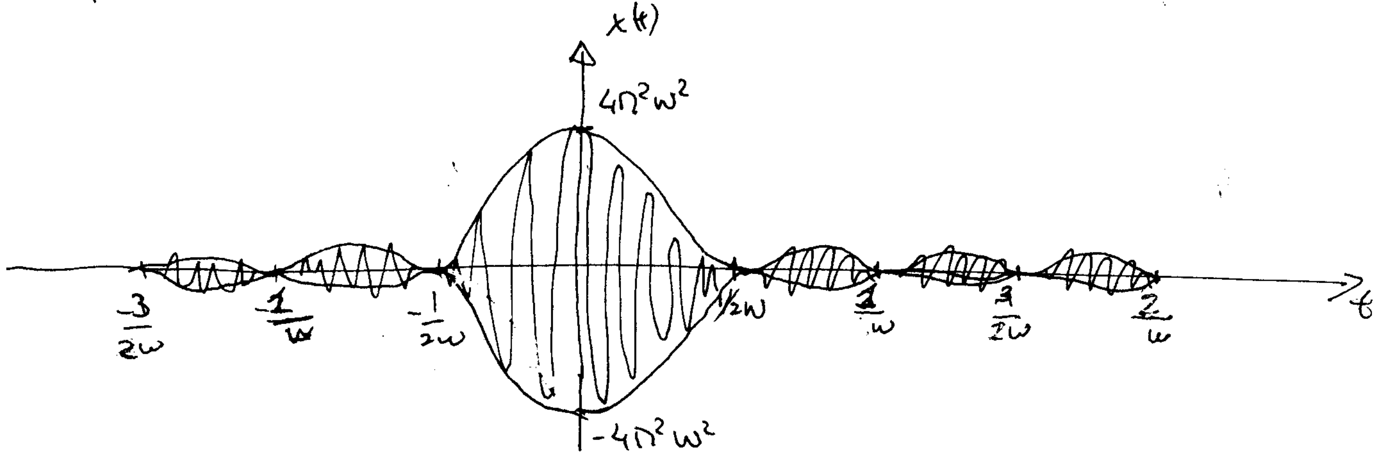
PROBLEMA 1:

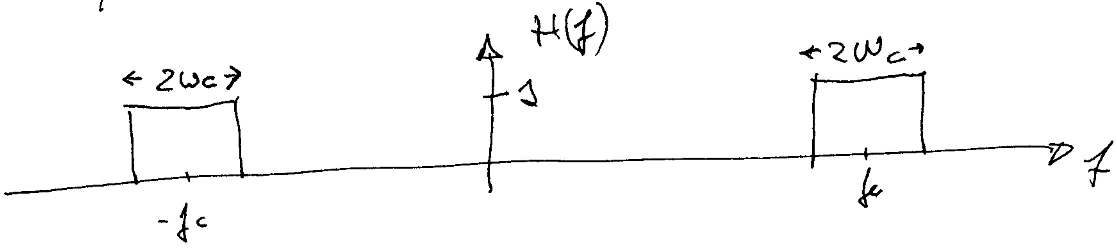
(a) $$x(t) = \frac{\operatorname{sen}^2(2\pi W t)\cos(2\pi f_c t)}{t^2} = \frac{(2\pi W)^2}{(2\pi W)^2}\,\frac{\operatorname{sen}^2(2\pi W t)\cos(2\pi f_c t)}{t^2}$$

$$= 4\pi^2 W^2 \operatorname{sinc}^2(2Wt)\cos(2\pi f_c t)$$

Se puede considerar una señal DSB con moduladora $4\pi^2 W^2 \operatorname{sinc}^2(2Wt)$ y portadora $\cos(2\pi f_c t)$. Dibujamos envolventes (moduladora positiva y negativa)



$$H(f) = \begin{cases} 1 & f_c - W_c \le |f| \le f_c + W_c \\ 0 & \text{resto} \end{cases}$$



(b) Mirando a las tablas:

$$a \operatorname{sinc}(at) \rightleftharpoons \Pi\left(\frac{f}{a}\right)$$

elevando al cuadrado en el dominio del tiempo:

$$a^2 \operatorname{sinc}^2(at) \rightleftharpoons \Pi\left(\frac{f}{a}\right) * \Pi\left(\frac{f}{a}\right) = a\,\Lambda\left(\frac{f}{a}\right) \Rightarrow a \operatorname{sinc}^2(at) \rightleftharpoons \Lambda\left(\frac{f}{a}\right)$$

también se puede obtener a partir de

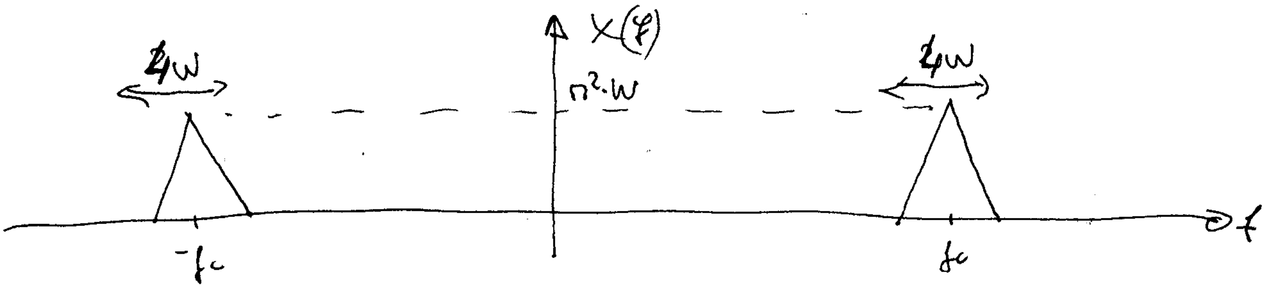
$$\Lambda\left(\frac{t}{T}\right) \rightleftharpoons T\,\mathrm{sinc}^2(fT)$$ aplicando la propiedad de dualidad

Entonces: con $a = 2W$.

$$4\pi^2 W^2 \,\mathrm{sinc}^2(2Wt) = 2\pi^2 W \cdot 2W \,\mathrm{sinc}^2(2Wt) \rightleftharpoons 2\pi^2 W \,\Lambda\left(\frac{f}{2W}\right)$$

$$x(t) = 4\pi^2 W^2 \,\mathrm{sinc}^2(2Wt)\cos(2\pi f_c t) \rightleftharpoons \cancel{2}\pi^2 W \cdot \Lambda\left(\frac{f}{2W}\right) * \left[\frac{1}{\cancel{2}}\delta(f-f_c) + \frac{1}{\cancel{2}}\delta(f+f_c)\right]$$

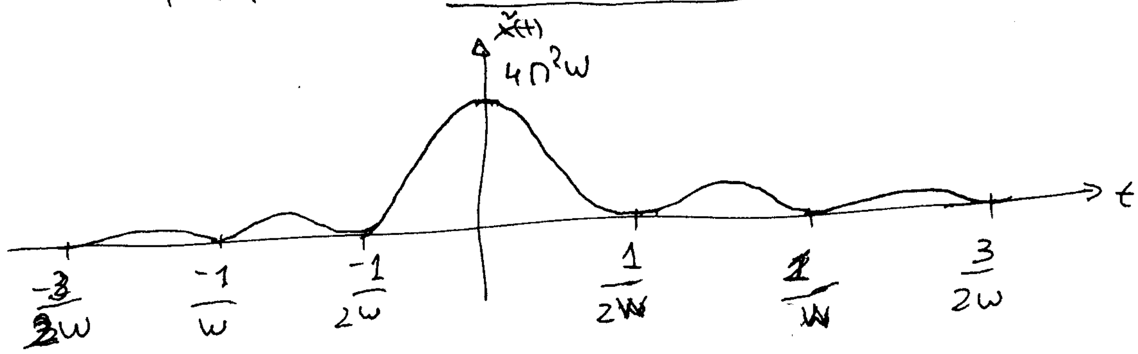
$$\boxed{X(f) = \pi^2 \cdot W \,\Lambda\left(\frac{f-f_c}{2W}\right) + \pi^2 W \,\Lambda\left(\frac{f+f_c}{2W}\right)}$$



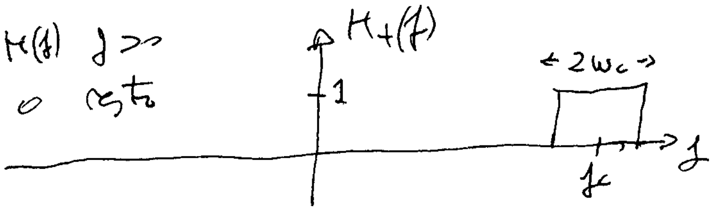
(c) $\hat{x}(t) = 4\pi^2 W^2 \,\mathrm{sinc}^2(2Wt)\,\mathrm{sen}(2\pi f_c t)$ misma moduladora y portadora en cuadratura

$$x_+(t) = 4\pi^2 W^2 \,\mathrm{sinc}^2(2Wt)\left[\cos(2\pi f_c t) + j\,\mathrm{sen}(2\pi f_c t)\right] = 4\pi^2 W^2 \,\mathrm{sinc}^2(2Wt)\exp(j2\pi f_c t)$$

$$\boxed{\tilde{x}(t) = x_+(t)\exp(-j2\pi f_c t) = 4\pi^2 W^2 \,\mathrm{sinc}^2(2Wt)}$$

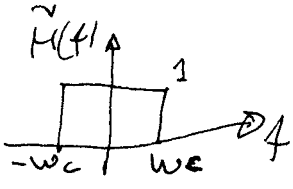


(d) $H_+(f) = \begin{cases} H(f) & f > 0 \\ 0 & \text{resto} \end{cases}$



$$H_+(f) = \begin{cases} 1 & f_c - W_c \le f \le f_c + W_c \\ 0 & \text{resto} \end{cases}$$

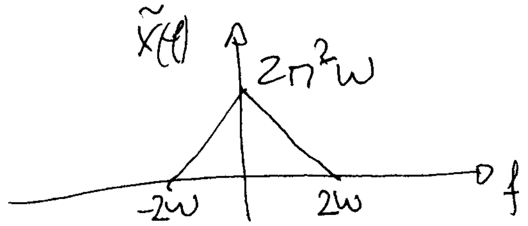
$\tilde{H}(f) = H_+(f + f_c)$



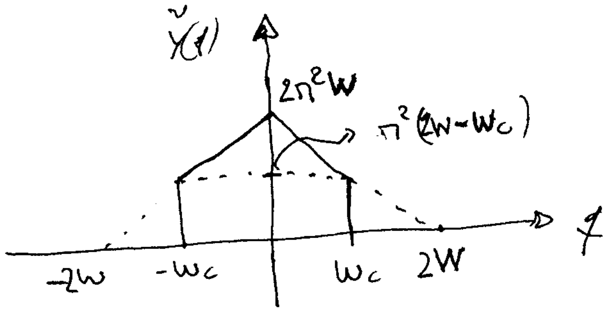
$$\boxed{\tilde{H}(f) = \Pi\left(\frac{f}{2W_c}\right)}$$

PROBLEMA 1 (CONT.)

(e) $\tilde{x}(t) = 4\pi^2 W \,\mathrm{sinc}^2(2Wt) \rightleftharpoons 2\pi^2 W \Lambda\left(\frac{f}{2W}\right) = \tilde{X}(f)$



(f) $\tilde{Y}(f) = \tilde{X}(f)\cdot \tilde{H}(f)$ como $W_c < W$ se recorta



Se puede poner como la suma de una función rectangular y otra triangular:

$$\tilde{Y}(f) = \pi^2 (2W - W_c)\,\Pi\left(\frac{f}{2W_c}\right) + \pi^2 W_c\,\Lambda\left(\frac{f}{W_c}\right)$$

(g) Haciendo transformada inversa:

teniendo en cuenta: $a\,\mathrm{sinc}(at) \rightleftharpoons \Pi\left(\frac{f}{a}\right)$ y $a\,\mathrm{sinc}^2(at) \rightleftharpoons \Lambda\left(\frac{f}{a}\right)$

$$\tilde{y}(t) = 2\pi^2 (2W - W_c) W_c\,\mathrm{sinc}(2W_c t) + \pi^2 W_c^2\,\mathrm{sinc}^2(W_c t)$$

(h) $\boxed{y(t)} = \mathrm{Re}\{\tilde{y}(t)\exp(j2\pi f_c t)\} = \pi^2 W_c\left[(4W - 2W_c)\,\mathrm{sinc}(2W_c t) + W_c\,\mathrm{sinc}^2(W_c t)\right]\cos(2\pi f_c t)$

PROBLEMA 2.

(a) $E[S(t)] = E\left[\frac{A_c}{2} M(t) \cos(2\pi f_c t + \Theta) - \frac{A_c}{2} \hat{M}(t) \operatorname{sen}(2\pi f_c t + \Theta)\right]$ (prop. indep)

$= \frac{A_c}{2} E[M(t)]\, E[\cos(2\pi f_c t + \Theta)] - \frac{A_c}{2} E[\hat{M}(t)]\, E[\operatorname{sen}(2\pi f_c t + \Theta)]$

puesto que $M(t) \longrightarrow \boxed{h(t) = \frac{1}{\pi t}} \longrightarrow \hat{M}(t)$ sabemos que $E[\hat{M}(t)] = \cancel{E[M(t)]}^{\,0} \cdot H(0) = 0$

$$\boxed{E[S(t)] = \frac{A_c}{2} \cdot 0 \cdot E[\cos(2\pi f_c t + \Theta)] - \frac{A_c}{2} \cdot 0 \cdot E[\operatorname{sen}(2\pi f_c t + \Theta)] = 0}$$

(b) $R_S(\tau) = E[S(t) S(t+\tau)] = E\left[\left(\frac{A_c}{2} M(t) \cos(2\pi f_c t + \Theta) - \frac{A_c}{2} \hat{M}(t) \operatorname{sen}(2\pi f_c t + \Theta)\right) \cdot \right.$

$\left.\left(\frac{A_c}{2} M(t+\tau) \cos(2\pi f_c (t+\tau) + \Theta) - \frac{A_c}{2} \hat{M}(t+\tau) \operatorname{sen}(2\pi f_c (t+\tau) + \Theta)\right)\right]$

prop. ind

$= \frac{A_c^2}{4} E[M(t) M(t+\tau)]\, E[\cos(2\pi f_c t + \Theta) \cos(2\pi f_c (t+\tau) + \Theta)]$

$- \frac{A_c^2}{4} E[M(t) \hat{M}(t+\tau)]\, E[\cos(2\pi f_c t + \Theta) \operatorname{sen}(2\pi f_c (t+\tau) + \Theta)]$

$- \frac{A_c^2}{4} E[\hat{M}(t) M(t+\tau)]\, E[\operatorname{sen}(2\pi f_c t + \Theta) \cos(2\pi f_c (t+\tau) + \Theta)]$

$+ \frac{A_c^2}{4} E[\hat{M}(t) \hat{M}(t+\tau)]\, E[\operatorname{sen}(2\pi f_c t + \Theta) \operatorname{sen}(2\pi f_c (t+\tau) + \Theta)]$

Sabemos que $R_M(\tau) = E[M(t) M(t+\tau)]$

$R_{\hat{M}}(\tau) = E[\hat{M}(t) M(t+\tau)]$

y que $M(t)$ y $\hat{M}(t)$ tienen la misma función de autocorrelación, puesto que $M(t)$ y $\hat{M}(t)$ se diferencian sólo a la fase y la autocorrelación no depende para nada de la fase, entonces $R_M(\tau) = R_{\hat{M}}(\tau)$

Además

$$R_{M\hat{M}}(\tau) = E[\hat{M}(t)\,M(t+\tau)]$$
$$R_{\hat{M}M}(\tau) = E[M(t)\,\hat{M}(t+\tau)]$$

también sabemos que:

$$R_{\hat{M}M}(\tau) = \hat{R}_M(\tau)$$
$$R_{M\hat{M}}(\tau) = -\hat{R}_M(\tau)$$

Entonces:

$$R_S(\tau) = \frac{A_c^2}{4} R_M(\tau)\left[\cos(2\pi f_c t+\Theta)\cos(2\pi f_c(t+\tau)+\Theta) + \operatorname{sen}(2\pi f_c t+\Theta)\operatorname{sen}(2\pi f_c(t+\tau)+\Theta)\right]$$
$$-\frac{A_c^2}{4}\hat{R}_M(\tau)\left[\cos(2\pi f_c t+\Theta)\operatorname{sen}(2\pi f_c(t+\tau)+\Theta) - \operatorname{sen}(2\pi f_c t+\Theta)\cos(2\pi f_c(t+\tau)+\Theta)\right]$$

aplicando las relaciones trigonométricas conocidas:

$$R_S(\tau) = \frac{A_c^2}{4} R_M(\tau)\cos(2\pi f_c\tau) - \frac{A_c^2}{4}\hat{R}_M(\tau)\operatorname{sen}(2\pi f_c\tau)$$

(c) Puesto que la media es cero y la autocorrelación sólo depende de τ (no depende de t), sí es estacionaria en sentido amplio.

(d) Tomando transformada de Fourier de la $R_S(\tau) \rightleftharpoons S_S(f)$

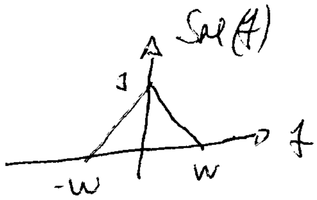
$$S_S(f) = \frac{A_c^2}{4} S_M(f) * \left[\frac{1}{2}\delta(f-f_c) + \frac{1}{2}\delta(f+f_c)\right]$$
$$-\frac{A_c^2}{4}\left[-j\,\mathrm{sgn}(f)\,S_M(f)\right] * \left[\frac{1}{2j}\delta(f-f_c) - \frac{1}{2j}\delta(f-f_c)\right]$$

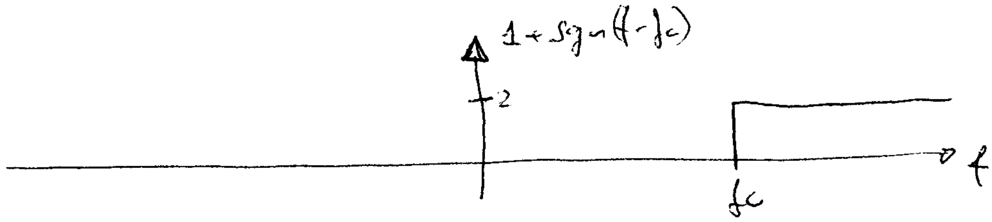
$$= \frac{A_c^2}{8} S_M(f-f_c) + \frac{A_c^2}{8} S_M(f+f_c) + \frac{A_c^2}{8}\mathrm{sgn}(f-f_c)S_M(f-f_c) - \frac{A_c^2}{8}\mathrm{sgn}(f+f_c)S_M(f+f_c)$$

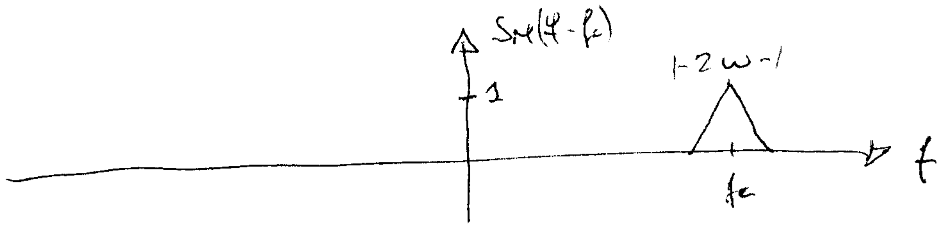
$$= \frac{A_c^2}{8}\left[1+\mathrm{sgn}(f-f_c)\right]S_M(f-f_c) + \frac{A_c^2}{8}\left[1-\mathrm{sgn}(f+f_c)\right]S_M(f+f_c)$$

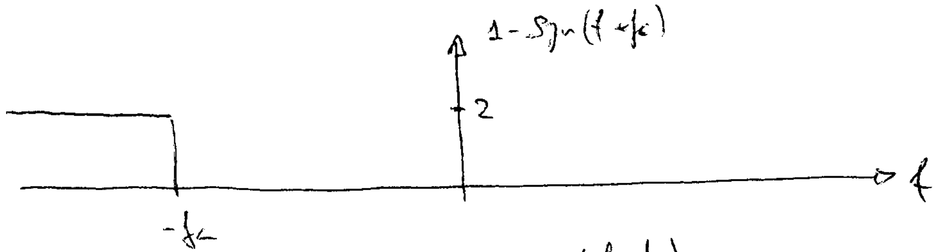
PROBLEMA 2 (CONT.)

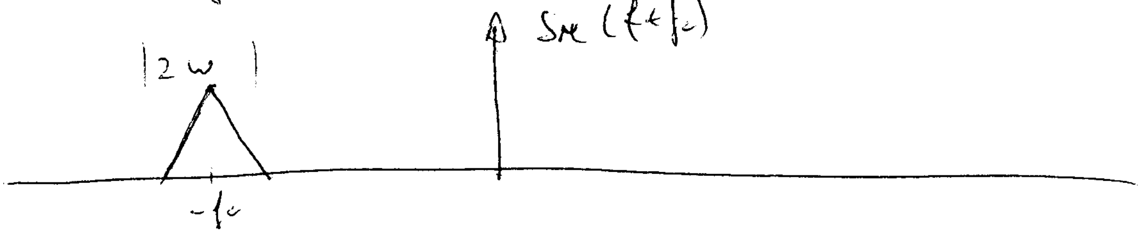
$$S_M(f) = \begin{cases} 1-\dfrac{|f|}{W} & |f|<W \\ 0 & \text{Resto} \end{cases} = \Lambda\left(\frac{f}{W}\right)$$

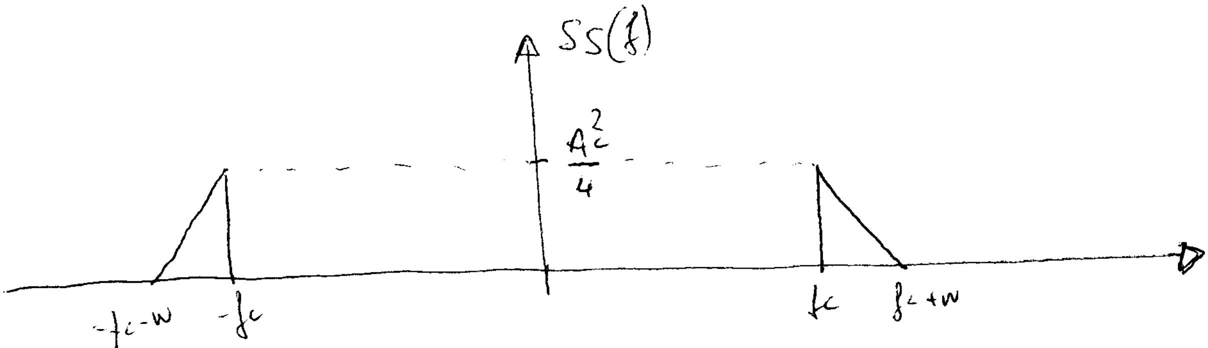












$$\boxed{S_S(f) = \frac{A_c^2}{8}\left[1+\text{sgn}(f-f_c)\right]\Lambda\left(\frac{f-f_c}{W}\right) + \frac{A_c^2}{8}\left[1-\text{sgn}(f+f_c)\right]\Lambda\left(\frac{f+f_c}{W}\right)}$$

Como se puede apreciar el ancho de banda es W.

PROBLEMA 3

(a.1) $S_M(f) = \begin{cases} 1 - \dfrac{f^2}{50} & |f| < 5 \\ \dfrac{f^2}{50} - \dfrac{2|f|}{5} + 2 & 5 < |f| < 10 \\ 0 & \text{Resto} \end{cases}$

Como es de esperar, $S_M(f)$ es positiva y par. Son tres parábolas unidas: una entre -10 y -5, otra entre -5 y 5 y una última entre 5 y 10. El valor de la función en los ptos relevantes:

$S_M(-10) = 0$

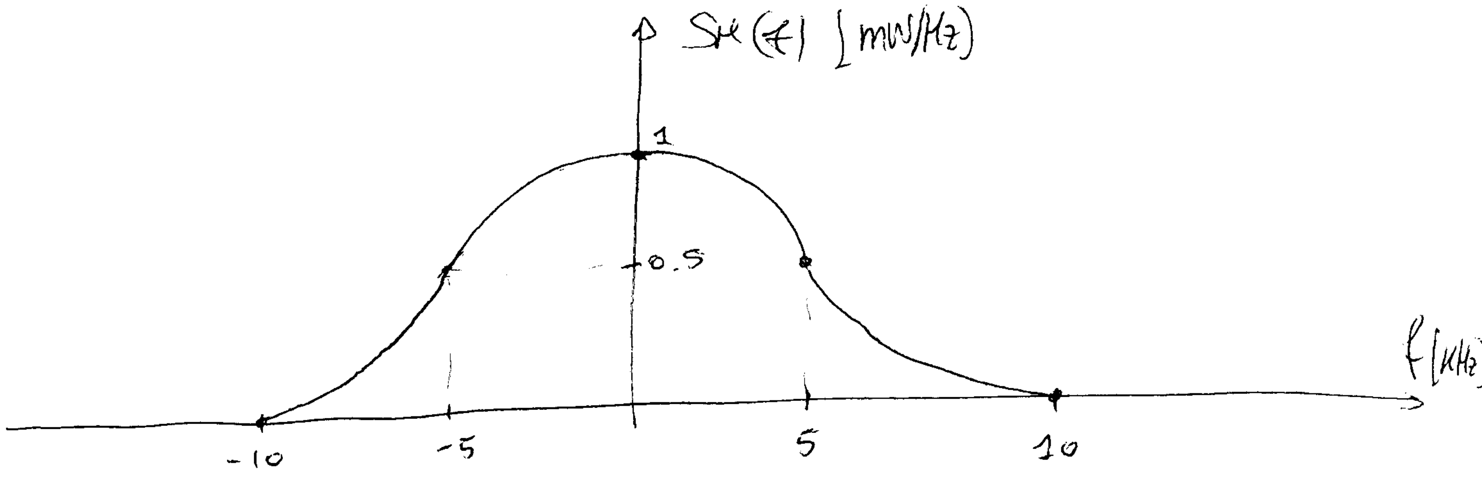
$S_M(-5) = S_M(5) = \frac{1}{2}$

$S_M(0) = 1$

* máximos y mínimos $1 - \frac{f^2}{50} \Rightarrow$ derivada $-\frac{2f}{50} = 0 \Rightarrow f = 0$, 2ª derivada negativa, entonces máximo para $f=0$.

* máximos y mínimos $\frac{f^2}{50} - \frac{2f}{5} + 2 \Rightarrow$ derivada $\frac{2f}{50} - \frac{2}{5} = 0 \Rightarrow f = 10$, 2ª derivada positiva, entonces mínimo para $f=10$.

* máximos y mínimos $\frac{f^2}{50} + \frac{2f}{5} + 2 \Rightarrow$ derivada $\frac{2f}{50} + \frac{2}{5} = 0 \Rightarrow f = -10$, 2ª derivada positiva, entonces mínimo para $f=-10$.



Vamos a calcular la potencia $P_M = \int_{-\infty}^{\infty} S_M(f)df$. Como es par:

$$\boxed{P_M} = 2\int_0^{\infty} S_M(f)\,df = 2\int_0^5 \left(1 - \frac{f^2}{50}\right)df + 2\int_5^{10}\left(\frac{f^2}{50} - \frac{2f}{5} + 2\right)df =$$

$$2\left[f - \frac{f^3}{150}\right]_0^5 + 2\left[\frac{f^3}{150} - \frac{f^2}{5} + 2f\right]_5^{10} =$$

$$2\left[5 - \frac{125}{150} + \frac{1000}{150} - \cancel{\frac{100}{5}} + \cancel{20} - \frac{125}{150} + \frac{25}{5} - 10\right] =$$

$$2\left[\cancel{5} - \frac{5}{6} + \frac{20}{3} - \frac{5}{6} + \cancel{5} - \cancel{10}\right] = 2\,\frac{-5 + 40 - 5}{6} = 2 \cdot 5 = 10 \text{ mW/Hz} \cdot \text{KHz}$$

$$\boxed{= 10\,W}$$

(a.2) $\Delta f = K_f \cdot \max_t |m(t)| = 5000\,Hz/V \cdot 10\,V = 50\,KHz$

$$D = \frac{\Delta f}{W} = \frac{50\,KHz}{10\,KHz} = 5$$

$$B_T^{CARSON} = 2\Delta f + 2W = 2 \cdot 50\,KHz + 2 \cdot 10\,KHz = 120\,KHz$$

$$B_T^{1\%} = 2n_{max} \cdot W = 16 \cdot 10\,KHz = 160\,KHz$$

$$\boxed{B_T} = \frac{B_T^{CARSON} + B_T^{1\%}}{2} = \boxed{140\,KHz}$$

(a.3) $\boxed{P_c = P_S^T = \frac{A_c^2}{2} = \frac{400^2}{2} = 80KW}$

(a.4) $\boxed{P_S^R = \frac{P_S^T}{Att} = \frac{80 \cdot 10^3\,W}{10^{55/10}} = 253mW}$

PROBLEMA 3 (CONT.)

(a.5) $N_0 = 3.5 \cdot 10^{-9}$

para estar por encima del umbral $P_S^R \geq 20 N_0 B_T$

por el apartado anterior $P_S^R = 253\,mW$

$$20 \cdot N_0 \cdot B_T = 20 \cdot 3{,}5 \cdot 10^{-9} \cdot 140 \cdot 10^3 = 9.8\,mW$$

Como $253\,mW \geq 9.8\,mW$ sí estamos operando por encima del umbral.

(a.6)
$$\boxed{SNR_I = CNR} = \frac{A_c^2(Rx)}{2 N_0 B_T} = \frac{P_S^R}{N_0 B_T} = \frac{253 \cdot 10^{-3}}{3.5 \cdot 10^{-9} \cdot 140 \cdot 10^3} = 516{,}33 \equiv \boxed{27{,}13\,dB}$$

$$\boxed{SNR_C} = \frac{A_c^2(Rx)}{2 N_0 W} = \frac{P_S^R}{N_0 W} = \frac{253 \cdot 10^{-3}}{3.5 \cdot 10^{-9} \cdot 10 \cdot 10^3} = 7228{,}57 \equiv \boxed{38{,}59\,dB}$$

(a.7)
$$\boxed{SNR_O} = \frac{3 A_c^2(Rx) K_f^2 P_M}{2 W^3 N_0} = \frac{3 \cdot 253 \cdot 10^{-3} \cdot 5000^2 \cdot 10}{(10 \cdot 10^3)^3 \cdot 3.5 \cdot 10^{-9}} = 54214{,}29 \equiv \boxed{47{,}34\,dB}$$

$$\boxed{FOM} = \frac{3 \cdot K_f^2 P_M}{W^2} = \frac{3 \cdot 5000^2 \cdot 10}{(10 \cdot 10^3)^2} = 7.5 \equiv \boxed{8{,}75\,dB}$$

$$\boxed{\frac{SNR_O}{SNR_I}} = \frac{3 B_T K_f^2 P_M}{W^3} = \frac{3 \cdot 140 \cdot 10^3 \cdot 5000^2 \cdot 10}{(10 \cdot 10^3)^3} = 105 \equiv \boxed{20{,}21\,dB}$$

(b.1) se tiene

$$m(t) \longrightarrow \boxed{H_{pe}(f)} \longrightarrow m_1(t)$$

a nivel densidad espectral: $S_{M_1}(f) = |H_{pe}(f)|^2 S_M(f)$

$S_{M_1}(f) = K^2 (|f| + 1) S_M(f)$ puesto que $S_M(f) = 0$ para $|f| > 10\,kHz$.

para que se conserve la potencia $P_M = 10 = S_M(f)$

$$10 = P_M = \int_{-\infty}^{\infty} |H_{pe}(f)|^2 S_M(f)\,df.$$

Como se puede ver tanto $H_{pe}(f)$ como $S_M(f)$ son pares, entonces:

$$10 = 2\int_0^{10} K^2(1+f)\,S_M(f)\,df = 2K^2\int_0^5 (1+f)\left(1-\frac{f^2}{50}\right)df +$$

$$+2K^2\int_5^{10}(1+f)\left(\frac{f^2}{50}-\frac{2f}{5}+2\right)df =$$

$$2K^2\int_0^5 \left(f-\frac{f^3}{50}+1-\frac{f^2}{50}\right)df + 2K^2\int_5^{10}\left(\frac{f^3}{50}-\frac{2f^2}{5}+2f+\frac{f^2}{50}-\frac{2f}{5}+2\right)df =$$

$$2K^2\left[\frac{f^2}{2}-\frac{f^4}{200}+f-\frac{f^3}{150}\right]_0^5 + 2K^2\left[\frac{f^4}{200}-\frac{2f^3}{15}+f^2+\frac{f^3}{150}-\frac{f^2}{5}+2f\right]_5^{10} =$$

$$2K^2\left[\frac{25}{2}-\frac{625}{200}+5-\frac{125}{150}+\frac{10000}{200}-\frac{2000}{15}+100+\frac{1000}{150}-\frac{100}{5}+20\right.$$

$$\left.-\frac{625}{200}+\frac{250}{15}-25-\frac{125}{150}+\frac{25}{5}-10\right] =$$

$$2K^2\left[\frac{25}{2}-\frac{25}{8}+\cancel{5}-\frac{5}{6}+50-\frac{400}{3}+100+\frac{20}{3}-\cancel{20}+\cancel{20}-\frac{25}{8}+\frac{50}{3}-25-\frac{5}{6}+\cancel{5}-\cancel{10}\right] =$$

$$2K^2\left[\frac{25}{2}-\frac{25}{4}-\frac{5}{3}+125-\frac{370}{3}\right] = \cancel{2}K^2\left[\frac{150-75-1340+1500}{\cancel{12}\ 6}\right] = \frac{235\,K^2}{6} = 10$$

$$\boxed{k=\sqrt{\frac{60}{235}} \simeq 0.5053}$$

PROBLEMA 3 (CONT.)

(b.2) La mejora a preénfasis está definida por:

$$D = \frac{2W^3}{3\int_{-W}^{W} f^2 |H_{de}(f)|^2 \, df} \qquad \text{frecuencia en kHz.}$$

con

$$H_{de}(f) = \frac{1}{H_{pe}(f)} = \begin{cases} \dfrac{1}{K\sqrt{|f|+1}} & |f| \leq 10 \\ 0 & \text{resto.} \end{cases}$$

Determinando la integral:

$$\int_{-10}^{10} \frac{f^2}{k^2(1+|f|)} df \overset{\text{par}}{=} 2\int_0^{20} \frac{f^2}{k^2(1+f)} df = \frac{2}{k^2}\int_0^{10} \frac{f^2}{1+f} df \quad \Bigg| \begin{array}{l} f+1=u \\ df = du \end{array}$$

$$= \frac{2}{k^2}\int_1^{11} \frac{(u-1)^2}{u} du = \frac{2}{k^2}\int_1^{11} \frac{u^2-2u+1}{u} du = \frac{2}{k^2}\int_1^{11}\left(u-2+\frac{1}{u}\right)du$$

$$= \frac{2}{k^2}\left[\frac{u^2}{2} - 2u + \ln u\right]_1^{11} = \frac{2}{k^2}\left[\frac{121}{2} - 22 + \ln(11) - \frac{1}{2} + 2 - \cancelto{0}{\ln 1}\right]$$

$$= \frac{2}{k^2}\left[60 - 20 + \ln(11)\right] = \frac{2(40+\ln(11))}{k^2} = \frac{80 + 2\ln(11)}{k^2}$$

Como $k^2 = \frac{60}{235}$

$$= \frac{80 + 2\ln(11)}{60/235} = \frac{235[40+\ln(11)]}{30} = \frac{235}{30}(40 + \ln(11))$$

$$\boxed{D = \frac{2\cdot 10^3}{3\,\frac{235}{30}(40+\ln(11))} = \frac{20000}{235(40+\ln(11))} \simeq 2 \equiv 3\,\text{dB}}$$