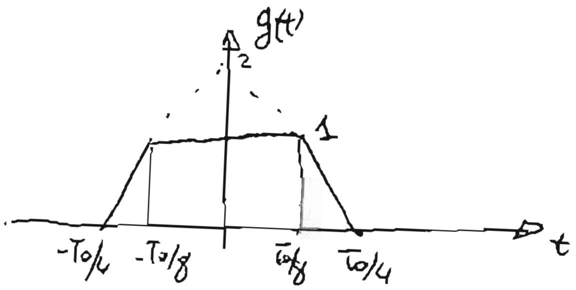
PROBLEMA 1.

Sabemos que la representación en serie de Fourier es

$$g_p(t) = \sum_{n=-\infty}^{\infty} c_n \exp\left(\frac{j2\pi n t}{T_0}\right)$$

siendo T_0 el periodo de la señal.

Para calcular c_n, usamos la función generadora $g(t)$



entonces $c_n = \frac{1}{T_0} G\left(\frac{n}{T_0}\right)$, siendo $G(f)$ la transformada de Fourier de $g(t)$.

Podemos poner $g(t)$ como la diferencia de dos funciones triangulares:

$$\boxed{g(t) = 2\Lambda\left(\frac{t}{T_0/4}\right) - \Lambda\left(\frac{t}{T_0/8}\right) = 2\Lambda\left(\frac{4t}{T_0}\right) - \Lambda\left(\frac{8t}{T_0}\right)}$$

Sabemos que

$$\Lambda\left(\frac{t}{T}\right) \rightleftharpoons T\,\mathrm{sinc}^2(fT)$$

entonces:

$$G(f) = 2\cdot\frac{T_0}{4}\,\mathrm{sinc}^2(fT_0/4) - \frac{T_0}{8}\,\mathrm{sinc}^2(fT_0/8) =$$

$$= \frac{T_0}{2}\left[\mathrm{sinc}^2\left(\frac{fT_0}{4}\right) - \frac{1}{4}\,\mathrm{sinc}^2\left(\frac{fT_0}{8}\right)\right] = \frac{T_0}{2}\left[\frac{\mathrm{sen}^2\left(\frac{fT_0}{4}\right)}{\frac{\pi^2 f^2 T_0^2}{16}} - \frac{\mathrm{sen}^2\left(\frac{fT_0}{8}\right)}{4\cdot\frac{\pi^2 f^2 T_0^2}{64}}\right]$$

$$= \frac{T_0}{2}\left[\frac{16\,\mathrm{sen}^2\left(\frac{fT_0}{4}\right)}{\pi^2 f^2 T_0^2} - \frac{16\,\mathrm{sen}^2\left(\frac{fT_0}{8}\right)}{\pi^2 f^2 T_0^2}\right] = \frac{8\left[\cancel{\frac{1}{2}} - \frac{1}{2}\cos\left(\frac{\pi f T_0}{2}\right) - \cancel{\frac{1}{2}} + \frac{1}{2}\cos\left(\frac{\pi f T_0}{4}\right)\right]}{\pi^2 f^2 T_0}$$

$$\boxed{G(f) = \frac{4}{\pi^2 f^2 T_0}\left[\cos\left(\frac{\pi f T_0}{4}\right) - \cos\left(\frac{\pi f T_0}{2}\right)\right]}$$

$$c_n = \frac{1}{T_0} G\left(\frac{n}{T_0}\right) = \frac{4}{\cancel{T_0} \cdot \pi^2 \cdot \frac{n^2}{\cancel{T_0^2}} \cancel{T_0}} \left[\cos\left(\frac{\pi n}{4}\right) - \cos\left(\frac{\pi n}{2}\right)\right]$$

$$c_n = \frac{4}{\pi^2 n^2} \left[\cos\left(\frac{\pi n}{4}\right) - \cos\left(\frac{\pi n}{2}\right)\right]$$

$$g(t) = \sum_{n=-\infty}^{\infty} \frac{4}{\pi^2 n^2} \left[\cos\left(\frac{\pi n}{4}\right) - \cos\left(\frac{\pi n}{2}\right)\right] \exp\left(j \frac{2\pi n t}{T_0}\right)$$

para $n=0$, aplicamos L'Hopital

$$c_0 = \frac{4}{\pi^2} \left. \frac{-\frac{\pi}{4} \operatorname{sen}\left(\frac{\pi n}{4}\right) + \frac{\pi}{2} \operatorname{sen}\left(\frac{\pi n}{2}\right)}{2n} \right|_{n=0}$$

Volvemos a aplicar L'Hopital

$$c_0 = \frac{4}{\pi^2} \left. \frac{-\frac{\pi^2}{16} \cos\left(\frac{\pi n}{4}\right) + \frac{\pi^2}{4} \cos\left(\frac{\pi n}{4}\right)}{2} \right|_{n=0} = \frac{4}{\pi^2} \frac{\frac{\pi^2}{4} - \frac{\pi^2}{16}}{2} = \frac{1}{2} - \frac{1}{8} = \frac{4-1}{8} = \frac{3}{8}$$

$$g(t) = \frac{3}{8} + \sum_{n=1}^{\infty} \frac{4}{\pi^2 n^2} \left[\cos\left(\frac{\pi n}{4}\right) - \cos\left(\frac{\pi n}{2}\right)\right] \left[\exp\left(j \frac{2\pi n t}{T_0}\right) + \exp\left(-j \frac{2\pi n t}{T_0}\right)\right]$$

$$\boxed{g(t) = \frac{3}{8} + \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{\cos\left(\frac{\pi n}{4}\right) - \cos\left(\frac{\pi n}{2}\right)}{n^2} \cos\left(\frac{2\pi n t}{T_0}\right)}$$

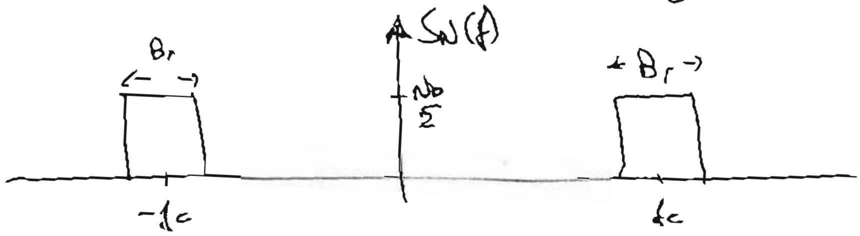
PROBLEMA 2:

a) $\sigma^2 = \int_{-\infty}^{\infty} S_N(f)\,df$ siendo $S_N(f)$ la densidad espectral de potencia de $n(t)$ dada por

$$S_N(f) = |H_{pb}(f)|^2 S_W(f) = \frac{N_0}{2}|H_{pb}(f)|^2$$

siendo $H_{pb}(f)$ el filtro paso banda con ancho de banda B_T y frecuencia central f_c. Lo suponemos ideal y de ganancia unidad

$$\boxed{\sigma^2 = \int_{-\infty}^{\infty}\frac{N_0}{2}|H_{pb}(f)|^2 df = \frac{N_0}{2}\int_{-f_c-\frac{B_T}{2}}^{-f_c+\frac{B_T}{2}} df + \frac{N_0}{2}\int_{f_c-\frac{B_T}{2}}^{f_c+\frac{B_T}{2}} df = 2\cdot\frac{N_0}{2}\cdot B_T = N_0 B_T}$$



b) Las componentes en fase y cuadratura son Gaussianas y con media cero. La varianza de cada componente será σ^2. Además como el ruido $n(t)$ es localmente simétrico en $\pm f_c$, las componentes serán independientes. La función de densidad conjunta será el producto de las marginales

$$f_{N_c N_s}(n_c, n_s) = f_{N_c}(n_c)\cdot f_{N_s}(n_s)$$

$$f_{N_c}(n_c) = \frac{1}{\sqrt{2\pi N_0 B_T}} \exp\left(-\frac{n_c^2}{2N_0 B_T}\right)$$

$$f_{N_s}(n_s) = \frac{1}{\sqrt{2\pi N_0 B_T}} \exp\left(-\frac{n_s^2}{2N_0 B_T}\right)$$

$$\boxed{f_{N_cN_s}(n_c, n_s) = \frac{1}{2\pi N_0 B_T} \exp\left(-\frac{n_c^2 + n_s^2}{2 N_0 B_T}\right)}$$

c) El detector de envolvente:

$r^2(t) = n_c^2(t) + n_s^2(t)$

$\psi(t) = \operatorname{atan}\left(\frac{n_s(t)}{n_c(t)}\right)$

$n_c(t) = r(t)\cos[\psi(t)]$

$n_s(t) = r(t)\,\mathrm{sen}[\psi(t)]$

$y(t) = r(t)$ salida detector de envolvente

Sabemos que es Rayleigh por teoría.

$$\boxed{f_R(r) = f_Y(y) = \frac{y}{N_0 B_T} \exp\left(-\frac{y^2}{2 N_0 B_T}\right) u(y)}$$

d) $z(t) = \ln[y(t)]$

$$\boxed{E[z(t)]} = E[\ln[y(t)]] = \int_{-\infty}^{\infty} \ln[y] \cdot f_Y(y)\, dy =$$

$$= \int_0^{\infty} \frac{\ln[y] \cdot y}{N_0 B_T} \exp\left[-\frac{y^2}{2 N_0 B_T}\right] dy$$

$u = \frac{y^2}{2N_0B_T}$ $\quad y = \sqrt{2uN_0B_T}$

$du = \frac{\cancel{2}y\, dy}{\cancel{2}N_0B_T}$

$$= \int_0^{\infty} \ln\sqrt{2uN_0B_T} \cdot \exp(-u)\, du$$

$$= \underbrace{\frac{1}{2}\int_0^{\infty} \ln(u) \exp(-u)\, du}_{-\gamma} + \frac{1}{2}\left[\ln 2 + \ln N_0 + \ln B_T\right] \underbrace{\int_0^{\infty} \exp(-u)\, du}_{[-e^{-u}]_0^{\infty} = 1}$$

$$= \frac{1}{2}\left[\ln(2N_0B_T) - \gamma\right] = \boxed{\frac{\ln(2N_0B_T) - \gamma}{2}}$$

e) $\quad var(z(t)) = E[z^2(t)] - E^2[z(t)]$

$$E[z^2(t)] = E[\ln^2[y(t)]] = \int_{-\infty}^{\infty} \ln^2[y]\, f_y(y)\, dy =$$

$$= \int_0^\infty \frac{\ln^2[y]\cdot y}{N_0 B_T} \exp\left[-\frac{y^2}{2N_0 B_T}\right] dy \quad \Bigg| \quad u = \frac{y^2}{2N_0B_T} \qquad du = \frac{\cancel{2}y\,dy}{\cancel{2}N_0B_T} \qquad y = \sqrt{2uN_0B_T}$$

$$= \int_0^\infty \ln^2\sqrt{2uN_0B_T}\, \exp(-u)\, du = \frac{1}{4}\int_0^\infty \left[\ln(u) + \ln(2N_0B_T)\right]^2 \exp(-u)\, du$$

$$= \frac{1}{4}\underbrace{\int_0^\infty \ln^2(u)\, e^{-u}\, du}_{=\gamma^2+\frac{\pi^2}{6}} + \frac{1}{4}\ln^2(2N_0B_T)\underbrace{\int_0^\infty e^{-u}\, du}_{=1} + \frac{1}{2}\ln(2N_0B_T)\underbrace{\int_0^\infty \ln(u)\, e^{-u}\, du}_{=-\gamma}$$

$$= \frac{\gamma^2}{4} + \frac{\pi^2}{24} + \frac{1}{4}\ln^2(2N_0B_T) - \frac{1}{2}\gamma\ln(2N_0B_T)$$

$$var[z(t)] = \frac{\gamma^2}{4} + \frac{\pi^2}{24} + \frac{1}{4}\ln^2(2N_0B_T) - \frac{1}{2}\gamma\ln(2N_0B_T) - \left[\frac{\ln(2N_0B_T) - \gamma}{2}\right]^2$$

$$= \cancel{\frac{\gamma^2}{4}} + \frac{\pi^2}{24} + \cancel{\frac{1}{4}\ln^2(2N_0B_T)} - \cancel{\frac{1}{2}\gamma\ln(2N_0B_T)} - \cancel{\frac{1}{4}\ln^2(2N_0B_T)} - \cancel{\frac{\gamma^2}{4}} + \cancel{\frac{1}{2}\gamma\ln(2N_0B_T)}$$

$$\boxed{var[z(t)] = \frac{\pi^2}{24}}$$

PROBLEMA 3:

a) $$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(2\pi(f_c + n f_m)t)$$

$$\boxed{\beta = \frac{\Delta f}{f_m} = \frac{A_m \cdot K_f}{f_m} = \frac{5 \text{ Volts} \cdot 1200 \text{ Hz/Volt}}{2600 \text{ Hz}} = 2{,}3077}$$

Los bandas laterales están separadas f_m.

El filtro paso banda tiene como frecuencia de corte:

$$f_{c1} = f_c + \frac{9 f_m}{2} = f_c + 4'5 f_m$$
$$f_{c2} = f_c - \frac{9 f_m}{2} = f_c - 4'5 f_m$$

Los bandas que caen dentro de este filtro:

$f_c - 4f_m$	f_c	$f_c + f_m$
$f_c - 3f_m$		$f_c + 2f_m$
$f_c - 2f_m$		$f_c + 3f_m$
$f_c - f_m$		$f_c + 4f_m$

9 componentes

Tras el filtro

$$S_1(f) = \frac{A_c}{2} \sum_{n=-4}^{n=4} J_n(\beta) \left[\delta(f - f_c - n f_m) + \delta(f + f_c + n f_m) \right]$$

Las amplitudes:

$$\frac{A_c}{2} J_n(\beta) = \frac{A_c}{2} J_n(2{,}3077)$$

Necesito: $J_0(2{,}3077)$, $J_1(2{,}3077)$, $J_2(2{,}3077)$, $J_3(2{,}3077)$ y $J_4(2{,}3077)$

ya que

$$J_{-1}(2{,}3077) = -J_1(2{,}3077) \qquad J_{-3}(2{,}3077) = -J_3(2{,}3077)$$
$$J_{-2}(2{,}3077) = J_2(2{,}3077) \qquad J_{-4}(2{,}3077) = J_4(2{,}3077)$$

Mediante interpolación lineal a partir de la tabla.

$\boxed{n=0}$ $J_0(2) = 0{,}2239$

$J_0(3) = -0{,}2601$

$J_0(2{,}3077) = [J_0(3) - J_0(2)]\, 0{,}3077 + J_0(2)$

$= 0{,}3077\, J_0(3) + 0{,}6923\, J_0(2)$

$\boxed{J_0(2{,}3077) = 0{,}3077\,[-0{,}2601] + 0{,}2239 \cdot 0{,}6923 = 0{,}075}$

$\boxed{n=1}$ $\boxed{J_1(2{,}3077)} = 0{,}3077\, J_1(3) + 0{,}6923\, J_1(2)$

$= 0{,}3077 \cdot 0{,}3391 + 0{,}6923 \cdot 0{,}5767 = \boxed{0{,}5036}$

$\boxed{n=2}$ $\boxed{J_2(2{,}3077)} = 0{,}3077\, J_2(3) + 0{,}6923\, J_2(2)$

$= 0{,}3077 \cdot 0{,}4861 + 0{,}6923 \cdot 0{,}3528 = \boxed{0{,}3938}$

$\boxed{n=3}$ $\boxed{J_3(2{,}3077)} = 0{,}3077 \cdot 0{,}3091 + 0{,}6923 \cdot 0{,}1289 = \boxed{0{,}1843}$

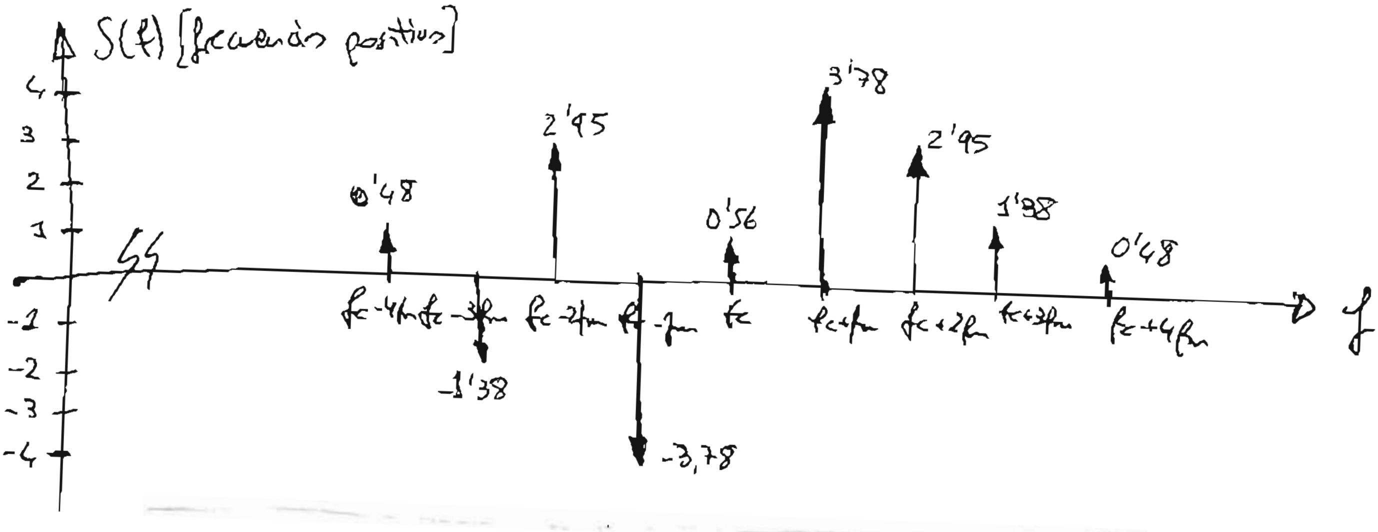
$\boxed{n=4}$ $\boxed{J_4(2{,}3077)} = 0{,}3077 \cdot 0{,}1320 + 0{,}6923 \cdot 0{,}0340 = \boxed{0{,}0642}$

$J_{-1}(2{,}3077) = -0{,}5036$ $\qquad J_{-2}(2{,}3077) = 0{,}3938$

$J_{-3}(2{,}3077) = -0{,}1843$ $\qquad J_{-4}(2{,}3077) = 0{,}0642$

Amplitudes multiplicadas por $\frac{A_c}{2} = 7'5$

$n=-4 \Rightarrow 0'4815$	$n=-1 \Rightarrow -3{,}777$	$n=2 \Rightarrow 2{,}9535$
$n=-3 \Rightarrow -1'3823$	$n=0 \Rightarrow 0{,}5625$	$n=3 \Rightarrow 1'3823$
$n=-2 \Rightarrow 2{,}9535$	$n=1 \Rightarrow 3{,}777$	$n=4 \Rightarrow 0'4815$



b) Potencia total: $P_T = A_c^2/2 = 112'5$ Watts.

Potencia tras el filtro paso banda:

$$P_{PB} = 2\left[0'5625^2 + 2\left(3'777^2 + 2{,}9535^2 + 1'3823^2 + 0'4815^2\right)\right]$$

$$= 101'16 \text{ Watts} \Rightarrow \boxed{89'92\%}$$