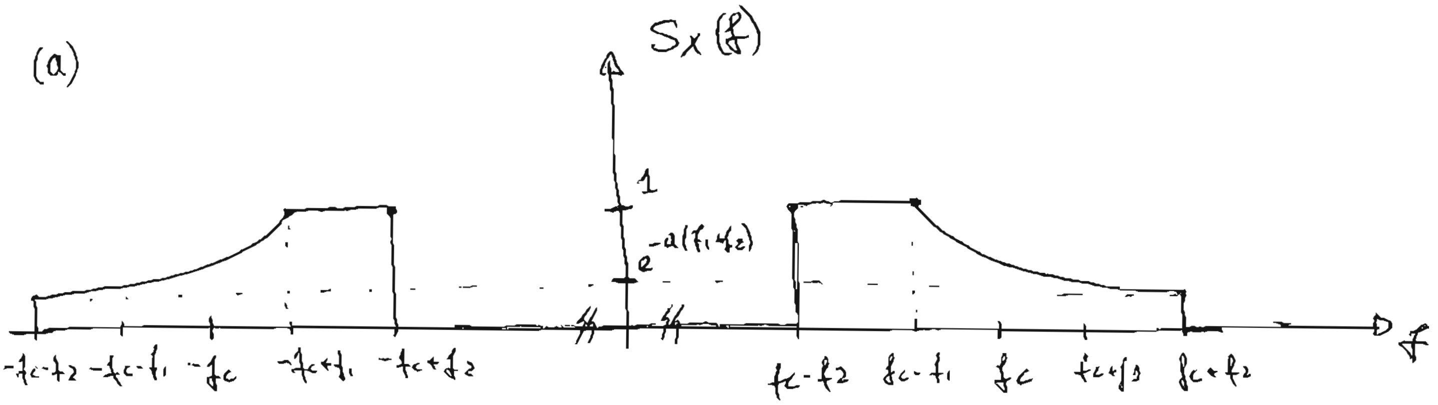
PROBLEMA 1:

(a)

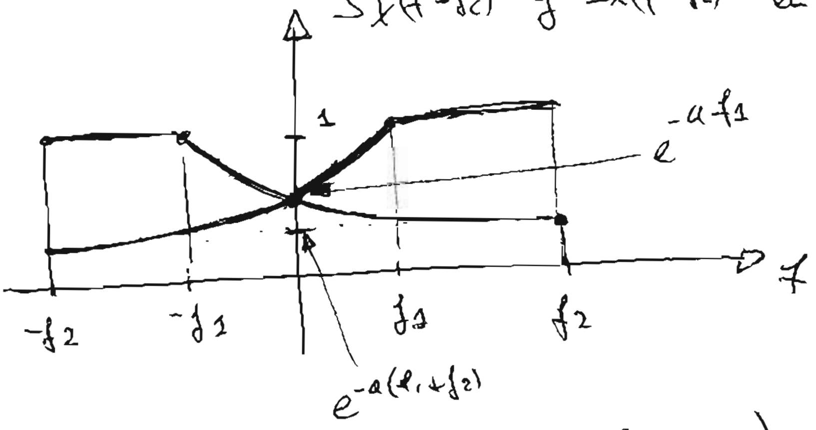


(b)

Sabemos que

$$S_{X_c}(f) = S_{X_s}(f) = \begin{cases} S_X(f-f_c) + S_X(f+f_c) & |f| \leq f_2 \\ 0 & \text{Resto} \end{cases}$$

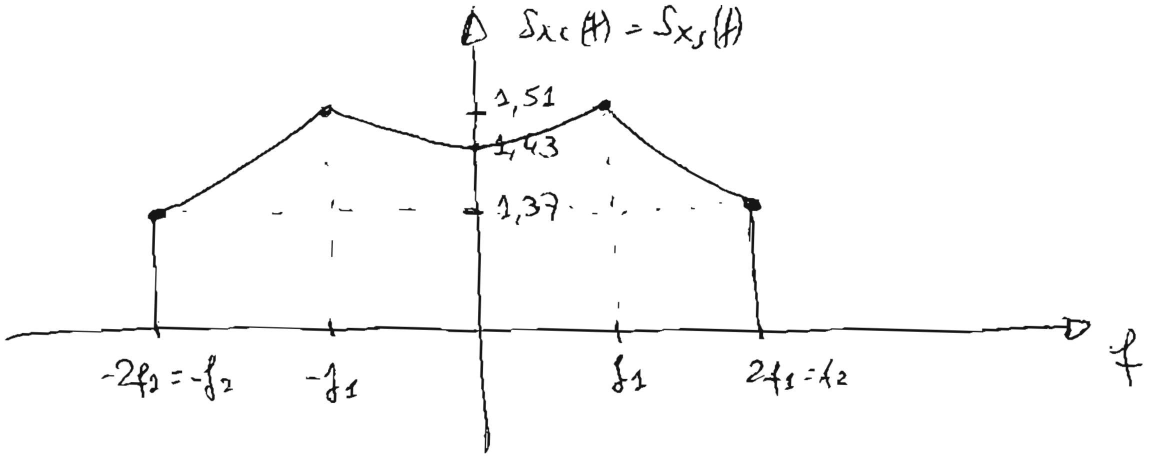
$S_X(f-f_c)$ y $S_X(f+f_c)$ en $|f| \leq f_2$



$$S_{X_c}(f) = S_{X_s}(f) = \begin{cases} e^{-a(|f|+f_1)} + e^{a(|f|-f_1)} & |f| < f_1 \\ 1 + e^{-a(|f|+f_1)} & f_2 < |f| < f_2 \\ 0 & \text{Resto} \end{cases}$$

Para $a = \frac{1}{f_1+f_2}$ y $f_2 = 2f_1 \Rightarrow \boxed{a = \frac{1}{f_1+2f_1} = \frac{1}{3f_1}}$

$$S_{X_c}(f) = S_{X_s}(f) = \begin{cases} e^{-\frac{|f|+f_1}{3f_1}} + e^{\frac{|f|-f_1}{3f_1}} & |f| < f_1 \\ 1 + e^{-\frac{|f|+f_1}{3f_1}} & f_1 \leq |f| \leq 2f_1 \\ 0 & \text{Resto} \end{cases}$$



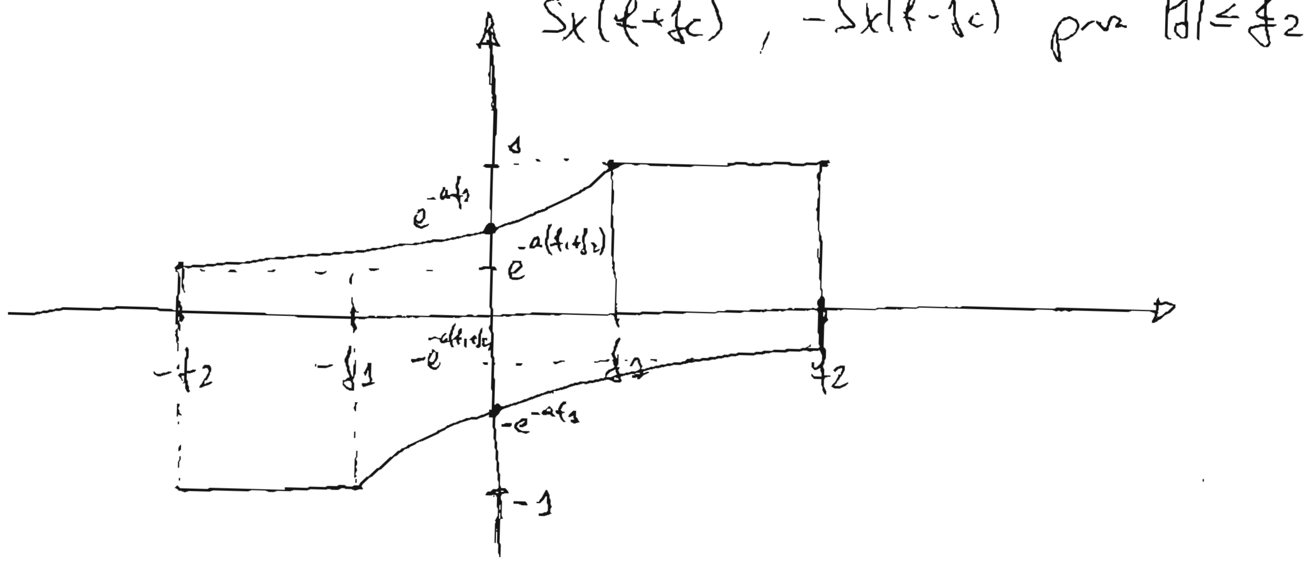
En $f=0 \Rightarrow 2e^{-\frac{1}{3}} = 1,43$

En $f=\pm f_1 \Rightarrow 1+e^{-\frac{2}{3}} = 1,51$

En $f=\pm 2f_1 = f_2 \Rightarrow 1+e^{-1} = 1,37$

(c)

$$S_{x_c x_s}(f) = \begin{cases} j[S_x(f+f_c) - S_x(f-f_c)] & |f| \le f_2 \\ 0 & \text{Resto} \end{cases}$$



$$S_{x_c x_s}(f) = \begin{cases} j\left(e^{a(f-f_1)} - e^{-a(f+f_1)}\right) & |f| \le f_1 \\ j - j e^{-a(f+f_1)} & f_1 < f < f_2 \\ j e^{a(f-f_1)} - j & -f_2 < f < -f_1 \\ 0 & \text{Resto} \end{cases}$$

PROBLEMA 1 (CONT.)

Para $a = \frac{5}{f_1+f_2}$ $\quad f_2 = 3f_1 \Rightarrow a = \frac{5}{f_1+3f_1} = \frac{5}{4f_1}$

En $f=0 \Rightarrow 0$

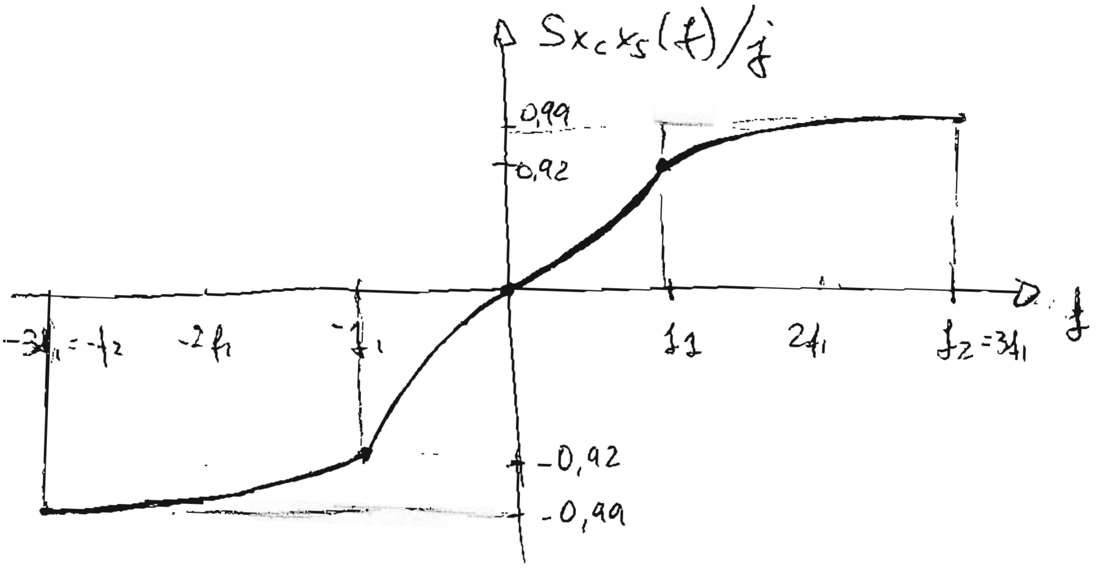
En $f = f_1 \Rightarrow j - je^{-5\frac{f_1+f_1}{4f_1}} = j - je^{-\frac{5}{2}} = j(1-e^{-\frac{5}{2}}) = j\,0'92$

En $f = -f_1 \Rightarrow -j\,0'92$

En $f = f_2 = 3f_1 \Rightarrow j - je^{-5\frac{3f_1+f_1}{4f_1}} = j - je^{-5} = j(1-e^{-5}) = j\,0'99$

En $f = -f_2 \Rightarrow -j\,0'99$

$$S_{x_cx_s}(f) = \begin{cases} j\left(e^{5\frac{f-f_1}{4f_1}} - e^{-5\frac{f+f_1}{4f_1}}\right) & |f| \leq f_1 \\ j - je^{-5\frac{f+f_1}{4f_1}} & f_1 < f < f_2 \\ je^{5\frac{f-f_1}{4f_1}} - j & -f_2 < f < -f_1 \\ 0 & \text{Resto} \end{cases}$$



(d) Incorreladas $\Rightarrow R_{x_cx_s}(\tau) = E[x_c(t+\tau)x_s(t)] = E[\overset{=0}{x_c(t+\tau)}]\,E[\overset{=0}{x_s(t)}] = 0$

$\Updownarrow$

$S_{x_cx_s}(f) = 0$ sólo si $\underline{a=0}$

(e) $Y_1(t) = 2\,X(t)\cos(2\pi f_c t + \Theta)$

$Y_2(t) = -2\,X(t)\,\text{sen}(2\pi f_c t + \Theta)$

$Z_1(t) = Y_1(t) * h(t) = \int_{-\infty}^{\infty} h(\lambda_1)\, y_1(t-\lambda_1)\, d\lambda_1$

$Z_2(t) = Y_2(t) * h(t) = \int_{-\infty}^{\infty} h(\lambda_2)\, y_2(t-\lambda_2)\, d\lambda_2$

$R_{Z_1}(\tau) = E[Z_1(t)\, Z_1(t+\tau)] =$

$= E\left[\int_{-\infty}^{\infty} h(\lambda_1)\, y_1(t-\lambda_1)\, d\lambda_1 \int_{-\infty}^{\infty} h(\lambda_2)\, y_1(t+\tau-\lambda_2)\, d\lambda_2\right]$

$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\lambda_1)\, h(\lambda_2)\, \underbrace{E[y_1(t-\lambda_1)\, y_1(t+\tau-\lambda_2)]}_{R_{y_1}(\tau-\lambda_2+\lambda_1)}\, d\lambda_1\, d\lambda_2 = R_{y_1}(\tau) * h(\tau) * h(-\tau)$

$R_{y_1}(\tau) = E[y_1(t)\, y_1(t+\tau)] = 4R_X(\tau)\, E[\cos(2\pi f_c t+\Theta)\cos(2\pi f_c(t+\tau)+\Theta)]$

$= 4R_X(\tau)\, E[\frac{1}{2}\cos(2\pi f_c\tau) + \frac{1}{2}\cancelto{0}{\cos(2\pi f_c(2t+\tau)+2\Theta)}]$

$= 2R_X(\tau)\cos(2\pi f_c\tau)$

$\boxed{R_{Z_1}(\tau) = 2[R_X(\tau)\cos(2\pi f_c\tau)] * h(\tau) * h(-\tau)}$

De igual forma:

$R_{Z_2}(\tau) = R_{Y_2}(\tau) * h(\tau) * h(-\tau)$

$R_{Y_2}(\tau) = E[Y_2(t)\, Y_2(t+\tau)] = 4R_X(\tau)\, E[\text{sen}(2\pi f_c t+\Theta)\,\text{sen}(2\pi f_c(t+\tau)+\Theta)]$

$= 4R_X(\tau)\, E[\frac{1}{2}\cos(2\pi f_c\tau) - \frac{1}{2}\cancelto{0}{\cos(2\pi f_c(2t+\tau)+2\Theta)}]$

$= 2R_X(\tau)\cos(2\pi f_c\tau)$ $\boxed{R_{Z_2}(\tau) = R_{Z_1}(\tau)}$

PROBLEMA 1 .(CONT)

(f) Tomando TF de

$$R_{Z_1}(\tau) = R_{Z_2}(\tau) = 2\left[R_X(\tau)\cos(2\pi f_c \tau)\right] * h(\tau) * h(-\tau)$$

$$S_{Z_1}(f) = S_{Z_2}(f) = \cancel{2}\left[S_X(f) * \left[\frac{1}{\cancel{2}}\delta(f-f_c)+\delta(f+f_c)\right]\right]\cdot H(f)\cdot H^*(-f)$$

$$= \left[S_X(f-f_c) + S_X(f+f_c)\right]|H(f)|^2$$

$$= \begin{cases} S_X(f-f_c)+S_X(f+f_c) & |f| \le f_2 \\ 0 & \text{Resto}\end{cases}$$

Lo mismo que en el apartado (b).

(g)

$$R_{Z_1 Z_2}(\tau) = E\left[Z_1(t+\tau)\,Z_2(t)\right] \cdot$$

$$= E\left[\int_{-\infty}^{\infty} h(\lambda_1)\, y_1(t+\tau-\lambda_1)\, d\lambda_1 \int_{-\infty}^{\infty} h(\lambda_2)\, y_2(t-\lambda_2)\, d\lambda_2\right]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\lambda_1)\, h(\lambda_2)\, \underbrace{E\left[y_1(t+\tau-\lambda_1)\, y_2(t-\lambda_2)\right]}_{R_{y_1 y_2}(\tau-\lambda_1+\lambda_2)}\, d\lambda_1\, d\lambda_2$$

$$= R_{y_1 y_2}(\tau) * h(\tau) * h(-\tau)$$

$$\boxed{R_{y_1 y_2}(\tau) = E\left[y_1(t+\tau)\, y_2(t)\right]} = -4R_X(\tau)\, E\left[\cos(2\pi f_c(t+\tau)+\Theta)\,\text{sen}(2\pi f_c t+\Theta)\right]$$

$$= -4R_X(\tau)\, E\left[-\frac{1}{2}\,\text{sen}(2\pi f_c\tau) + \cancelto{0}{\frac{1}{2}\,\text{sen}(2\pi f_c(2t+\tau)+2\Theta)}\right]$$

$$= 4R_X(\tau)\,\frac{1}{2}\,\text{sen}(2\pi f_c\tau) = \boxed{2R_X(\tau)\,\text{sen}(2\pi f_c\tau)}$$

$$\boxed{R_{Z_1 Z_2}(\tau) = \left[2R_X(\tau)\,\text{sen}(2\pi f_c\tau)\right] * h(\tau) * h(-\tau)}$$

(h) Haciendo transformada de Fourier:

$$S_{z_1 z_2}(f) = 2\left[S_X(f) * \left[\frac{1}{2j}\,\delta(f-f_c) - \frac{1}{2j}\,\delta(f+f_c)\right]\right] \cdot H(f)\cdot H^*(f)$$

$$= \cancel{2}\,\frac{1}{4j}\left(S_X(f-f_c) - S_X(f+f_c)\right) |H(f)|^2$$

$$= j\left(S_X(f+f_c) - S_X(f-f_c)\right) |H(f)|^2$$

$$= \begin{cases} j\left[S_X(f+f_c) - S_X(f-f_c)\right] & |f| \le f_2 \\ 0 & \text{Resto} \end{cases}$$

Lo mismo que en el apartado (c).

PROBLEMA 2:

(a) $$\boxed{P_X} = \int_{-\infty}^{\infty} S_X(f)\, df = \frac{4000\ \text{Hz} \cdot 10^{-4}\ \text{W/Hz}}{2} = 200\ \text{mW} \equiv \boxed{23\ \text{dBm}}$$

$\mu = 0,5$

$$K_a \cdot |m(t)|_{max} = \mu = 0,5$$

$$\boxed{K_a} = \frac{\mu}{|m(t)|_{max}} = \frac{0,5}{2} = \boxed{0,25}$$

(b) El filtro IF equivalente no modifica la componente de señal modulada s(t).

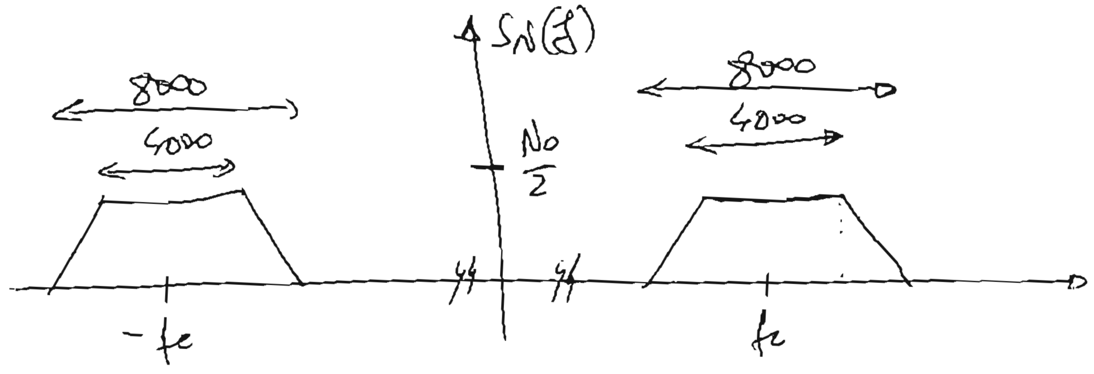
$P_R = \frac{P_T}{A_t}$, siendo A_t la atenuación del canal a

unidades naturales:

$$A_t = 10^{\frac{80}{10}} = 10^8$$

$$\boxed{P_R} = \frac{600W}{108} = 6 \cdot 10^{-3}\ \text{mW} = 6\ \mu\text{W} \equiv \boxed{-22,22\ \text{dBm}}$$

$$S_N(f) = S_W(f) \cdot |H_{IF}(f)|^2 = \frac{N_0}{2} |H_{IF}(f)|^2$$



$$\boxed{P_N} = \int_{-\infty}^{\infty} S_N(f)\, df = N_0 \cdot \left(\frac{4000}{2} + 4000\right) = 6000\ N_0 = 6000\ \text{Hz}\ 5 \cdot 10^{-14}\ \text{W/Hz}$$

$$= 3 \cdot 10^{-10}\ \text{W} = 3 \cdot 10^{-7}\ \text{mW} = 300\ \text{pW} \equiv \boxed{-65,23\ \text{dBm}}$$

$$\boxed{SNR_I\,[dB] = P_R\,[dBm] - P_N\,[dBm] = -22,22 - [-65,23] = \boxed{43,01\,dB}}$$

(c) $$P_R = \frac{A_c^2}{2}(1+Ka^2 P_x) \Rightarrow A_c^2 = \frac{2P_R}{1+Ka^2 P_x}$$

$$\boxed{A_c} = \sqrt{\frac{2P_R}{1+Kc^2 P_x}} = \sqrt{\frac{2\cdot 6\cdot 10^{-6}}{1+0{,}25^2\cdot 0{,}2}} = 3'44\cdot 10^{-3}\,\text{Vd} = \boxed{3'44\,mV}$$

$s_R(t) = A_c \cos(2\pi f_c t) + A_c Ka\, x(t) \cos(2\pi f_c t)$ señal tras $H_{IF}(f)$

tras modulador producto:

$$v(t) = 2A_c + 2A_c \cos(4\pi f_c t) + 2A_c Ka\, x(t) + 2A_c Ka\, x(t) \cos(4\pi f_c t)$$

tras el filtro paso bajo:

$$y(t) = 2A_c + 2A_c Ka\, x(t)$$

Sin la componente continua:

$$y_1(t) = 2A_c Ka\, x(t)$$

$$\boxed{P_{S_o}} = 4A_c^2 Ka^2 P_x = 4\cdot(3'44\cdot 10^{-3})^2\cdot 0'25^2\cdot 0'2 = 5{,}9168\cdot 10^{-7}\,W$$

$$= 5{,}9168\cdot 10^{-4}\,mW = 591{,}68\,nW = \boxed{-32{,}27\,dBm}$$

Ruido de banda estrecha tras $H_{IF}(f)$:

$$n(t) = n_c(t)\cos(2\pi f_c t) - n_s(t)\,\mathrm{sen}(2\pi f_c t)$$

tras el modulador producto:

$$n_1(t) = n(t)\cdot 4\cos(2\pi f_c t) = 2n_c(t) + 2n_c(t)\cos(4\pi f_c t) - 2n_s(t)\,\mathrm{sen}(2\pi f_c t)$$

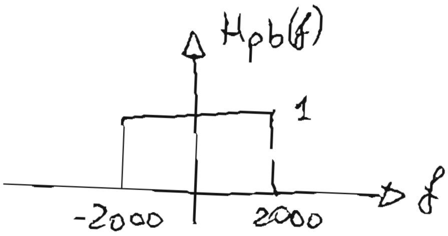
tras el filtro paso bajo.

$n_o(t) = 2n_c(t) * h_{pb}(t)$ siendo $h_{pb}(t)$ la respuesta al impulso del filtro paso bajo

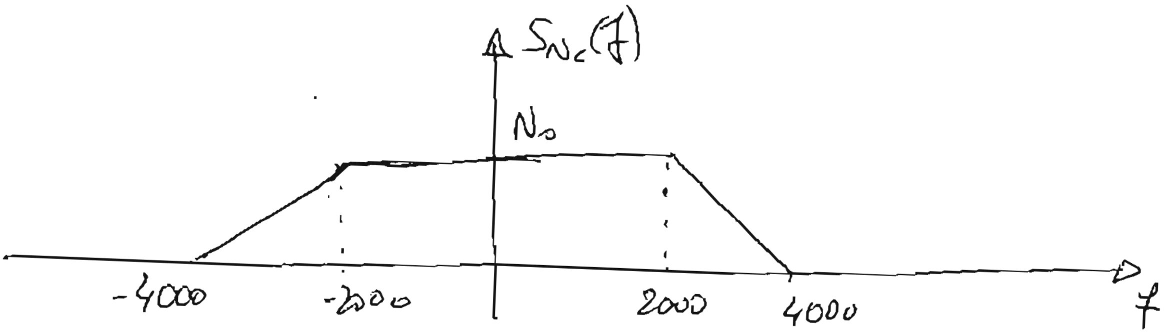
PROBLEMA 2: (CONT.)

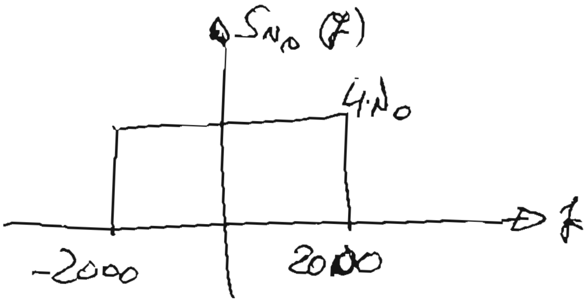
En densidades espectrales:

$$S_{No}(f) = 4\, S_{Nc}(f)\, |H_{pb}(f)|^2$$



$$S_{Nc}(f) = \begin{cases} S_N(f-f_c) + S_N(f+f_c) & |f| < 4000 \\ 0 & \text{resto} \end{cases}$$





$$\boxed{P_{No}} = \int_{-\infty}^{\infty} S_{No}(f)\, df = 4 \cdot 4000\, N_o = 4000 \cdot 4 \cdot 5 \cdot 10^{-14} = 8 \cdot 10^{-10}\, W = 8 \cdot 10^{-7}\, mW =$$

$$= \boxed{800\,pW = -60'97\ dBm}$$

$$\boxed{SNR_o = P_{So}[dBm] - P_{No}[dBm] = 28'7\ dB}$$

(d) $\boxed{P_R = 6\,\mu W = -22{,}22\ dBm}$

Como el receptor no cambia, el ruido se demodula igual, entonces

$\boxed{P_N = -65{,}23\ dBm = 300\,pW}$ y $\boxed{SNR_I = 43'01\ dB}$

$s_R(t) = \frac{A_c}{2} x(t) \cos(2\pi f_c t) - \frac{A_c}{2} \hat{x}(t) \operatorname{sen}(2\pi f_c t)$ tras filtro $H_{IF}(f)$

$P_R = \frac{A_c^2 P_x}{4}$ (Señal SSB)

$A_c^2 = \frac{4 P_R}{P_x} \Rightarrow A_c = \sqrt{\frac{4 P_R}{P_x}} = 10'95 \, mV$

tras el modulador producto:

$v(t) = 4 \cos(2\pi f_c t) \cdot s_R(t) = A_c x(t) + A_c x(t) \cos(4\pi f_c t) - A_c \hat{x}(t) \operatorname{sen}(4\pi f_c t)$

tras el filtro paso bajo:

$y(t) = A_c x(t)$

$\boxed{P_{S_o} = A_c^2 \cdot P_x = (10'95 \cdot 10^{-3})^2 \cdot 0,2 = 24 \cdot 10^{-3} \, mW = 24 \mu W \equiv -16'2 \, dBm}$

Con respecto al ruido como el receptor es el mismo:

$\boxed{P_{N_o} = 800 \, pW \equiv -60'97 \, dBm}$

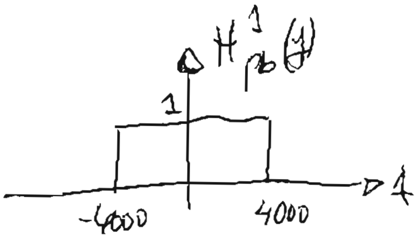
$\boxed{SNR_o [dB] = P_{S_o} [dBm] - P_{N_o} [dBm] = 44'77 \, dB}$

(e) AM. La potencia de señal a la salida no cambia:

$\boxed{P_{S_o}^{AM} = 591,68 \, nW \equiv -32,27 \, dBm}$

Con respecto al ruido:

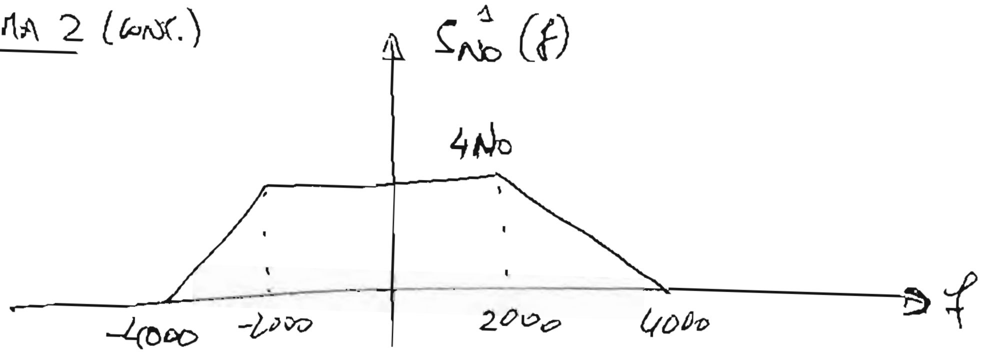
$S_{N_o}^1(f) = 4 S_{N_c}(f) \, |H_{pb}^1(f)|^2$ con



por lo que:

$S_{N_o}^1(f) = 4 S_c(f)$

PROBLEMA 2 (CONT.)



$$\boxed{P_{No}^1} = \int_{-\infty}^{\infty} S_{No}^1(f)df = 4\cdot No[4000+2000] = 24000\,No =$$

$$= 24000\cdot 5\cdot 10^{-14} = 1,2\cdot 10^{-9}\,W = 1,2\cdot 10^{-6}\,mW = 1,2\,nW = -59,21\,dBm$$

$$\boxed{SNR_o^{AM} = P_{So}^{AM}(dBm) - P_{No}^1[dBm] = \boxed{26'94\,dB}}$$

SSB. La potencia de señal a la salida no cambia:

$$\boxed{P_{So}^{SSB} = 24\mu W = -16'2\,dBm}$$

P_{No}^1 es la misma que para AM.

$$\boxed{P_{No}^1 = 1,2\,nW = -59,21\,dBm}$$

$$\boxed{SNR_o^{SSB} = P_{So}^{SSB}[dBm] - P_{No}^1[dBm] = \boxed{43,01\,dB}}$$