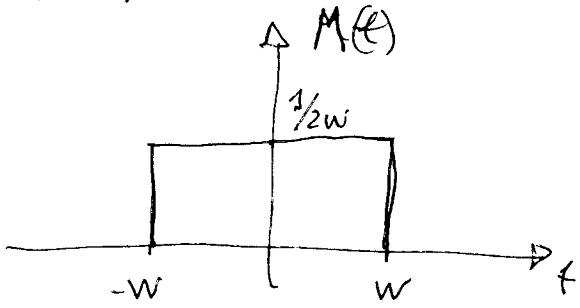
UNIVERSIDAD DE VALLADOLID
DPTO. TEORÍA DE LA SEÑAL Y COMUNICACIONES E INGENIERIA TELEMÁTICA

ASIGNATURA		FECHA		
APELLIDOS		CURSO		CALIFICACIÓN
NOMBRE		GRUPO		

1

a) $m(t) = \text{sinc}(2Wt)$ $\qquad c(t) = A_c \cos(2\pi f_c t)$

$$M(f) = \frac{1}{2W}\Pi\left(\frac{f}{2W}\right)$$

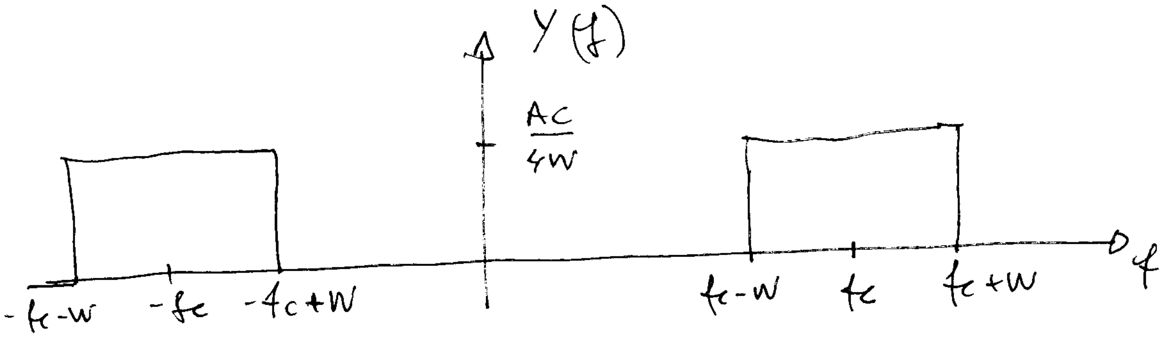


b) $y(t) = m(t)\cdot c(t) = A_c \text{sinc}(2Wt)\cos(2\pi f_c t)$

$$C(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]$$

$$Y(f) = M(f) * C(f) = \frac{A_c}{2}\frac{1}{2W}\Pi\left(\frac{f}{2W}\right) * \left[\delta(f-f_c) + \delta(f+f_c)\right]$$

$$= \frac{A_c}{4W}\left[\Pi\left(\frac{f-f_c}{2W}\right) + \Pi\left(\frac{f+f_c}{2W}\right)\right]$$



c) $z(t) = y^2(t) = m^2(t)\, c^2(t) = \mathrm{sinc}^2(2Wt)\, A_c^2 \cos^2(2\pi f_c t)$

$$z(t) = A_c^2\, \mathrm{sinc}^2(2Wt)\left[\frac{1}{2} + \frac{1}{2}\cos[4\pi f_c t]\right]$$

$$z(t) = \frac{A_c^2}{2}\,\mathrm{sinc}^2(2Wt) + \frac{A_c^2}{2}\,\mathrm{sinc}^2(2Wt)\cos(4\pi f_c t)$$

$\Lambda\left(\frac{t}{T}\right) \rightleftharpoons T\,\mathrm{sinc}^2(fT)$ Aplicando dualidad y para $\boxed{T=2W}$

$2W\,\mathrm{sinc}^2(2Wt) \rightleftharpoons \Lambda\left(\frac{f}{2W}\right)$ Dividiendo entre $2W$

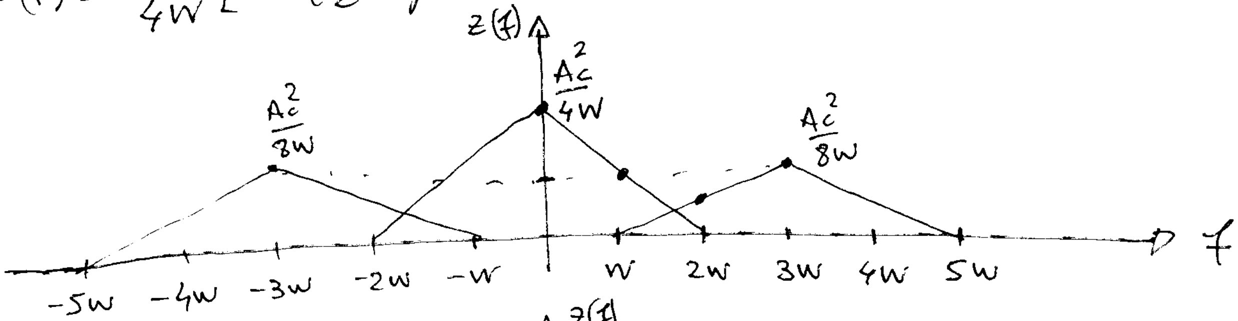
$$\mathrm{sinc}^2(2Wt) \rightleftharpoons \frac{1}{2W}\,\Lambda\left(\frac{f}{2W}\right)$$

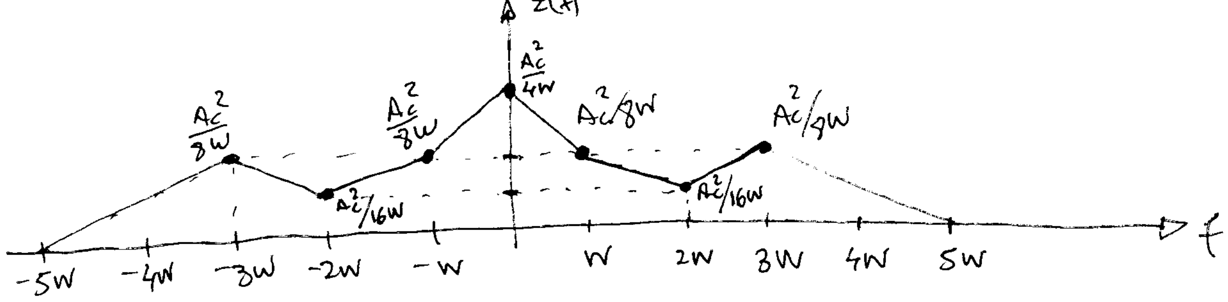
$$Z(f) = \frac{A_c^2}{2}\,\frac{1}{2W}\,\Lambda\left(\frac{f}{2W}\right) + \frac{A_c^2}{2}\,\frac{1}{2W}\,\Lambda\left(\frac{f}{2W}\right) * \left[\frac{\delta(f-2f_c)}{2} + \frac{\delta(f+2f_c)}{2}\right]$$

$$Z(f) = \frac{A_c^2}{4W}\left[\Lambda\left(\frac{f}{2W}\right) + \frac{1}{2}\Lambda\left(\frac{f-2f_c}{2W}\right) + \frac{1}{2}\Lambda\left(\frac{f+2f_c}{2W}\right)\right]$$

c.1) $f_c = \frac{3W}{2}$.

$$Z(f) = \frac{A_c^2}{4W}\left[\Lambda\left(\frac{f}{2W}\right) + \frac{1}{2}\Lambda\left(\frac{f-3W}{2W}\right) + \frac{1}{2}\Lambda\left(\frac{f+3W}{2W}\right)\right]$$





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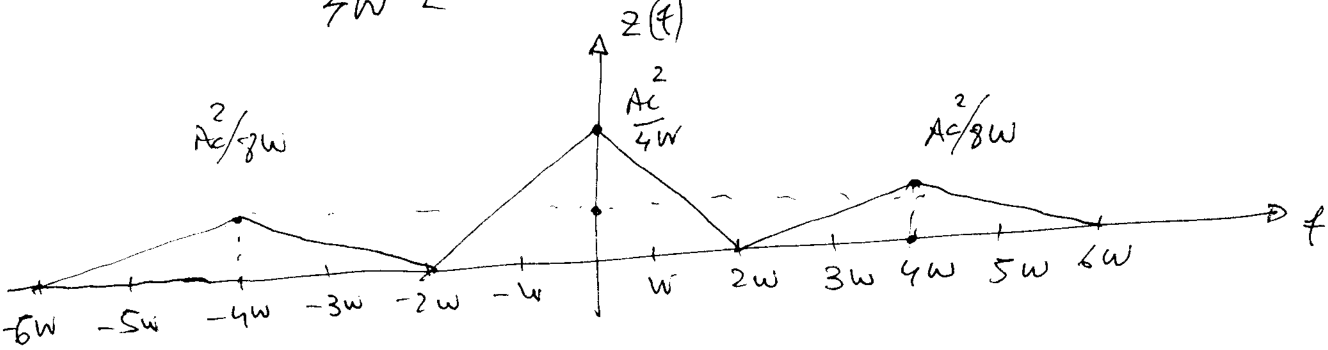
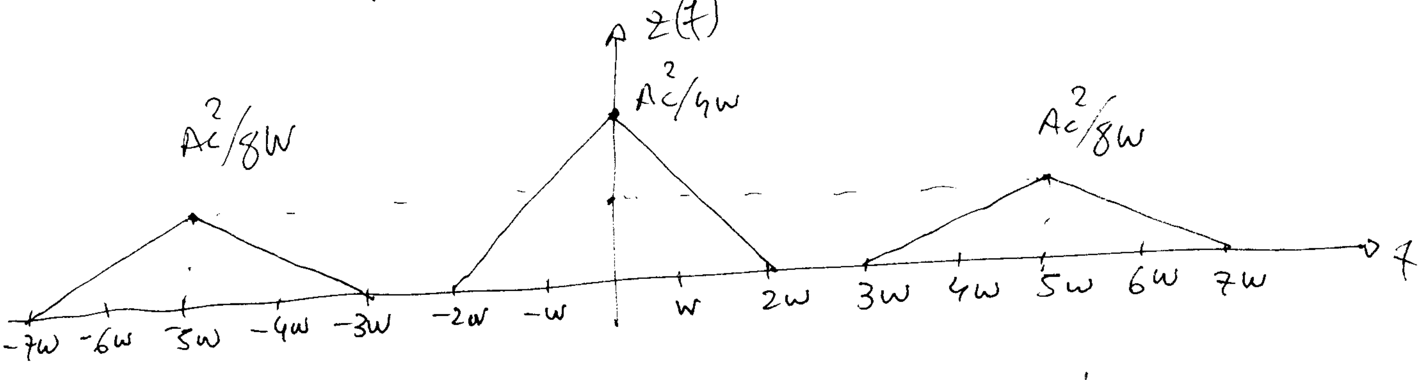
c.2) $f_c = 2W$

$$Z(f) = \frac{A_c^2}{4W}\left[\Lambda\left(\frac{f}{2W}\right) + \frac{1}{2}\Lambda\left(\frac{f-4W}{2W}\right) + \frac{1}{2}\Lambda\left(\frac{f+4W}{2W}\right)\right]$$



c.3) $f_c = \frac{5W}{2}$

$$Z(f) = \frac{A_c^2}{4W}\left[\Lambda\left(\frac{f}{2W}\right) + \frac{1}{2}\Lambda\left(\frac{f-5W}{2W}\right) + \frac{1}{2}\Lambda\left(\frac{f+5W}{2W}\right)\right]$$



d) NO. $m(t)$ y $|m(t)|$ dan lugar a la misma señal $y(t)$ por lo que no se puede recuperar el signo (FASE)

e) En FM elevar al cuadrado la señal significa multiplicar la fase por 2. Siempre que no ocurra el caso c.1), es decir, para $f_c \gg$ (suficientemente grande) en FM se podrá recuperar $m(t)$ a partir de $y(t)$.

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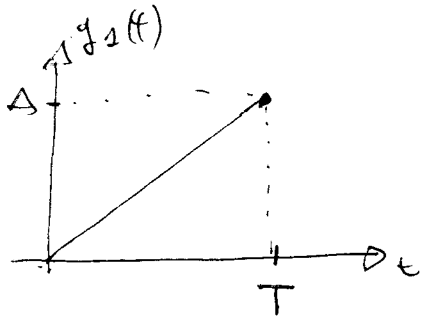
2. a)

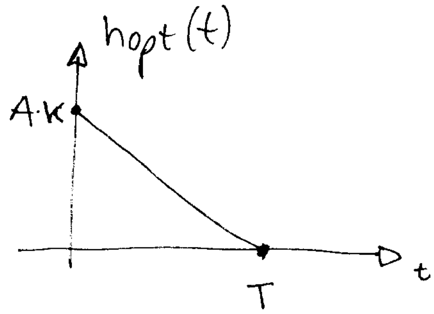


$$\underline{E} = \int_0^T g_1^2(t)\,dt = \int_0^T \left(\frac{A}{T}t\right)^2 dt$$

$$= \frac{A^2}{T^2}\int_0^T t^2\,dt = \frac{A^2}{\cancel{T^2}}\frac{\cancel{T^3}^{\,T}}{3} = \boxed{\frac{A^2T}{3}}$$

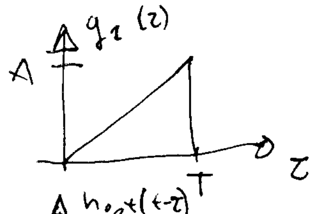
b) $h_{opt}(t) = g_1(T-t)$ para $0 \le t \le T$

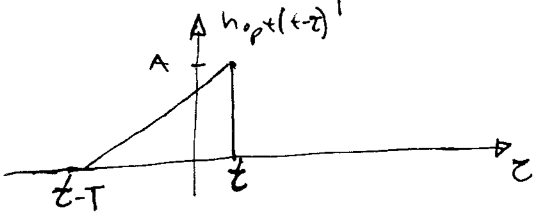
$$h_{opt}(t) = \begin{cases} \frac{A(T-t)}{T}\cdot K & 0 \le t \le T \\ 0 & \text{resto} \end{cases}$$



K: cte arbitraria.

c) $y_1(t) = g_1(t) * h_{opt}(t) = \int_0^T g_1(\tau)\, h_{opt}(t-\tau)\, d\tau$ $\boxed{0 \le t \le T}$





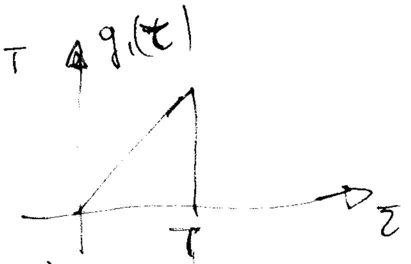
$$\underline{y_1(t)} = \int_0^t \frac{A\cdot\tau}{T}\cdot k\,\frac{A(T-t+\tau)}{T}\,d\tau$$

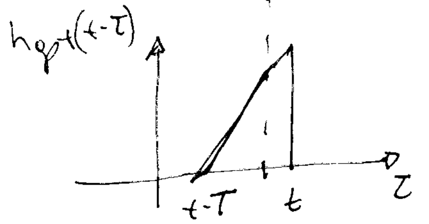
$$= \int_0^t k\frac{A^2}{T^2}[T-t]\,\tau\,d\tau + \int_0^t k\frac{A^2}{T^2}\tau^2\,d\tau$$

$$= \frac{A^2k}{T^2}[T-t]\frac{t^2}{2} + \frac{A^2}{T^2}\frac{t^3}{3} = \frac{A^2k}{T^2}\left[\frac{Tt^2}{2} - \frac{t^3}{2} + \frac{t^3}{3}\right] \cancel{= \frac{A^2k}{T^2}\left[T - \frac{t^3}{6}\right]}$$

$$\boxed{= \frac{A^2k}{T^2}\,\frac{3Tt^2 - t^3}{6} \qquad 0 \le t \le T}$$

$T \le t \le 2T$





$$y_1(t) = \int_{t-T}^{T} \frac{A\tau}{T} k \frac{A}{T}[T-t+\tau]\, d\tau$$

$$= \frac{A^2}{T^2}k\int_{t-T}^{T}(T-t)\,\tau\, d\tau + \frac{A^2}{T^2}k\int_{t-T}^{T}\tau^2\, d\tau$$

$$\boxed{y_1(t) = \frac{A^2k}{T^2}[T-t]\left[\frac{T^2}{2}-\frac{(t-T)^2}{2}\right] + \frac{A^2k}{T^2}\left[\frac{T^3}{3}-\frac{(t-T)^3}{3}\right]}$$

$$= \frac{A^2k}{T^2}[T-t]\,\frac{2Tt-t^2}{2} + \frac{A^2k}{T^2}\,\frac{-3T^2t+3Tt^2-t^3+2T^3}{3}$$

$$= \frac{A^2k}{T^2}\,\frac{2T^2t-Tt^2-2Tt^2+t^3}{2} + \frac{A^2k}{T^2}\,\frac{-3T^2t+3Tt^2-t^3+2T^3}{3}$$

$$= \frac{A^2k}{T^2}\,\frac{\cancel{6T^2t}-3Tt^2-\cancel{6Tt^2}+3t^3-\cancel{6T^2t}+\cancel{6Tt^2}-2t^3+4T^3}{6}$$

$$\cancel{= \frac{A^2}{T^2}\,\frac{12T^2t-15Tt^2+5t^3}{6}}$$

$$\boxed{= \frac{A^2k}{T^2}\,\frac{t^3-3Tt^2+4T^3}{6}} \quad \boxed{T \le t \le 2T}$$

$24 - 15\cdot 4 + 5\cdot 8$

$24 - 60 + 40$

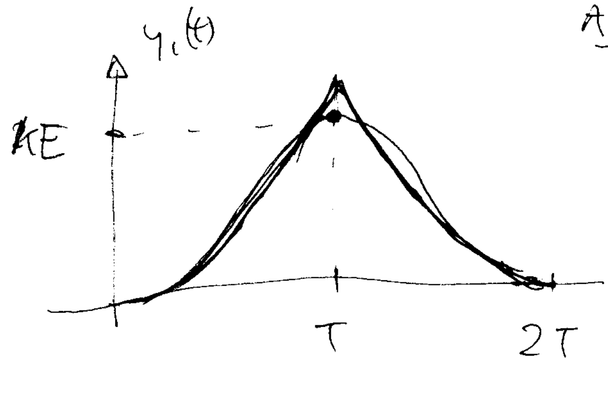
$8 - 3\cdot 4 + 4$

$8 - 12 + 4 = 0$

$1 - 3 + 4 = 2$

$$\boxed{y_1(T) = k\cdot E} = \frac{kA^2T}{3}$$

$$\frac{A^2k2T^3}{T^2 6} = \frac{A^2k\cdot T}{3}$$



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$g_\emptyset(t) = 0 \Rightarrow y_0(t) = 0 \quad y_0(T) = 0.$

d) $\sigma_N^2 = K^2 \frac{N_0 E}{2} = \frac{K^2 N_0 A^2 T}{6}$ // $\mu_N = 0$ es GAUSSIANO pero no blanco.

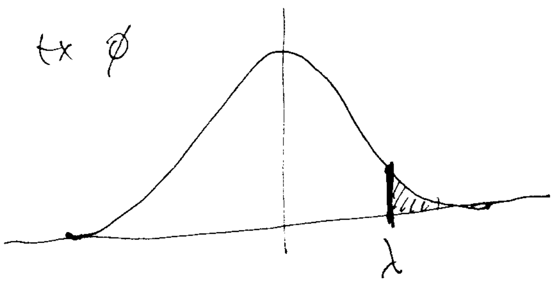
e) $\boxed{SNR^{\emptyset}_{pico} = \frac{|y_0(T)|^2}{\sigma_N^2} = 0}$

$\boxed{SNR^{1}_{pico} = \frac{|y_1(T)|^2}{\sigma_N^2} = \frac{K^2E^2}{\frac{K^2 N_0 E}{2}} = \frac{2K^2E^2}{K^2 N_0 E} = \frac{2E}{N_0}} = \frac{2A^2T}{3N_0}$

f) tx $\emptyset$ $\quad y_0(T) = 0$ $\quad \boxed{\lambda = \frac{KE}{2}}$ $\quad \sigma_N^2 = \frac{K^2 N_0 E}{2}$

tx 1 $\quad y_1(T) = KE$ $\quad \lambda = \frac{KA^2T}{6}$

g) tx $\emptyset$



$$P\emptyset = \int_\lambda^\infty \frac{1}{\sqrt{2\pi}\,\sigma_N} \exp\left(-\frac{1}{2}\frac{(y-0)^2}{\sigma_N^2}\right) dy$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma_N} \int_\lambda^\infty \exp\left(-\frac{1}{2}\frac{y^2}{\sigma_N^2}\right) dy$$

$z = \frac{\lambda}{\sqrt{2}\,\sigma_N}$

$$= \frac{1}{\sqrt{\pi}} \int_{\frac{\lambda}{\sqrt{2}\sigma_N}}^\infty \exp(-z^2)\, dz = \frac{1}{2}\operatorname{erfc}\left(\frac{\lambda}{\sqrt{2}\,\sigma_N}\right) = \frac{1}{2}\operatorname{erfc}\left(\frac{KE\cancel{\sqrt{2}}}{2\cancel{\sqrt{2}}\,K\sqrt{N_0E}}\right)$$

$$= \frac{1}{2}\operatorname{erfc}\left(\frac{1}{2}\sqrt{\frac{E}{N_0}}\right)$$

1 $\quad P_1 = \frac{1}{2} \operatorname{erfc}\left(\frac{1}{2}\sqrt{\frac{E}{N_0}}\right)$

$$P_e = \frac{1}{2} P_0 + \frac{1}{2} P_1 = \frac{1}{2} \operatorname{erfc}\left(\frac{1}{2}\sqrt{\frac{E}{N_0}}\right) = \frac{1}{2} \operatorname{erfc}\left(\frac{1}{2}\sqrt{\frac{A^2 T}{3}}\right)$$

$$\boxed{P_e = \frac{1}{2} \operatorname{erfc}\left(\frac{A}{2}\sqrt{\frac{T}{3}}\right)}$$