Igual para el ruido ya que el filtro es el mismo)

$\boxed{SNR_I = 49,95\,dB}$

Veamos a la salida:

$$s(t) = \frac{A_c}{2} x(t)\cos(2\pi f_c t) - \frac{A_c\,\hat{x}(t)}{2}\,\mathrm{sen}(2\pi f_c t) + n_c(t)\cos(2\pi f_c t) - n_s(t)\,\mathrm{sen}(2\pi f_c t)$$

$$v(t) = 2\cos(2\pi f_c t)\,s(t)$$

$$= \left[\frac{A_c\,x(t)}{2} + n_c(t)\right] 2\cos^2(2\pi f_c t) - \left[\frac{A_c\,\hat{x}(t)}{2} + n_s(t)\right] 2\cos(2\pi f_c t)\,\mathrm{sen}(2\pi f_c t)$$

$$= \left[\frac{A_c\,x(t)}{2} + n_c(t)\right]\left[1 + \cos(4\pi f_c t)\right] - \left[\frac{A_c\,\hat{x}(t)}{2} + n_s(t)\right]\mathrm{sen}(4\pi f_c t)$$

$$y(t) = \frac{A_c\,x(t)}{2} + n_o(t)$$, con $n_c(t) \xrightarrow{\ \boxed{F.P.B.}\ (4\,KHz)\ } n_o(t)$

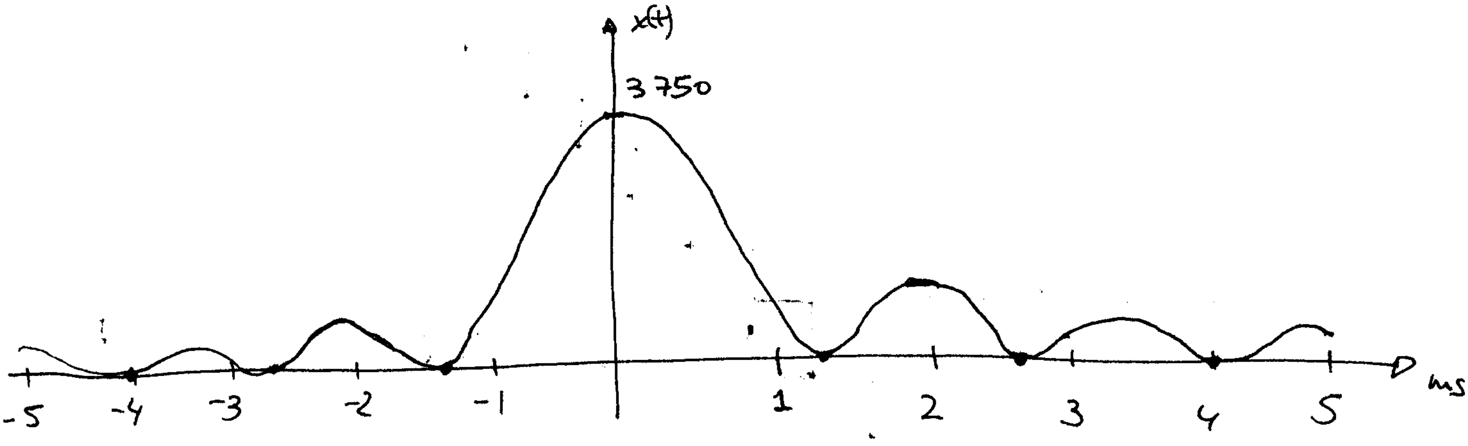
$\boxed{P_{No} = -68,24\,dBm}$ (No cambia)

$$P_{So} = \frac{A_c^2\,P_x}{4}$$ para calcular $A_c \Rightarrow P_R = \dfrac{A_c^2\,P_x}{4} \Rightarrow A_c^2 = \dfrac{4P_R}{P_x}$

$\boxed{P_{So} = P_R = -18,01\,dBm}$ $\boxed{SNR_o = P_{So}(dBm) - P_{No}(dBm) = -18,01 + 68,24 = 50,23\ dB}$

PROBLEMA 3

(a) $X(f) = 5\,\Lambda\left(\frac{f}{750}\right) \rightleftharpoons x(t) = 5 \cdot 750 \cdot \text{sinc}^2(750t) = 3750\,\text{sinc}^2(750t)$



ceros de la función $\frac{n}{750} <^{1'333}_{2'666}$

(b) $W = 750\ Hz \Rightarrow f_s = 1'3 \cdot 2W = 2'6 \cdot 750 = 1950\ Hz$

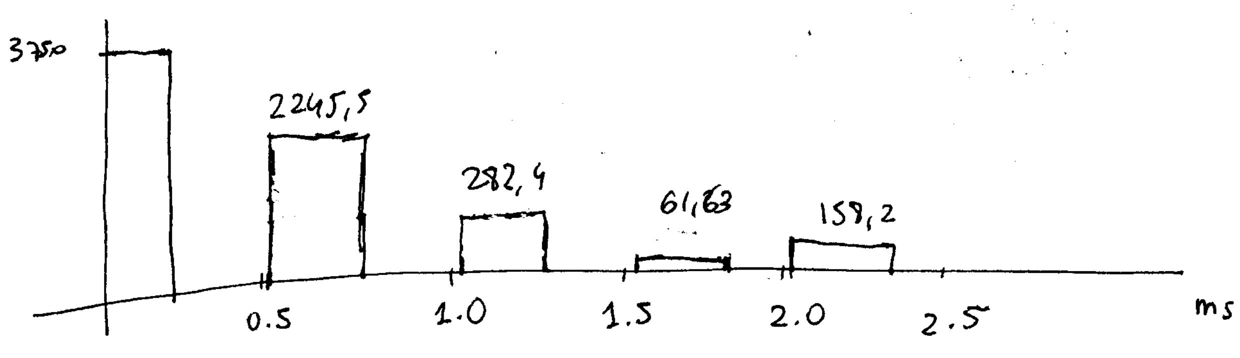
(c) $t = 0 \qquad x(t) = 3750$

$t = T_s = 0'5128\,ms \qquad x(t) = 3750\left(\frac{\text{sen}(750 \cdot \pi \cdot 0'5128 \cdot 10^{-3})}{750 \cdot \pi \cdot 0'5128 \cdot 10^{-3}}\right)^2 = 2245,5$

$t = 2T_s = 1'0256\,ms \qquad x(t) = 3750\left(\frac{\text{sen}(750 \cdot \pi \cdot 1'0256 \cdot 10^{-3})}{750 \cdot \pi \cdot 1'0256 \cdot 10^{-3}}\right)^2 = 282,4$

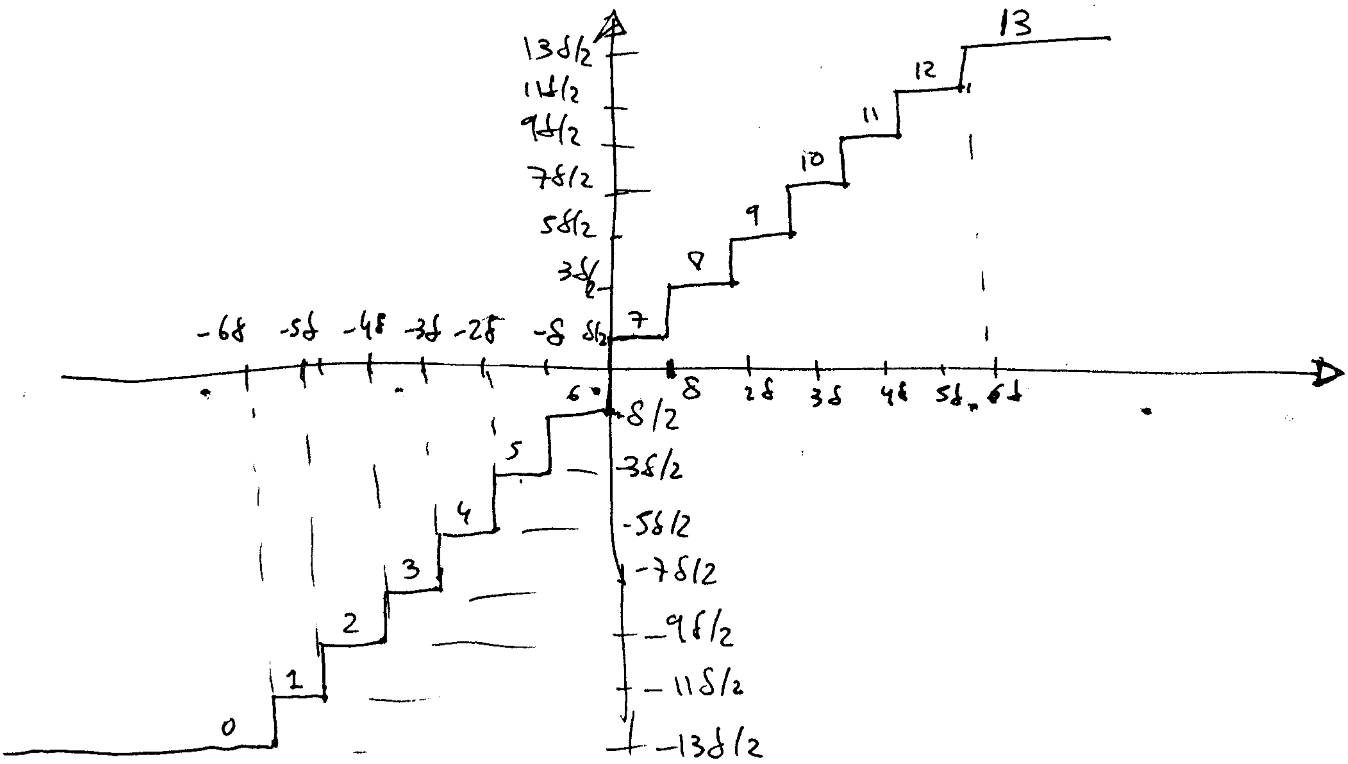
$t = 3T_s = 1'538\,ms \qquad x(t) = 3750\left(\frac{\text{sen}(750 \cdot \pi \cdot 1'538 \cdot 10^{-3})}{750 \cdot \pi \cdot 1'538 \cdot 10^{-3}}\right)^2 = 61'63$

$t = 4T_s = 2'051\,ms \qquad x(t) = 3750\left(\frac{\text{sen}(750 \cdot \pi \cdot 2,051 \cdot 10^{-3})}{750 \cdot \pi \cdot 2,051 \cdot 10^{-3}}\right)^2 = 158,2$



d) $A_{max} = 3{,}6 \cdot 3750 = 13500$

$$\delta = \frac{2 \cdot A_{max}}{L} = \frac{2 \cdot 13500}{14} = 1928{,}57$$



Nº escalón	Int. entrada	Valor salida
0	$[-\infty, -11571{,}42]$	$-12535{,}71$
1	$[-11571{,}42, -9642{,}85]$	$-10607{,}14$
2	$[-9642{,}85, -7714{,}28]$	$-8678{,}57$
3	$[-7714{,}28, -5785{,}71]$	$-6750{,}00$
4	$[-5785{,}71, -3857{,}14]$	$-4821{,}43$
5	$[-3857{,}14, -1928{,}57$	$-2892{,}86$
6	$[-1928{,}57, 0]$	$-964{,}29$
7	$[0, 1928{,}57]$	$964{,}29$
8	$[1928{,}57, 3857{,}14]$	$2892{,}86$
9	$[3857{,}14, 5785{,}71]$	$4821{,}43$
10	$[5785{,}71, 7714{,}28]$	$6750{,}00$
11	$[7714{,}28, 9642{,}85$	$8678{,}57$
12	$[9642{,}85, 11571{,}42]$	$10607{,}14$
13	$[11571{,}42, \infty]$	$12535{,}71$

PROBLEMA 3 (CONT)

$x(0) = 3750 \quad \hat{x}(0) = 2892{,}86 \quad (8)$

$x(T_s) = 2245{,}5 \quad \hat{x}(T_s) = 2892{,}86 \quad (8)$

$x(2T_s) = 282{,}4 \quad \hat{x}(2T_s) = 964{,}29 \quad (7)$

$x(3T_s) = 61{,}63 \quad \hat{x}(3T_s) = 964{,}29 \quad (7)$

$x(4T_s) = 158{,}2 \quad \hat{x}(4T_s) = 964{,}29 \quad (7)$

(e)

$$\boxed{SQNR} = \frac{\frac{1}{5}\left[3750^2 + 2245{,}5^2 + 282{,}4^2 + 61{,}63^2 + 158{,}2^2\right]}{\frac{1}{5}\left[(3750-2892{,}86)^2 + (2245{,}5-2892{,}86)^2 + (282{,}4-964{,}29)^2 + \dots\right]}$$

$$= 6{,}2314 \equiv \boxed{7{,}946 \text{ dB}}$$

(f)

8 8 7 7 7

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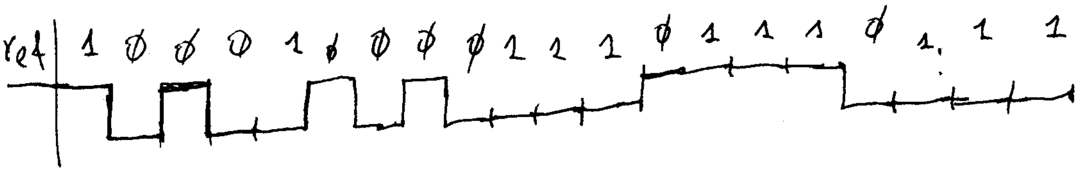


(g) para la ley $\mu \Rightarrow \quad |v_2| = \dfrac{\ln(1+\mu|v_1|)}{\ln(1+\mu)}$ y $|v_1| = \dfrac{e^{|v_2|\ln(1+\mu)} - 1}{\mu}$

para $\mu = 250 \quad |v_2| = \dfrac{\ln(1+250|v_1|)}{\ln(251)}$

primero hay que normalizar la entrada respecto a $A_{max} = 13500$

$\dfrac{x(0)}{A_{max}} = 0{,}2778 = |v_1|$

$\dfrac{x(T_s)}{A_{max}} = 0{,}16633 = |v_1|$

$\dfrac{x(2T_s)}{A_{max}} = 0{,}0209 = |v_1|$

$\dfrac{x(3T_s)}{A_{max}} = 0{,}004565 = |v_1|$

$\dfrac{x(4T_s)}{A_{max}} = 0{,}01172 = |v_1|$

Ahora $|v_2| = \dfrac{\ln(1+250|v_1|)}{\ln(251)}$

$t=0$	0,77
$t=T_s$	0,679
$t=2T_s$	0,3311
$t=3T_s$	0,1378
$t=4T_s$	0,2477

multiplicamos por Δmax

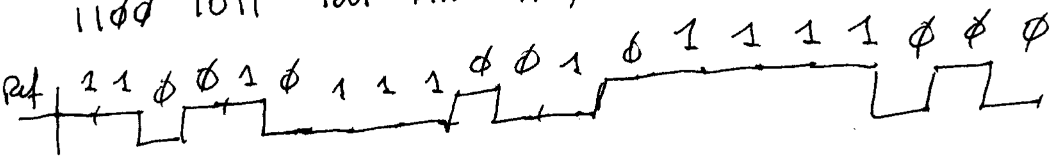
$t=0$	10395
$t=T_s$	9166,5
$t=2T_s$	4469,85
$t=3T_s$	1860,3
$t=4T_s$	3343,95

tras el cuantificador		Dividimos Δmax	expansor	Mult Δmax
$\hat{x}[0] = 10607,14$	(12)	0,7857	0,3033	4094,55
$\hat{x}(T_s) = 8678,57$	(11)	0,6429	0,1355	1829,25
$\hat{x}(2T_s) = 4821,43$	(9)	0,3571	0,0248	334,8
$\hat{x}(3T_s) = 964,29$	(7)	0,0714	0,0019	25,65
$\hat{x}(4T_s) = 2892,86$	(8)	0,2143	0,0091	122,85

$$\boxed{SQNR = \frac{1/5\left(3750^2 + 2245'5^2 + 282'4^2 + 61'63^2 + (158'2)^2\right)}{1/5\left((4094,55-3750)^2 + (2245'5-1829'25)^2 + (282,4-334,8)^2 + \dots\right)}} = 64,8052$$

$$\boxed{= 18,11 dB}$$

12	11	9	7	8
1100	1011	1001	0111	1000



GANANCIA EN dB ≡ 10,17 dB

puesto que la señal $x(t)$ se puede considerar débil ($\Delta max = 3,6 \cdot max(x(t))$) el introducir compansión da lugar a una ganancia de compansión.