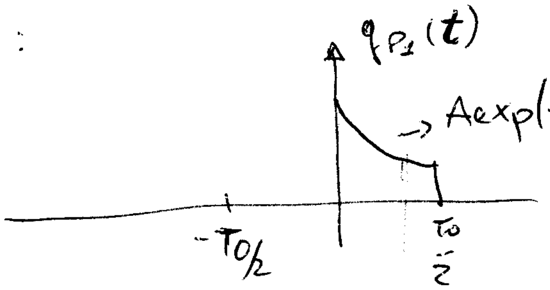
## PROBLEMA 1

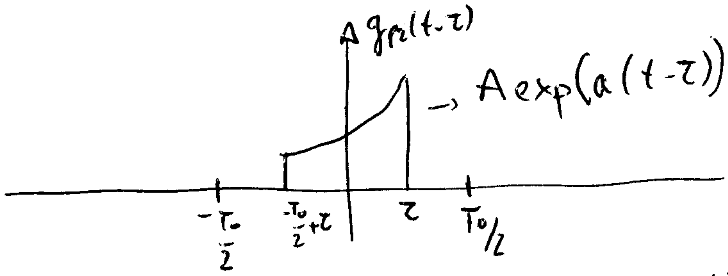
Para unas señales periódicas ambas con periodo $T_0$:

$$R_{12}(\tau) = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_{p1}(t)\, g_{p2}^{*}(t-\tau)\,dt$$

Sabemos por las propiedades que $R_{12}(\tau)$ va a ser periódica también con periodo $T_0$, por lo que basta con determinar su valor en el intervalo $-T_0/2 \le \tau \le \frac{T_0}{2}$. Como lo que ocurre para $\tau$ negativo y $\tau$ positivo es diferente vamos a considerar dos casos:

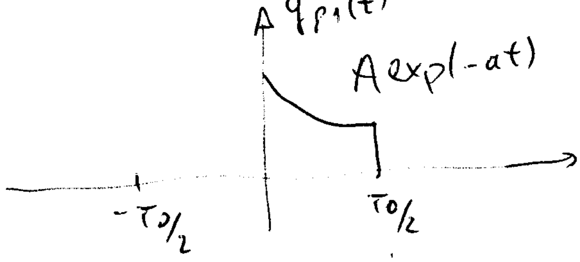
$\frac{T_0}{2} > \tau > 0 \Rightarrow$

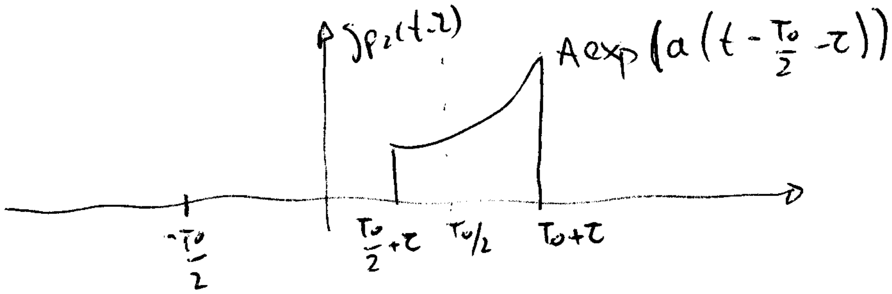



$$\boxed{R_{12}(\tau) = \frac{1}{T_0}\int_0^{\tau} Ae^{-at}\cdot Ae^{at}e^{-a\tau}\,dt = \frac{A^2}{T_0}e^{-a\tau}\int_0^{\tau} dt = \frac{A^2e^{-a\tau}}{T_0}\cdot\tau \quad 0\le\tau\le\frac{T_0}{2}}$$

$-\frac{T_0}{2} < \tau < 0 \Rightarrow$



$$R_{12}(\tau) = \frac{1}{T_0}\int_{T_0/2+\tau}^{T_0/2} A\exp(-at)\, A e^{at} e^{-aT_0} e^{-a\tau}\, dt$$

$$= \frac{A^2}{T_0} e^{-aT_0} e^{-a\tau}\int_{T_0/2+\tau}^{T_0/2} dt = \frac{A^2}{T_0} e^{-aT_0} e^{-a\tau}(-\tau) \qquad -\frac{T_0}{2} \le \tau \le 0.$$

Entonces:

$$\boxed{\begin{aligned} R_{12}(\tau) = &\sum_{n=-\infty}^{\infty} \frac{A^2}{T_0} e^{-a(\tau-nT_0)}(\tau-nT_0)\,\Pi\left(\frac{t-T_0/4-nT_0}{T_0/2}\right) \\ &-\sum_{n=-\infty}^{\infty} \frac{A^2}{T_0} e^{-a(\tau+T_0-nT_0)}(\tau+nT_0)\,\Pi\left(\frac{t+T_0/4-nT_0}{T_0/2}\right)\end{aligned}}$$

Para dibujarlo; basta con dibujar intervalo $-T_0/2 < \tau < \frac{T_0}{2}$ y repetirlo periódicamente:

para $0 \le \tau \le \frac{T_0}{2}$ $\qquad R_{12}(0) = 0$ , $R_{12}\left(\frac{T_0}{2}\right) = \frac{A^2}{T_0} e^{-aT_0/2}\cdot\frac{T_0}{2} = \frac{A^2}{2}e^{-aT_0/2}$

miramos si tenemos puntos singulares (extremos en este intervalo)

$$\frac{dR_{12}(\tau)}{d\tau} = \frac{A^2}{T_0}e^{-a\tau} + \frac{A^2}{T_0}\tau\cdot e^{-a\tau}(-a) = \frac{A^2}{T_0}e^{-a\tau}[1-a\tau] = 0$$

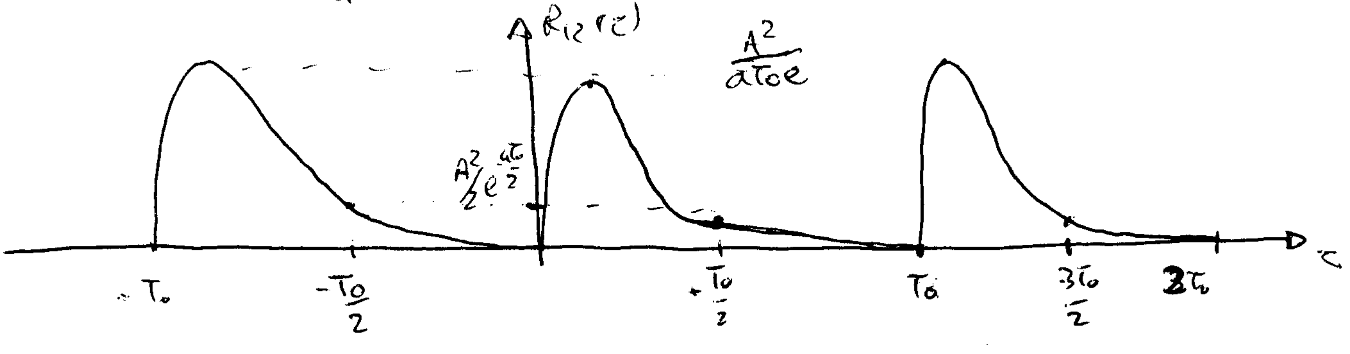
$a\tau = 1 \qquad \boxed{\tau = \frac{1}{a}}$ si $a>0$ y cae en el intervalo

$0 \le \tau \le \frac{T_0}{2}$ , se puede comprobar que es un máximo.

para $-\frac{T_0}{2} \le \tau \le 0 \qquad R_{12}(0) = 0$ , $R_{12}\left(-\frac{T_0}{2}\right) = \frac{A^2}{T_0}e^{-aT_0}e^{aT_0/2}\cdot\frac{T_0}{2} = \frac{A^2}{2}e^{-aT_0/2}$

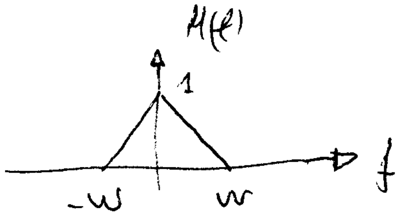
extremos: $\frac{dR_{12}(\tau)}{d\tau} = \frac{A^2}{T_0}e^{-aT_0}(-1)e^{-a\tau} + \frac{A^2}{T_0}e^{-aT_0}(-\tau)e^{-a\tau}(-a) = \frac{A^2}{2}e^{-aT_0}e^{-a\tau}[-1+a\tau]$

$a\tau = 1 \quad \tau = \frac{1}{a}$ cae fuera del intervalo.



PROBLEMA 2:

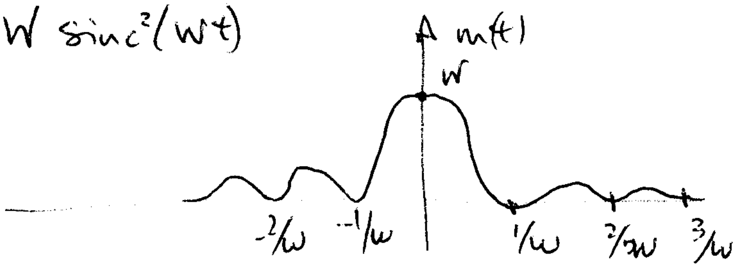
$$M(f) = \Lambda\left(\frac{f}{W}\right)$$




(a)

$$s(t) = A_c\,[\,1 + k_a m(t)]\cos(2\pi f_c t) \quad \text{sin sobremodulación.}$$

$$m(t) = W\,\text{sinc}^2(Wt)$$

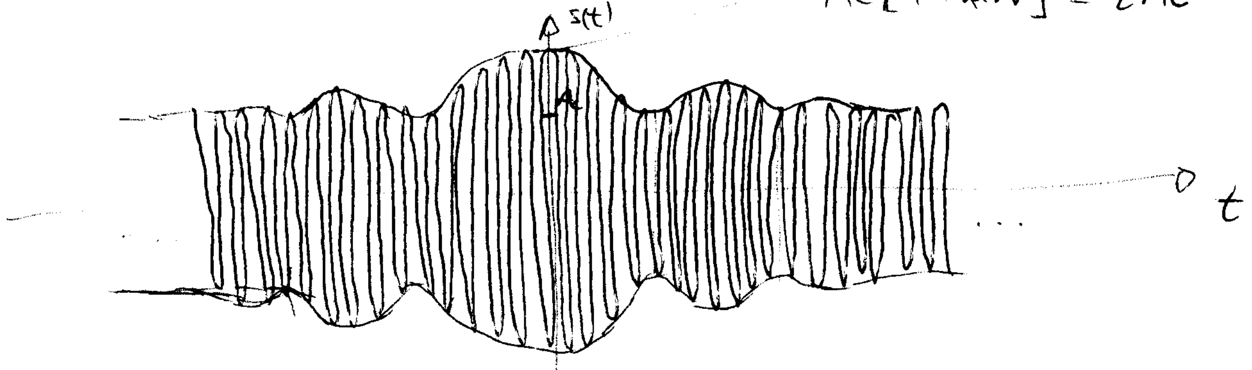



$$\max\{m(t)\} = W$$

$$\boxed{s(t) = A_c[1 + k_a W\,\text{sinc}^2(Wt)]\cos(2\pi f_c t)}$$

Entonces $k_a \cdot W \le 1 \Rightarrow \boxed{k_a \le \frac{1}{W}}$

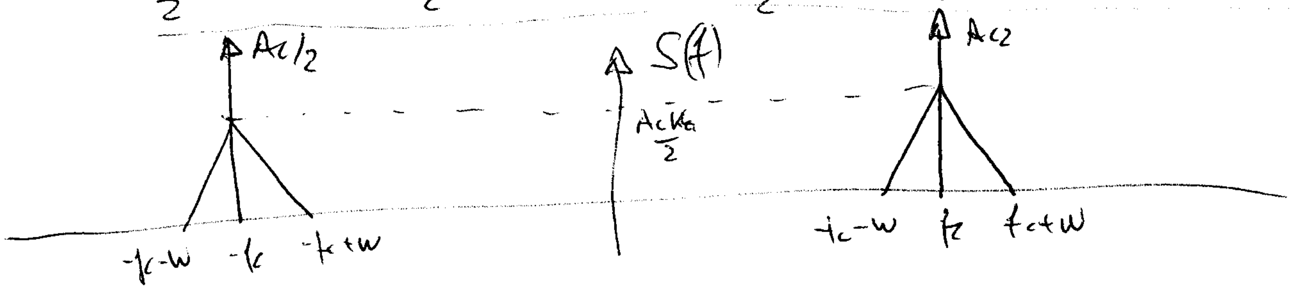
$$A_c[1 + k_a W] \le 2A_c$$



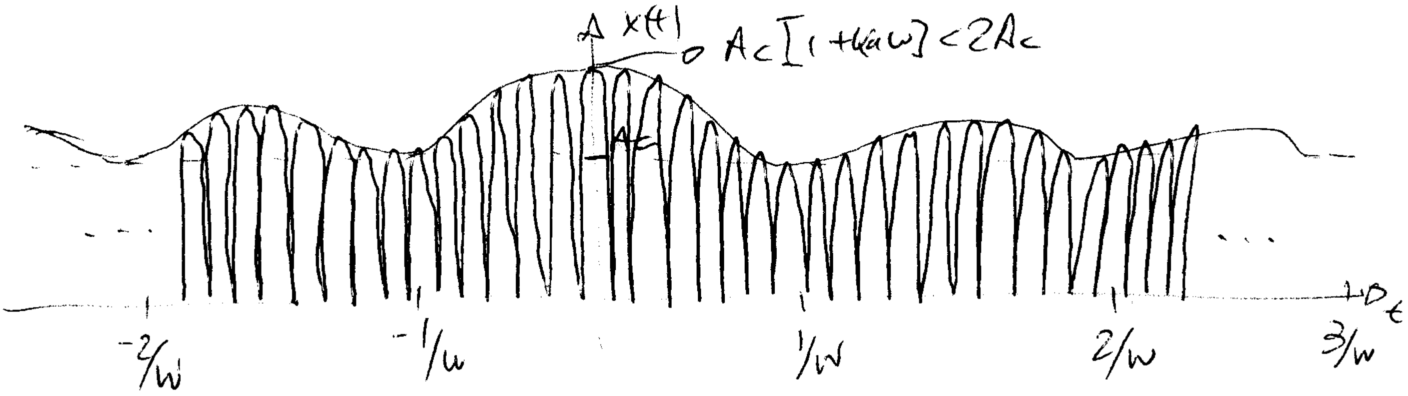

$$\boxed{S(f) = A_c\left[1 + k_a M(f)\right] * \left[\frac{1}{2}\delta(f - f_c) + \frac{1}{2}\delta(f + f_c)\right]} =$$

$$= \frac{A_c}{2}\delta(f - f_c) + \frac{A_c}{2}\delta(f + f_c) + \frac{k_a A_c}{2}M(f - f_c) + \frac{k_a A_c}{2}M(f + f_c)$$

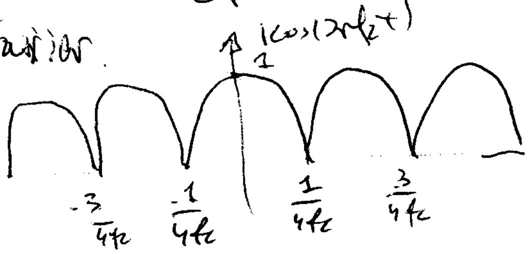
$$= \frac{A_c}{2}\delta(f - f_c) + \frac{A_c}{2}\delta(f + f_c) + \frac{A_c k_a}{2}\Lambda\left(\frac{f - f_c}{W}\right) + \frac{A_c k_a}{2}\Lambda\left(\frac{f + f_c}{W}\right)$$



(b) $x(t) = |s(t)| = \left|A_c[1+k_a m(t)]\cos(2\pi f_c t)\right| = A_c[1+K_a W \operatorname{sinc}^2(Wt)]\,|\cos(2\pi f_c t)|$




(c) El operador |·| es no lineal, por lo que no se puede hacer su TF directamente, pero resulta que $|\cos(2\pi f_c t)|$ es una señal periódica con periodo $\frac{1}{2f_c} = T_0$ (doble frecuencia), podemos hacer su serie de Fourier.




$f_0 = 2f_c$

Como es par los términos $b_n = 0$. Hacemos las cuentas en $-\frac{1}{4f_c} \le t \le \frac{1}{4f_c}$ (periodo centrado en $t=0$)

$$a_0 = 2f_c \int_{-\frac{1}{4f_c}}^{\frac{1}{4f_c}} \cos(2\pi f_c t)\,dt =$$

$$= 2f_c \left[\frac{\operatorname{sen}(2\pi f_c t)}{2\pi f_c}\right]_{-\frac{1}{4f_c}}^{\frac{1}{4f_c}} = \frac{2}{\pi}\operatorname{sen}\left(\frac{\pi}{2}\right) = \boxed{\frac{2}{\pi}}$$

$$a_n = 2f_c \int_{-\frac{1}{4f_c}}^{\frac{1}{4f_c}} \cos(2\pi f_c t)\cos(4\pi f_c n t)\,dt = \frac{2f_c}{2}\int_{-\frac{1}{4f_c}}^{\frac{1}{4f_c}} \left[\cos[2\pi f_c(2n-1)t] + \cos[2\pi f_c(2n+1)t]\right]dt$$

$$= f_c\left[\frac{\operatorname{sen}(2\pi f_c(2n-1)t)}{2\pi f_c(2n-1)}\right]_{-\frac{1}{4f_c}}^{\frac{1}{4f_c}} + f_c\left[\frac{\operatorname{sen}(2\pi f_c(2n+1)t)}{2\pi f_c(2n+1)}\right]_{-\frac{1}{4f_c}}^{\frac{1}{4f_c}}$$

$$= \frac{1}{2\pi(2n-1)}\,2\operatorname{sen}\left(\frac{\pi}{2}(2n-1)\right) + \frac{1}{2\pi(2n+1)}\,2\cdot\operatorname{sen}\left(\frac{\pi}{2}(2n+1)\right)$$

$$= \frac{\left[\operatorname{sen}(\pi n)\cos\left(\frac{\pi}{2}\right) - \cos(\pi n)\operatorname{sen}\frac{\pi}{2}\right](2n+1) + \left[\operatorname{sen}(\pi n)\cos\left(\frac{\pi}{2}\right) + \cos(\pi n)\operatorname{sen}\left(\frac{\pi}{2}\right)\right](2n-1)}{\pi(4n^2-1)}$$

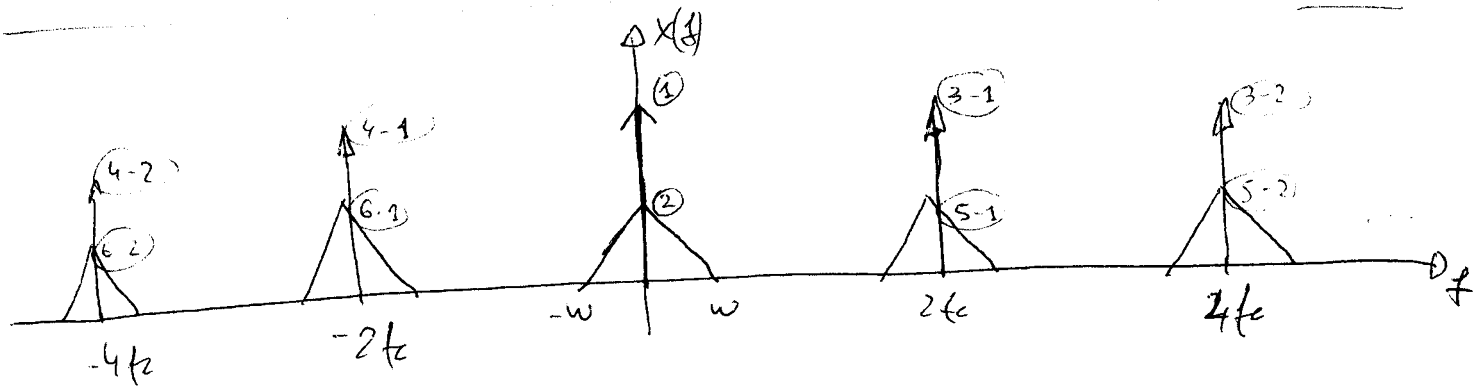
$$\boxed{a_n = \frac{-\cos(\pi n)(2n+1) + \cos(\pi n)(2n-1)}{\pi(4n^2-1)} = \frac{\cos(\pi n)}{\pi(4n^2-1)}\left[2n-1-2n-1\right] = \frac{2(-1)^n}{\pi(1-4n^2)}}$$

$$|\cos(2\pi f_c t)| = \frac{2}{\pi} + 2\sum_{n=1}^{\infty} a_n \cos(4\pi f_c n t) = \frac{2}{\pi} + \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{(1-4n^2)}\cos(4\pi f_c n t)$$

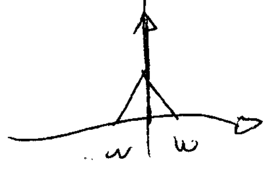
$$\boxed{x(t) = A_c\left[1 + k_a W \operatorname{sinc}^2(Wt)\right]\frac{2}{\pi} + \frac{A_c 4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{(1-4n^2)}\left(1+k_a W \operatorname{sinc}^2(Wt)\right)\cos(4\pi f_c n t)}$$

Ahora podemos hacer la TF:

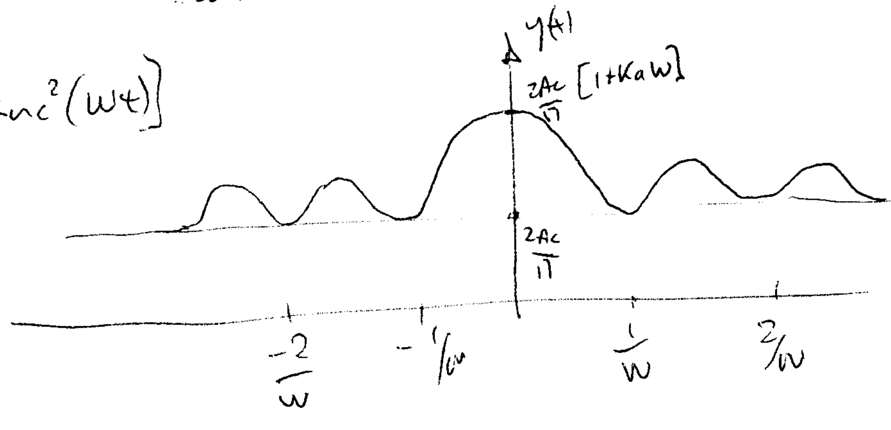
$$\boxed{\begin{aligned} X(f) &= \underbrace{\frac{2A_c}{\pi}\delta(f)}_{1} + \underbrace{\frac{2A_c k_a}{\pi}\Lambda\left(\frac{f}{W}\right)}_{2} + \frac{2A_c}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{(1-4n^2)}\left[\underbrace{\delta(f-2nf_c)}_{3n} + \underbrace{\delta(f+2nf_c)}_{4n}\right] \\ &+ \frac{2A_c k_a}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{(1-4n^2)}\left[\underbrace{\Lambda\left(\frac{f-2nf_c}{W}\right)}_{5\cdot n} + \underbrace{\Lambda\left(\frac{f+2nf_c}{W}\right)}_{6\cdot n}\right]\end{aligned}}$$




(d) Con un filtro pasa bajo con f-corte = W solo pasarían los términos 1 y 2 $\Rightarrow$ $Y(f) = \frac{2A_c}{\pi}\delta(f) + \frac{2A_c k_a}{\pi}\Lambda\left(\frac{f}{W}\right)$



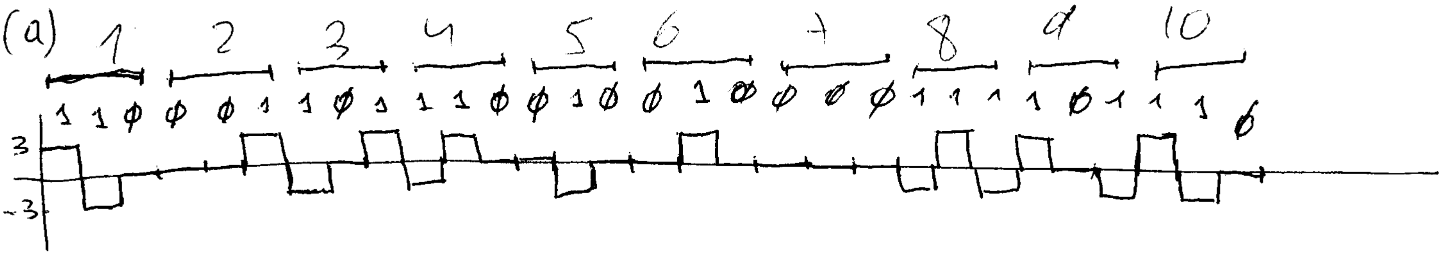

$$y(t) = \frac{2A_c}{\pi}\left[1 + k_a W \operatorname{sinc}^2(Wt)\right]$$




Faltaría eliminar la componente dc.

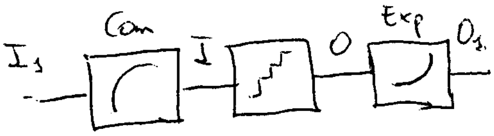
PROBLEMA 3

(a)

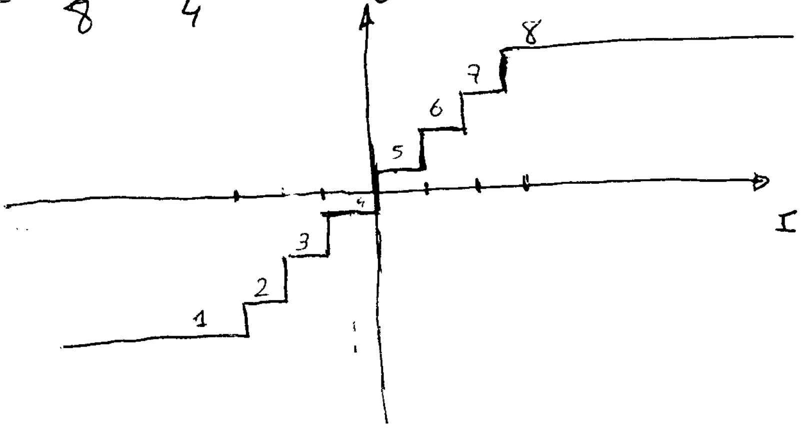



(b) ley $\mu$. $\mu = 10$ $n = 3$ bit $\Rightarrow L = 8$ niveles. $A_{max} = 5$

Comenzamos dibujando el uniforme.




$$\delta = \frac{2 \cdot A_{max}}{L} = \frac{2 \cdot 5}{8} = \frac{5}{4} = 1.25$$




Los valores los ponemos en una tabla

| Nº esc | I | O |
|---|---|---|
| 1 | [-∞, -3'75] | -4'375 |
| 2 | [-3,75, -2'5) | -3'125 |
| 3 | [-2'5, -1'25] | -1'875 |
| 4 | [-1'25, 0] | -0'625 |
| 5 | [0, 1'25] | 0'625 |
| 6 | [1'25, 2'5] | 1'875 |
| 7 | [2'5, 3'75] | 3'125 |
| 8 | [3'75, ∞] | 4'375 |

Ley $\mu$:

$$|v_2| = \frac{\ln(1 + \mu |v_1|)}{\ln(1+\mu)}$$

Compresor delante de cuantificador uniforme.

Entonces las entradas se transforman según:

$$\frac{I}{A_{max}} = \frac{\ln\left(1+\mu \frac{I_1}{A_{max}}\right)}{\ln(1+\mu)}$$ despejando $I_1$:

$$\frac{I \cdot \ln(1+\mu)}{A_{max}} = \ln\left(1+\mu \frac{I_1}{A_{max}}\right)$$

$$1+\mu \frac{I_1}{A_{max}} = \exp\left(I \frac{\ln(1+\mu)}{A_{max}}\right)$$

$$I_1 = \left[\exp\left(I \frac{\ln(1+\mu)}{A_{max}}\right) - 1\right] \frac{A_{max}}{\mu}$$

siendo $I$ los valores de los intervalos de entrada:

1'25, 2'5, 3'75 (los negativos son iguales cambiando de signo).

$$I_1 = \left[\exp\left(I \cdot \frac{\ln 11}{5}\right) - 1\right] \cdot \frac{1}{2}$$

$I_1$ toma los valores: 0.4106, 1.1583 y 2.5201.

Vamos ahora a la parte del expansor:

$$\frac{O}{A_{max}} = \frac{\ln\left(1+\mu \frac{O_1}{A_{max}}\right)}{\ln(1+\mu)}$$ despejando $O_1$ (lo mismo que antes)

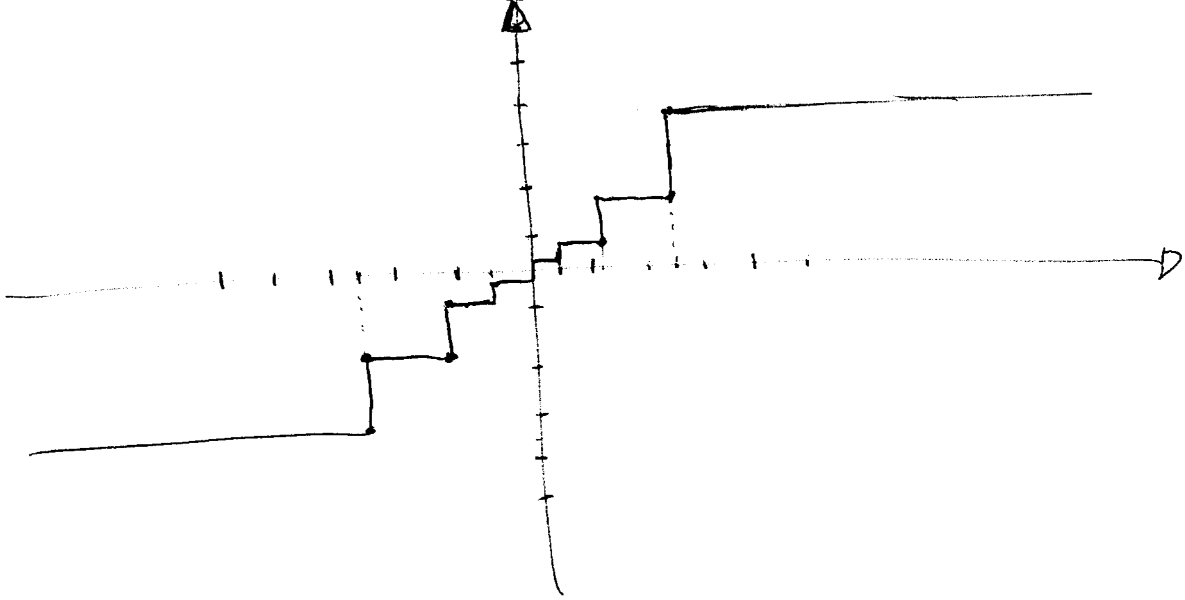
$$O_1 = \left[\exp\left(O \frac{\ln 11}{5}\right) - 1\right] \frac{1}{2}$$

Para $O$: 0.625, 1.875, 3.125, 4.375

$O_1$ toma los valores: 0.1748, 0.7288, 1.7379 3.5756

La tabla queda ahora:

| Nº ESC | $I_1$ | $O_1$ |
|---|---|---|
| 1 | [−∞, −2.5201] | −3.5756 |
| 2 | [−2.5201, −1.1583] | −1.7379 |
| 3 | [−1.1583, −0.4106] | −0.7288 |
| 4 | [−0.4106, 0) | −0.1748 |
| 5 | [0, 0.4106] | 0.1748 |
| 6 | [0.4106, 1.1583] | 0.7288 |
| 7 | [1.1583, 2.5201] | 1.7379 |
| 8 | [2.5201, ∞] | 3.5756 |



(c)

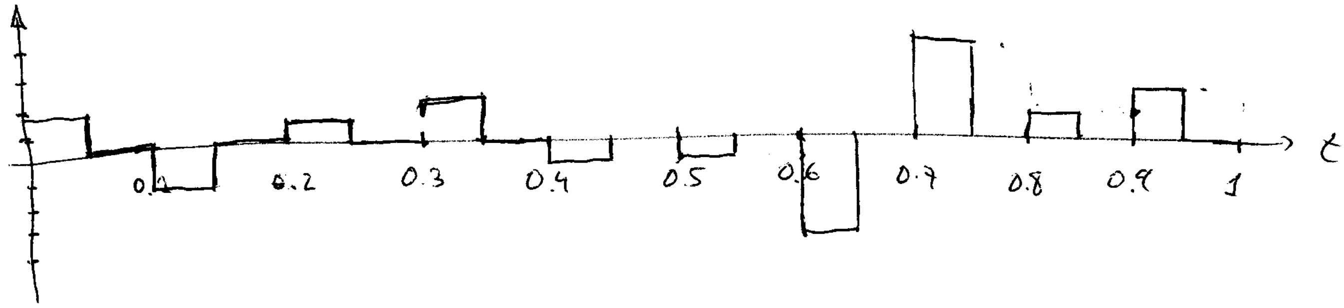
| Código | valor salida |
|---|---|
| 000 | -3.5756 |
| 001 | -1.7379 |
| 010 | -0.7288 |
| 011 | -0.1748 |
| 100 | 0.1748 |
| 101 | 0.7288 |
| 110 | 1.7379 |
| 111 | 3.5756 |

30 bits, cogiendolos de 3 en 3

habrá 10 muestras: (Niveles a Volt)

1.7379, -1.7379, 0.7288, 1.7379, -0.7288

-0.7288, -3.5756, 3.5756, 0.7288, 1.7379




(d) $$\frac{d|V_1|}{d|V_2|} = \frac{\ln(1+\mu)}{\mu}\left(1+\mu|V_1|\right)$$

señales débiles $|V_1| = 0 \Rightarrow \boxed{\text{Ganancia} = \frac{\mu}{\ln(1+\mu)} = 4.1703 = 12.4\,\text{dB}}$

señales fuertes $|V_1| = 1 \Rightarrow \boxed{\text{Pérdida}: \frac{\mu}{\ln(1+\mu)(1+\mu)} = 0.3791 = -8.4\,\text{dB}}$