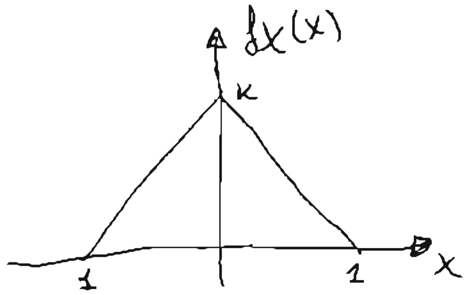
PROBLEMA 1:

(a)



$K>0$ ya que $f_X(x) \geq 0 \quad \forall x$

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1 \Rightarrow \boxed{K=1}$$

La variable aleatoria X está definida en el intervalo $[-1,1]$

(b) $$f_X(x) = \begin{cases} 1+x & -1<x<0 \\ 1-x & 0<x<1 \\ 0 & \text{Resto} \end{cases}$$

$$\boxed{P(|X|<0.5)} = \int_{-1/2}^{1/2} f_X(x)dx = \int_{-1/2}^{0} (1+x)dx + \int_{0}^{1/2} (1-x)\,dx =$$

$$= \left(x+\frac{x^2}{2}\right)_{-1/2}^{0} + \left(x-\frac{x^2}{2}\right)_{0}^{1/2} = 0 - \left[-\frac{1}{2}+\frac{1}{8}\right] + \left[\frac{1}{2}-\frac{1}{8}\right] - 0 = \frac{3}{8}+\frac{3}{8} = \frac{3}{4} = \boxed{0.75}$$

(c) $$\boxed{E[X]} = \int_{-\infty}^{\infty} x f_X(x)dx = \int_{-1}^{0} x(1+x)\,dx + \int_{0}^{1} x(1-x)\,dx = \int_{-1}^{0}(x^2+x)dx + \int_{0}^{1}(x-x^2)dx =$$

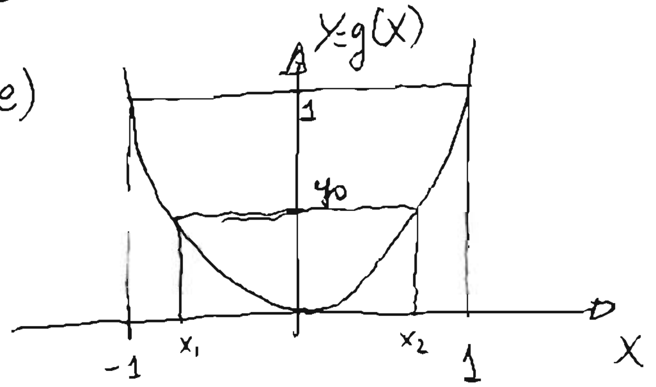
$$= \left[\frac{x^3}{3}+\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}-\frac{x^3}{3}\right]_{0}^{1} = 0 - \left[-\frac{1}{3}+\frac{1}{2}\right] + \left[\frac{1}{2}-\frac{1}{3}\right] - 0 = -\frac{1}{6}+\frac{1}{6} = \boxed{0}$$

(d) Como $E[X]=0 \Rightarrow var[X] = E[X^2]$

$$\boxed{var[X]} = E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)dx = \int_{-1}^{0} x^2(1+x)\,dx + \int_{0}^{1} x^2(1-x)\,dx = \int_{-1}^{0}(x^2+x^3)\,dx + \int_{0}^{1}(x^2-x^3)dx =$$

$$= \left[\frac{x^3}{3}+\frac{x^4}{4}\right]_{-1}^{0} + \left[\frac{x^3}{3}-\frac{x^4}{4}\right]_{0}^{1} = 0 - \left[-\frac{1}{3}+\frac{1}{4}\right] + \left[\frac{1}{3}-\frac{1}{4}\right] - 0 = \frac{1}{12}+\frac{1}{12} = \boxed{\frac{1}{6}}$$

(e)



Dado $Y=y_0$, $g(x_1)=g(x_2)=y_0$

Dos valores:

$$\boxed{x_1 = -\sqrt{y_0} = -\sqrt{Y}}$$

$$\boxed{x_2 = +\sqrt{y_0} = \sqrt{Y}}$$

Rango para Y: $[0,1]$

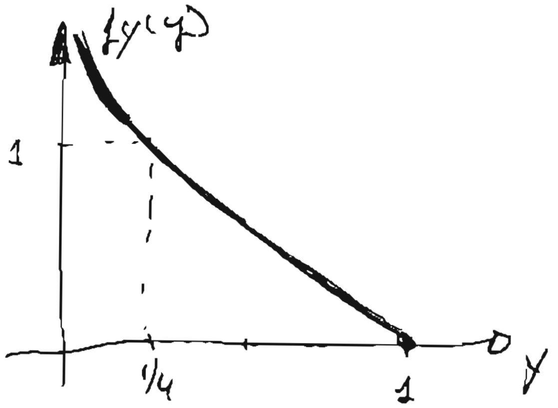
(f) $f_Y(y) = \sum_i \frac{f_X(x_i)}{\left|\frac{dg(x)}{dx}\right|_{x=x_i}} = \frac{f_X(x_1)}{\left|\frac{dg(x)}{dx}\right|_{x=x_1}} + \frac{f_X(x_2)}{\left|\frac{dg(x)}{dx}\right|_{x=x_2}} = \frac{f_X(-\sqrt{y})}{\left|\frac{dg(x)}{dx}\right|_{x=-\sqrt{y}}} + \frac{f_X(\sqrt{y})}{\left|\frac{dg(x)}{dx}\right|_{x=\sqrt{y}}}$

$\frac{dg(x)}{dx} = 2x \quad , \quad \left|\frac{dg(x)}{dx}\right|_{x=-\sqrt{y}} = |2(-\sqrt{y})| = 2\sqrt{y}$

$\left|\frac{dg(x)}{dx}\right|_{x=\sqrt{y}} = |2\sqrt{y}| = 2\sqrt{y}$

$$\boxed{f_Y(y) = \frac{1+(-\sqrt{y})}{2\sqrt{y}} + \frac{1-(\sqrt{y})}{2\sqrt{y}} = \frac{1-\sqrt{y}}{\sqrt{y}} = \frac{1}{\sqrt{y}} - 1 \quad 0 \le y \le 1}$$

(g)



$f_Y(0) = +\infty$

$f_Y(1/4) = \frac{1}{\sqrt{1/4}} - 1 = 2 - 1 = 1$

$f_Y(1) = 1 - 1 = 0$

$\boxed{f_Y(y) \geq 0 \ \forall y}$, faltaría comprobar si $\int_{-\infty}^{\infty} f_Y(y)\,dy = 1$

$$\boxed{\int_0^1 f_Y(y)\,dy = \int_0^1 \left(\frac{1}{\sqrt{y}} - 1\right) dy = \int_0^1 (y^{-1/2} - 1)\,dy = \left[\frac{y^{1/2}}{1/2} - y\right]_0^1 = \left[2\sqrt{y} - y\right]_0^1 = 2 - 1 = \underline{1}}$$

$$\boxed{P(Y > 0{,}5)} = \int_{1/2}^{1} f_Y(y)\,dy = \left[2\sqrt{y} - y\right]_{1/2}^{1} = [2-1] - [2\sqrt{1/2} - 1/2] = 1 - \sqrt{2} + \frac{1}{2} = \boxed{0'0858}$$

(h) $m_Z(t) = E[Z(t)] = E[\cos(2\pi f_0 t + \pi X)] = \int_{-\infty}^{\infty} \cos(2\pi f_0 t + \pi x) f_X(x)\,dx =$

$\int_{-1}^{0} \cos(2\pi f_0 t + \pi x)(1+x)\,dx + \int_0^1 \cos(2\pi f_0 t + \pi x)(1-x)\,dx = \underbrace{\int_{-1}^{0} x \cos(2\pi f_0 t + \pi x)\,dx}_{I_1} +$

$\underbrace{\int_{-1}^{1} \cos(2\pi f_0 t + \pi x)\,dx}_{I_2} - \underbrace{\int_0^1 x \cos(2\pi f_0 t + \pi x)\,dx}_{I_3}$

PROBLEMA 1 (CONT.)

$$I_1 = \int_{-1}^{0} x\cos(2\pi f_0 t + \pi x)\,dx = \left[\frac{1}{\pi^2}\cos(2\pi f_0 t + \pi x) + \frac{x}{\pi}\,\mathrm{sen}(2\pi f_0 t + \pi x)\right]_{-1}^{0} =$$

$$= \left[\frac{1}{\pi^2}\cos(2\pi f_0 t)\right] - \left[\frac{1}{\pi^2}\cos(2\pi f_0 t - \pi) - \frac{1}{\pi}\,\mathrm{sen}(2\pi f_0 t - \pi)\right] =$$

$$= \frac{1}{\pi^2}\cos(2\pi f_0 t) + \frac{1}{\pi^2}\cos(2\pi f_0 t) - \frac{1}{\pi}\,\mathrm{sen}(2\pi f_0 t) = \frac{2}{\pi^2}\cos(2\pi f_0 t) - \frac{1}{\pi}\,\mathrm{sen}(2\pi f_0 t)$$

$$I_2 = \int_{-1}^{1} \cos(2\pi f_0 t + \pi x)\,dx = \left[\frac{1}{\pi}\,\mathrm{sen}(2\pi f_0 t + \pi x)\right]_{-1}^{1} = \frac{1}{\pi}\,\mathrm{sen}(2\pi f_0 + \pi) - \frac{1}{\pi}\,\mathrm{sen}(2\pi f_0 t - \pi) = 0$$

$$I_3 = \int_{0}^{1} x\cos(2\pi f_0 t + \pi x)\,dx = \left[\frac{1}{\pi^2}\cos(2\pi f_0 t + \pi x) + \frac{x}{\pi}\,\mathrm{sen}(2\pi f_0 t + \pi x)\right]_{0}^{1} =$$

$$= \left[\frac{1}{\pi^2}\cos(2\pi f_0 t + \pi) + \frac{1}{\pi}\,\mathrm{sen}(2\pi f_0 t + \pi)\right] - \left[\frac{1}{\pi^2}\cos(2\pi f_0 t)\right] =$$

$$= \frac{-2}{\pi^2}\cos(2\pi f_0 t) - \frac{1}{\pi}\,\mathrm{sen}(2\pi f_0 t)$$

$$\boxed{m_Z(t)} = I_1 + I_2 - I_3 = \frac{2}{\pi^2}\cos(2\pi f_0 t) - \frac{1}{\pi}\,\mathrm{sen}(2\pi f_0 t) + 0 - \left[\frac{-2}{\pi^2}\cos(2\pi f_0 t) - \frac{1}{\pi}\,\mathrm{sen}(2\pi f_0 t)\right] =$$

$$= \boxed{\frac{4}{\pi^2}\cos(2\pi f_0 t)}$$ No es estacionario ya que depende de t.

(i) $$R_Z(t,u) = E[Z(t)Z(u)] = E[\cos(2\pi f_0 t + \pi x)\cos(2\pi f_0 u + \pi x)] =$$

$$= \frac{1}{2}E[\cos(2\pi f_0(t+u) + 2\pi x)] + \frac{1}{2}E[\cos(2\pi f_0(t-u))] =$$

$$= \frac{1}{2}\cos(2\pi f_0(t-u)) + \frac{1}{2}\int_{-\infty}^{\infty}\cos(2\pi f_0(t+u) + 2\pi x)\,f_X(x)\,dx =$$

$$= \frac{1}{2}\int_{-1}^{0}\cos(2\pi f_0(t+u) + 2\pi x)(1+x)\,dx + \frac{1}{2}\int_{0}^{1}\cos(2\pi f_0(t+u) + 2\pi x)(1-x)\,dx + \frac{1}{2}\cos(2\pi f_0(t-u))$$

$$= \underbrace{\frac{1}{2}\int_{-1}^{0}x\cos(2\pi f_0(t+u) + 2\pi x)\,dx}_{I_1} + \underbrace{\frac{1}{2}\int_{-1}^{1}\cos(2\pi f_0(t+u) + 2\pi x)\,dx}_{I_2} - \underbrace{\frac{1}{2}\int_{0}^{1}x\cos(2\pi f_0(t+u) + 2\pi x)\,dx}_{I_3} +$$

$$+ \frac{1}{2}\cos(2\pi f_0(t-u))$$

$$I_1 = \int_{-1}^{0} x\cos(2\pi f_0(t+u)+2\pi x)\,dx = \left[\frac{1}{4\pi^2}\cos(2\pi f_0(t+u)+2\pi x) + \frac{x}{2\pi}\,\mathrm{sen}(2\pi f_0(t+u)+2\pi x)\right]_{-1}^{0} =$$

$$= \left[\frac{1}{4\pi^2}\cos(2\pi f_0(t+u))\right] - \left[\frac{1}{4\pi^2}\cos(2\pi f_0(t+u)-2\pi) - \frac{1}{2\pi}\,\mathrm{sen}(2\pi f_0(t+u)-2\pi)\right] =$$

$$= \frac{1}{4\pi^2}\cos(2\pi f_0(t+u)) - \frac{1}{4\pi^2}\cos(2\pi f_0(t+u)) + \frac{1}{2\pi}\,\mathrm{sen}(2\pi f_0(t+u)) = \frac{1}{2\pi}\,\mathrm{sen}(2\pi f_0(t+u))$$

$$I_2 = \int_{-1}^{1} \cos(2\pi f_0(t+u)+2\pi x)\,dx = \left[\frac{1}{2\pi}\,\mathrm{sen}(2\pi f_0(t+u)+2\pi x)\right]_{-1}^{1} =$$

$$= \left[\frac{1}{2\pi}\,\mathrm{sen}(2\pi f_0(t+u)+2\pi)\right] - \left[\frac{1}{2\pi}\,\mathrm{sen}(2\pi f_0(t+u)-2\pi)\right] =$$

$$= \frac{1}{2\pi}\,\mathrm{sen}(2\pi f_0(t+u)) - \frac{1}{2\pi}\,\mathrm{sen}(2\pi f_0(t+u)) = 0$$

$$I_3 = \int_{0}^{1} x\cos(2\pi f_0(t+u)+2\pi x)\,dx = \left[\frac{1}{4\pi^2}\cos(2\pi f_0(t+u)+2\pi x) + \frac{x}{2\pi}\,\mathrm{sen}(2\pi f_0(t+u)+2\pi x)\right]_{0}^{1} =$$

$$= \left[\frac{1}{4\pi^2}\cos(2\pi f_0(t+u)+2\pi) + \frac{1}{2\pi}\,\mathrm{sen}(2\pi f_0(t+u)+2\pi)\right] - \left[\frac{1}{4\pi^2}\cos(2\pi f_0(t+u))\right] =$$

$$= \frac{1}{4\pi^2}\cos(2\pi f_0(t+u)) + \frac{1}{2\pi}\,\mathrm{sen}(2\pi f_0(t+u)) - \frac{1}{4\pi^2}\cos(2\pi f_0(t+u)) = \frac{1}{2\pi}\,\mathrm{sen}(2\pi f_0(t+u))$$

$$\boxed{R_Z(t,u)} = \frac{1}{2}I_1 + \frac{1}{2}I_2 - \frac{1}{2}I_3 + \frac{1}{2}\cos(2\pi f_0(t-u)) =$$

$$= \frac{1}{\pi}\,\mathrm{sen}(2\pi f_0(t+u)) + 0 - \frac{1}{\pi}\,\mathrm{sen}(2\pi f_0(t+u)) + \frac{1}{2}\cos(2\pi f_0(t-u)) =$$

$$\boxed{= \frac{1}{2}\cos(2\pi f_0(t-u))}$$

ES estacionaria ya que depende sólo de $\tau = t-u$

$$\boxed{R_Z(\tau) = \frac{1}{2}\cos(2\pi f_0\tau)}$$

PROBLEMA 2:

(a) $g(t) = \dfrac{\text{sinc}(2\cdot 10^4 t)}{10^{-2} - 4\cdot 10^6 t^2}$ sabemos que $p(t) = \text{Sinc}(2B_T t)\dfrac{\cos(2\pi p B_T t)}{1-16p^2B_T^2t^2}$

Entonces:

$p(t) = \dfrac{\text{sen}(2\pi B_T t)}{2\pi B_T t} \cdot \dfrac{\cos(2\pi p B_T t)}{1-16p^2B_T^2t^2}$ y que $\text{sen}\,\alpha \cos\beta = \frac{1}{2}\text{sen}(\alpha-\beta) + \frac{1}{2}\text{sen}(\alpha+\beta)$

$$p(t) = \frac{\text{sen}(2\pi(1-p)B_T t) + \text{sen}(2\pi(1+p)B_T t)}{4\pi B_T t\,(1-16p^2B_T^2t^2)}$$

Si $p=0$ $\quad p(t) = \text{Sinc}(2B_T t)$ no vale

Si $p = 1/2$ $\quad p(t) = \dfrac{\text{sen}(\pi B_T t) + \text{sen}(3\pi B_T t)}{4\pi B_T t(1-4B_T^2t^2)}$ no vale

Si $\boxed{p=1}$ $\quad p(t) = \dfrac{\text{Sinc}(4B_T t)}{1+16B_T^2t^2}$ si vale

$g(t) = 100\,\dfrac{\text{Sinc}(2\cdot 10^4 t)}{1-4\cdot 10^8 t^2}$ $\quad 2\cdot 10^4 = 4B_T \Rightarrow \boxed{B_T = 5\,KHz}$

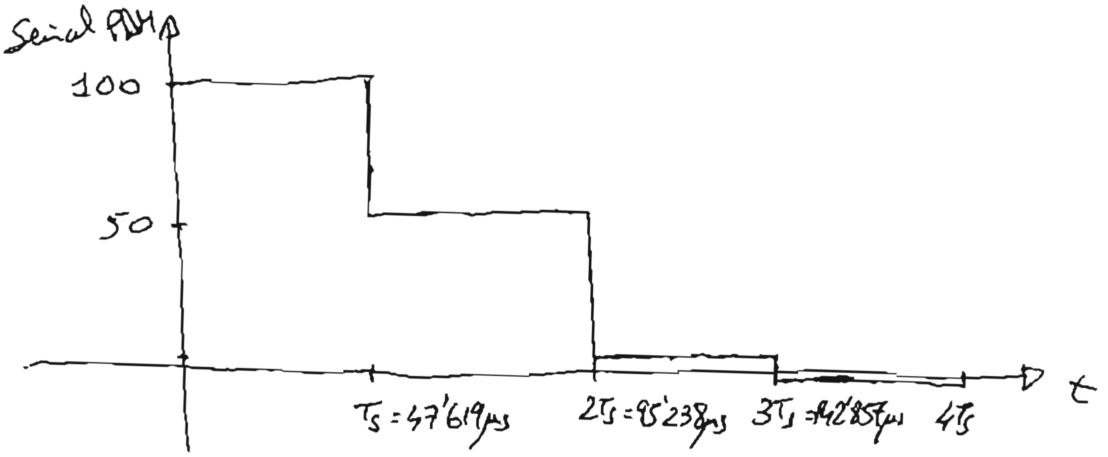
Además $16B_T^2 = 16\cdot(5000)^2 = 4\cdot 10^8$

Valor máximo en $t=0$, $\boxed{A_{max} = g(0) = 100}$

(b) $\boxed{B_W = B_T(1+p) = 2B_T = 10\,KHz}$

(c) $f_{Nyq} = 2\cdot B_W = 20KHz$. $\boxed{f_s = 1'05 f_{Nyq} = 21\,KHz}$, $\boxed{T_s = 47'619\mu s}$

g(t)	100	53'58	1'874	-0'675
t	0	47'619µs	95'238µs	142'857µs



(d) Ley μ $\quad |v_2| = \dfrac{\ln(1+\mu|v_1|)}{\ln(1+\mu)}$

para $\mu = 25$ $\quad |v_2| = \dfrac{\ln(1+25|v_1|)}{\ln(26)}$

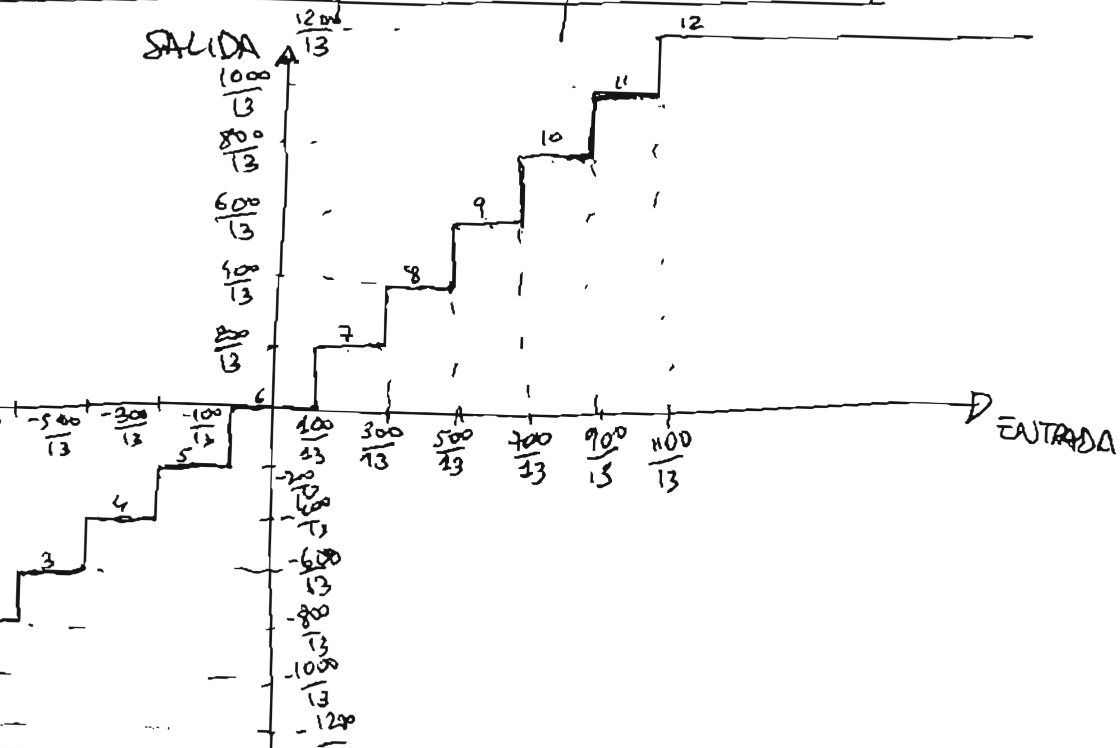
v_1 y v_2 están normalizados: $|g_1| = A_{max}|v_1|$ y $|g_2| = A_{max}|v_2|$

$$\boxed{|g_2| = 100|v_2| = \frac{\ln\left(1+25\cdot\frac{|g_1|}{100}\right)}{\ln(26} = \frac{\ln\left(1+\frac{|g_1|}{4}\right)}{\ln(26}}$$

g_1	100	53'58	1'874	-0'675
g_2	100	81'85	11'79	-4'78

(e)

$$\delta = \frac{2A_{max}}{L} = \frac{200}{13}$$



PROBLEMA 2 (CONT.)

(f)

g_2	100	81'85	11'79	-4'78
g_3	92'31	76'92	15'38	0
ESCALON	12	11	7	6

(g)

ESCALON	CODIGO
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100

- Son necesarios 4 bits
- La codificación no es eficiente ya que los códigos 1101, 1110 y 1111 no se utilizan. Se podría haber usado 16 escalones con la misma tasa binaria resultante mejorando la calidad al disminuir el error de cuantificación

(h)

ESCALON	12	11	7	6
CODIGO	1100	1011	0111	0110

Señal binaria con codificación Manchester