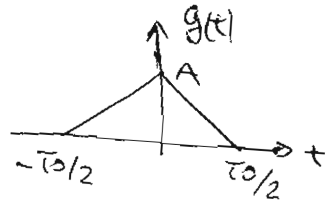
PROBLEMA 1.

a) Función generadora $g(t)$



$$g(t) = A \cdot \Lambda\left(\frac{t}{T_0/2}\right) = A \cdot \Lambda\left(\frac{2t}{T_0}\right)$$

Su transformada de Fourier es inmediata:

$$G(f) = \frac{AT_0}{2}\,\mathrm{sinc}^2\left(\frac{fT_0}{2}\right)$$

Sabemos que los coeficientes de la serie de $g_p(t)$ se relacionan con la transformada $G(f)$ mediante:

$$c_n = \frac{1}{T_0}\,G\left(\frac{n}{T_0}\right) = \frac{A\cancel{T_0}}{2\cancel{T_0}}\,\mathrm{sinc}^2\left(\frac{n\cancel{T_0}}{2\cancel{T_0}}\right) = \frac{A}{2}\,\mathrm{sinc}^2\left(\frac{n}{2}\right)$$

Entonces $g_p(t)$ quedaría:

$$g_p(t) = \frac{A}{2}\sum_{n=-\infty}^{\infty} \mathrm{sinc}^2\left(\frac{n}{2}\right)\exp(j2\pi n f_0 t)$$

Puesto que nos piden pasarlo usando serie real, teniendo en cuenta que:

$$\cos(2\pi n f_0 t) = \frac{1}{2}\exp(j2\pi n f_0 t) + \frac{1}{2}\exp(-j2\pi n f_0 t)$$

$$\boxed{g_p(t) = \frac{A}{2} + A\sum_{n=1}^{\infty}\mathrm{sinc}^2\left(\frac{n}{2}\right)\cos(2\pi n f_0 t)}$$

b) Primero calculamos la transformada de Fourier para $g_p(t)$

$$G_p(f) = \frac{A}{2}\delta(f) + \frac{A}{2}\sum_{n=1}^{\infty}\mathrm{sinc}^2\left(\frac{n}{2}\right)\left[\delta(f-nf_0) + \delta(f+nf_0)\right]$$

Ahora la densidad espectral de potencia será:

$$S_{gp}(f) = \frac{A^2}{4}\delta(f) + \frac{A^2}{4}\sum_{n=1}^{\infty}\mathrm{sinc}^4\left(\frac{n}{2}\right)\left[\delta(f-nf_0) + \delta(f+nf_0)\right]$$

Como sumar la serie anterior no parece sencillo, la potencia se calcula mejor en el dominio del tiempo:

$$P_{gp} = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} g_p^2(t)\,dt$$

como la señal es par, entonces:

$$\boxed{P_{gp}} = \frac{2}{T_0}\int_0^{T_0/2} g_p^2(t)\,dt = \frac{2}{T_0}\int_0^{T_0/2}\left[A\left(1-\frac{t}{T_0/2}\right)\right]^2 dt =$$

$$= \frac{2A^2}{T_0}\int_0^{T_0/2}\left(1-\frac{2t}{T_0}\right)^2 dt \;\left|\; \begin{matrix} u = \frac{2t}{T_0} \\ du = \frac{2\,dt}{T_0} \end{matrix} \;\right| = \frac{2A^2}{\cancel{T_0}}\int_0^1 (1-u)^2 \frac{\cancel{T_0}}{\cancel{2}}\,du =$$

$$= A^2\left[-\frac{(1-u)^3}{3}\right]_0^1 = A^2\cdot\frac{1}{3} = \boxed{\frac{A^2}{3}}$$

c) La expresión de $y(t)$ tras el filtro paso bajo

$$y(t) = \frac{A}{2} + A\sum_{n=1}^{3} \mathrm{sinc}^2\left(\frac{n}{2}\right)\cos(2\pi n f_0 t)$$

$$= \frac{A}{2} + A\,\mathrm{sinc}^2\left(\frac{1}{2}\right)\cos(2\pi f_0 t) + A\,\mathrm{sinc}^2(1)\cos(4\pi f_0 t) + A\,\mathrm{sinc}^2\left(\frac{3}{2}\right)\cos(6\pi f_0 t)$$

$$= \frac{A}{2} + A\,\mathrm{sinc}^2\left(\frac{1}{2}\right)\cos(2\pi f_0 t) + A\,\mathrm{sinc}^2\left(\frac{3}{2}\right)\cos(6\pi f_0 t)$$

$$\mathrm{sinc}\left(\frac{1}{2}\right) = \frac{\mathrm{sen}(\pi/2)}{\pi/2} = \frac{2}{\pi} \quad\Big/\quad \mathrm{sinc}\left(\frac{3}{2}\right) = \frac{\mathrm{sen}\left(\frac{3\pi}{2}\right)}{\frac{3\pi}{2}} = -\frac{2}{3\pi}$$

$$y(t) = \frac{A}{2} + \frac{4A}{\pi^2}\cos(2\pi f_0 t) + \frac{4A}{9\pi^2}\cos(6\pi f_0 t)$$

La densidad espectral de potencia:

$$\boxed{S_y(f) = \frac{A^2}{4}\delta(f) + \frac{4A^2}{\pi^4}\left[\delta(f-f_0)+\delta(f+f_0)\right] + \frac{4A^2}{81\pi^4}\left[\delta(f-3f_0)+\delta(f+3f_0)\right]}$$

d) $$P_y = \int_{-\infty}^{\infty} S_y(f)\,df = \frac{A^2}{4} + \frac{8A^2}{\pi^4} + \frac{8A^2}{81\pi^2} = \frac{A^2}{4} + \frac{8A^2}{\pi^4}\left[\frac{81+1}{81}\right] = \frac{A^2}{4} + \frac{656A^2}{81\pi^4} = \frac{81\pi^4+2624}{324\pi^4}A^2$$

$$1 - \frac{P_y}{P_{Gp}} = 1 - \frac{81\pi^4+2624}{324\pi^4}\cdot\cancel{A^2}\cdot\frac{3}{\cancel{A^2}} = 1 - \frac{81\pi^4+2624}{108\pi^4} = \frac{27\pi^4-2624}{108\pi^4} = 5'7\cdot 10^{-4}$$

En tanto por ciento $0{,}0575\%$ de potencia elimina el filtro

PROBLEMA 2

a)

$$R_y(\tau) = R_x(\tau) * h(\tau) * h(-\tau) = \delta(\tau) * h(\tau) * h(-\tau)$$
$$= h(\tau) * h(-\tau)$$

si llamamos $h_1(\tau) = h(-\tau)$

$$R_y(\tau) = h(\tau) * h_1(\tau) = \int_{-\infty}^{\infty} h(\lambda) h_1(\tau - \lambda) d\lambda =$$
$$= \int_{-\infty}^{\infty} h(\lambda) h(\lambda - \tau) d\lambda \qquad \text{con } h(t) = e^{-t} u(t)$$

entonces $$R_y(\tau) = \int_{-\infty}^{\infty} e^{-\lambda} u(\lambda) e^{-\lambda+\tau} u(\lambda - \tau) d\lambda$$

para $\tau > 0$ $\quad u(\lambda) \cdot u(\lambda - \tau) = u(\lambda - \tau)$ entonces

$$R_y(\tau) = e^{\tau} \int_{-\infty}^{\infty} e^{-2\lambda} u(\lambda - \tau) d\lambda = e^{\tau} \int_{\tau}^{\infty} e^{-2\lambda} d\lambda =$$
$$= e^{\tau} \left[\frac{e^{-2\lambda}}{-2} \right]_{\tau}^{\infty} = e^{\tau} \frac{e^{-2\tau}}{2} = \frac{1}{2} e^{-\tau} \qquad \tau > 0$$

para $\tau < 0$ $\quad u(\lambda) \cdot u(\lambda - \tau) = u(\lambda)$ entonces

$$R_y(\tau) = e^{\tau} \int_{-\infty}^{\infty} e^{-2\lambda} u(\lambda) d\lambda = e^{\tau} \int_{0}^{\infty} e^{-2\lambda} d\lambda =$$
$$= e^{\tau} \left[\frac{e^{-2\lambda}}{-2} \right]_{0}^{\infty} = e^{\tau} \cdot \frac{1}{2} = \frac{1}{2} e^{\tau} \qquad \tau < 0$$

Juntando ambas expresiones: $\boxed{R_y(\tau) = \frac{1}{2} e^{-|\tau|}}$

La densidad espectral será la transformada de Fourier. Como es una transformada inmediata, entonces $\boxed{S_y(f) = \frac{1}{2} \cdot \frac{2}{1+4\pi^2 f^2} = \frac{1}{1+4\pi^2 f^2}}$

②

b) El ancho de banda a 3dB es cuando la densidad espectral de potencia ha caído 3dB con respecto al valor máximo. Se puede ver que la señal (t) es paso bajo y el valor máximo está en el origen:

$$S_Y(f)\Big|_{max} = S_Y(0) = 1$$

Además 3dB equivale a un factor de 2, entonces

$$S_Y(W_3) = \frac{S_Y(0)}{2} = \frac{1}{2}$$

$$\frac{1}{1+4\pi^2 W_3^2} = \frac{1}{2} \Rightarrow 2 = 1 + 4\pi^2 W_3^2 \Rightarrow 4\pi^2 W_3^2 = 1$$

$$\boxed{W_3 = \frac{1}{2\pi} = 0{,}1592 \text{ Hz}}$$

c) La potencia total corresponde a:

$$Pot_Y = R_Y(0) = \int_{-\infty}^{\infty} S_Y(f)\,df = \frac{1}{2}$$

La potencia dentro de W_3:

$$Pot_Y^{W_3} = \int_{-\frac{1}{2\pi}}^{\frac{1}{2\pi}} \frac{1}{1+4\pi^2 f^2}\,df \quad \left| \begin{array}{l} u = 2\pi f \\ du = 2\pi\, df \end{array} \right| \int_{-1}^{1} \frac{1}{1+u^2}\,\frac{du}{2\pi} =$$

$$= \frac{1}{2\pi}\Big[\operatorname{atan}(u)\Big]_{-1}^{1} = \frac{1}{2\pi}\Big[\operatorname{atan}(1) - \operatorname{atan}(-1)\Big] = \frac{1}{\pi}\operatorname{atan}(1) = \frac{1}{\pi}\frac{\pi}{4} = \frac{1}{4}$$

$$\boxed{\%\text{ Potencia} = \frac{Pot_Y^{W_3}}{Pot_Y} = \frac{1/4}{1/2} = 0{,}5 = 50\%}$$

PROBLEMA 3.

a) Sabemos que la serie de taylor en $x = x_0$

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!} x - x_0 + \frac{f''(x_0)}{2!}(x-x_0)^2 + \cdots = \sum_{n=0}^{\infty} \left.\frac{d^n f(x)}{dx^n}\right|_{x=x_0} \cdot \frac{(x-x_0)^n}{n!}$$

para $f(x) = \ln(1+x)$ en $x = 0$: $\ln(1) = 0$

$$f'(x) = \frac{1}{1+x} \quad \text{en } x=0 \quad \frac{1}{1+0} = 1$$

$$f''(x) = \frac{-1}{(1+x)^2} \quad \text{en } x=0 \quad \frac{-1}{(1+0)^2} = -1$$

$$f'''(x) = \frac{2}{(1+x)^3} \quad \text{en } x=0 \quad \frac{2}{(1+0)^3} = 2$$

$$\boxed{\ln(1+x) = x - \frac{x^2}{2!} + \frac{2x^3}{3!} + o(x^4) = x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^4)}$$

b)

$$x_1(t) \cong c(t) + m(t) - \frac{[c(t)+m(t)]^2}{2} + \frac{[c(t)+m(t)]^3}{3} =$$

$$c(t)+m(t) - \frac{c^2(t)}{2} - \frac{m^2(t)}{2} - c(t)m(t) + \frac{c^3(t)}{3} + c^2(t)m(t) + c(t)m^2(t) + \frac{m^3(t)}{3}$$

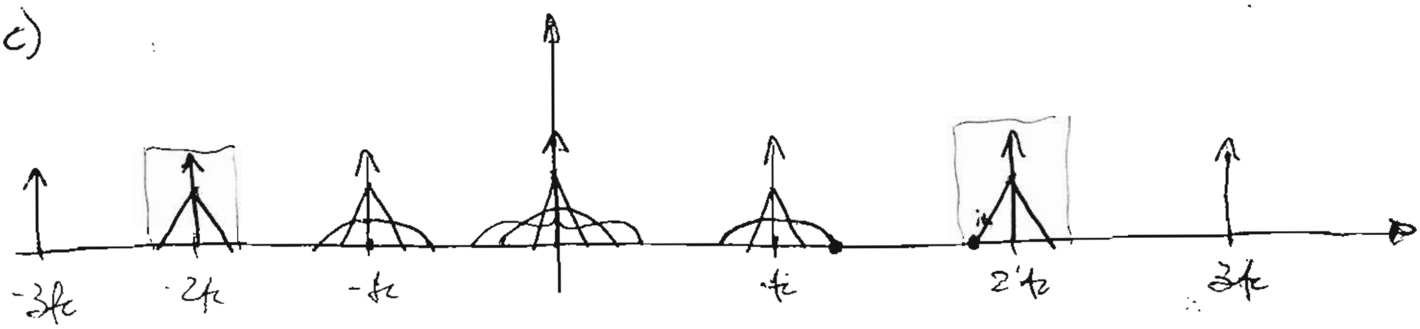
$$c(t) = A_c \cos(2\pi f_c t)$$

$$c^2(t) = \frac{A_c^2}{2} + \frac{A_c^2}{2}\cos(4\pi f_c t)$$

$$c^3(t) = \frac{A_c^3}{2}\cos(2\pi f_c t) + \frac{A_c^3}{4}\cos(6\pi f_c t) + \frac{A_c^3}{4}\cos(2\pi f_c t) = \frac{3A_c^3}{4}\cos(2\pi f_c t) + \frac{A_c^3}{4}\cos(6\pi f_c t)$$

$$\boxed{\begin{aligned} x_1(t) \cong\ & A_c\cos(2\pi f_c t) + m(t) - \frac{A_c^2}{4} - \frac{A_c^2}{4}\cos(4\pi f_c t) - \frac{m^2(t)}{2} - A_c\cos(2\pi f_c t)\, m(t) \\ & + \frac{A_c^3}{4}\cos(2\pi f_c t) + \frac{A_c^3}{12}\cos(6\pi f_c t) + \frac{A_c^2}{2} m(t) + \frac{A_c^2}{2} m(t)\cos(4\pi f_c t) + \\ & + A_c\cos(2\pi f_c t)\, m^2(t) + \frac{m^3(t)}{3} \end{aligned}}$$

c)



d) Usando el filtro paso banda de la figura anterior con frecuencia central $2f_c$ y ancho de banda $2W$ se puede extraer una señal AM.

$$\boxed{y_1(t)} = -\frac{A_c^2}{4}\cos(4\pi f_c t) + \frac{A_c^2}{2} m(t)\cos(4\pi f_c t)$$

$$= \boxed{-\frac{A_c^2}{4}\left[1 - 2m(t)\right]\cos(4\pi f_c t)}$$

La frecuencia de la portadora es $2f_c$ y $K_a = 2$.

Para que no haya distorsión:

$$2f_c - W > f_c + 2W \qquad \boxed{f_c > 3W}$$

e) $$\boxed{x_2(t)} = c(t) - m(t) - \frac{[c(t)-m(t)]^2}{2} + \frac{[c(t)-m(t)]^3}{3} =$$

$$c(t) - m(t) - \frac{c^2(t)}{2} - \frac{m^2(t)}{2} + c(t)m(t) + \frac{c^3(t)}{3} - c^2(t)m(t) + c(t)m^2(t) - \frac{m^3(t)}{3}$$

$$= A_c\cos(2\pi f_c t) - m(t) - \frac{A_c^2}{4} - \frac{A_c^2}{4}\cos(4\pi f_c t) - \frac{m^2(t)}{2} + A_c\cos(2\pi f_c t)m(t)$$
$$+ \frac{A_c^3}{4}\cos(2\pi f_c t) + \frac{A_c^3}{12}\cos(6\pi f_c t) - \frac{A_c^2}{2}m(t) - \frac{A_c^2}{2}m(t)\cos(4\pi f_c t) + A_c\cos(2\pi f_c t)m^2(t) - \frac{m^3(t)}{3}$$

La ocupación frecuencial es igual que en el apartado c)

$$\boxed{y_2(t) = -\frac{A_c^2}{4}\cos(4\pi f_c t) - \frac{A_c^2}{2}m(t)\cos(4\pi f_c t) = -\frac{A_c^2}{4}\left[1 + 2m(t)\right]\cos(4\pi f_c t)}$$

es señal AM.

Si definimos $\boxed{y(t) = y_1(t) - y_2(t)} = \frac{A_c^2}{4}2m(t)\cos(4\pi f_c t) + \frac{A_c^2}{4}2m(t)\cos(4\pi f_c t) =$ $\boxed{= A_c^2 m(t)\cos(4\pi f_c t)}$ que es DSB

PROBLEMA 4:

a) En el intervalo $[0, T_0]$ $\quad m(t) = a \cdot t$. La señal PM:

$$\boxed{s(t) = A_c \cdot \cos(2\pi f_c t + K_p m(t)) = A_c \cos(2\pi f_c t + K_p a t) \text{ en } [0, T_0]}$$

b)

$$\hat{s}(t) = A_c \, \text{sen}(2\pi f_c t + K_p a t) \text{ en } [0, T_0]$$

$$s_+(t) = s(t) + j\,\hat{s}(t) = A_c \exp(j 2\pi f_c t + j K_p a t) \text{ en } [0, T_0]$$

$$\boxed{\tilde{s}(t) = s_+(t) \cdot \exp(-j 2\pi f_c t) = A_c \exp(j K_p a t) \text{ en } [0, T_0]}$$

c) $\tilde{s}(t)$ es periódica con periodo T_0

La función generadora $\tilde{s}_{gen}(t) = A_c \exp(j K_p a t) \Pi\left(\frac{t - T_0/2}{T_0}\right)$

La transformada de Fourier:

$$\tilde{S}_{gen}(f) = A_c\, \delta\left(f - \frac{K_p \cdot a}{2\pi}\right) * \left[T_0 \, \text{sinc}(f T_0) \exp(-j\pi T_0 f)\right]$$

$$= A_c T_0 \, \text{sinc}\left(f T_0 - \frac{K_p a T_0}{2\pi}\right) \exp\left(-j\pi T_0 f + j\frac{K_p a T_0}{2}\right)$$

Los coeficientes de la serie para $\tilde{s}(t)$ serán entonces:

$$c_n = \frac{1}{T_0} \tilde{S}_{gen}\left(\frac{n}{T_0}\right) = \frac{A_c \cancel{T_0}}{\cancel{T_0}} \text{sinc}\left(\frac{n \cancel{T_0}}{\cancel{T_0}} - \frac{K_p a T_0}{2\pi}\right) \exp\left(-j\pi \frac{\cancel{T_0} n}{\cancel{T_0}}\right) \exp\left(j\frac{K_p a T_0}{2}\right)$$

$$= A_c \frac{\text{sen}\left(n\pi - \frac{K_p a T_0}{2}\right)}{n\pi - \frac{K_p a T_0}{2}} (-1)^n \exp\left(j\frac{K_p a T_0}{2}\right) =$$

$$= 2A_c \frac{\cancel{\text{sen}(n\pi)} \cos\left(\frac{K_p a T_0}{2}\right) - \cos(n\pi)\,\text{sen}\left(\frac{K_p a T_0}{2}\right)}{2\pi n - K_p a T_0} (-1)^n \exp\left(j\frac{K_p a T_0}{2}\right) =$$

$$= 2A_c \frac{\text{sen}\left(\frac{K_p a T_0}{2}\right) \cancel{(-1)^n}}{K_p a T_0 - 2\pi n} \cancel{(-1)^n} \exp\left(j\frac{K_p a T_0}{2}\right) = 2A_c \frac{\text{sen}\left(\frac{K_p a T_0}{2}\right)}{K_p a T_0 - 2\pi n} \exp\left(j\frac{K_p a T_0}{2}\right)$$

Entonces:

$$\boxed{\tilde{S}(t) = \sum_{n=-\infty}^{\infty} c_n \exp(j2\pi n f_0 t) = 2A_c \operatorname{sen}\left(\frac{K_{pa}T_0}{2}\right) \exp\left(j\frac{K_{pa}T_0}{2}\right) \sum_{n=-\infty}^{\infty} \frac{\exp(j2\pi n f_0 t)}{K_{pa}T_0 - 2\pi n}}$$

d) $$s(t) = \operatorname{Re}\left\{\tilde{S}(t)\, e^{j2\pi f_c t}\right\} = 2A_c \operatorname{sen}\left(\frac{K_{pa}T_0}{2}\right) \operatorname{Re}\left\{\exp\left(j\frac{K_{pa}T_0}{2}\right) \sum_{n=-\infty}^{\infty} \frac{\exp(j2\pi n f_0 t)}{K_{pa}T_0 - 2\pi n} e^{j2\pi f_c t}\right\}$$

$$\boxed{= 2A_c \operatorname{sen}\left(\frac{K_{pa}T_0}{2}\right) \sum_{n=-\infty}^{\infty} \frac{\cos\left(2\pi f_c t + 2\pi n f_0 t + \frac{K_{pa}T_0}{2}\right)}{K_{pa}T_0 - 2\pi n}}$$

e) Desarrollando el coseno:

$$s(t) = 2A_c \operatorname{sen}\left(\frac{K_{pa}T_0}{2}\right) \cos\left(\frac{K_{pa}T_0}{2}\right) \sum_{n=-\infty}^{\infty} \frac{\cos(2\pi(f_c + n f_0)t)}{K_{pa}T_0 - 2\pi n}$$

$$- 2A_c \operatorname{sen}^2\left(\frac{K_{pa}T_0}{2}\right) \sum_{n=-\infty}^{\infty} \frac{\operatorname{sen}(2\pi(f_c + n f_0)t)}{K_{pa}T_0 - 2\pi n} =$$

$$= A_c \operatorname{sen}(K_{pa}T_0) \sum_{n=-\infty}^{\infty} \frac{\cos(2\pi(f_c + n f_0)t)}{K_{pa}T_0 - 2\pi n} + A_c\left[\cos(K_{pa}T_0) - 1\right] \sum_{n=-\infty}^{\infty} \frac{\operatorname{sen}(2\pi(f_c + n f_0)t)}{K_{pa}T_0 - 2\pi n}$$

tomando transformada de Fourier:

$$\boxed{S(f) = \frac{A_c}{2} \operatorname{sen}(K_{pa}T_0) \sum_{n=-\infty}^{\infty} \frac{\delta(f - f_c - n f_0) + \delta(f + f_c + n f_0)}{K_{pa}T_0 - 2\pi n} + j\frac{A_c}{2}\left[1 - \cos(K_{pa}T_0)\right] \sum_{n=-\infty}^{\infty} \frac{\delta(f - f_c - n f_0) - \delta(f + f_c + n f_0)}{K_{pa}T_0 - 2\pi n}}$$

Simplificando, otra expresión

$$S(f) = j\frac{A_c}{2}\left[1 - e^{jK_{pa}T_0}\right] \sum_{n=-\infty}^{\infty} \frac{\delta(f - f_c + n f_0)}{K_{pa}T_0 - 2\pi n} + j\frac{A_c}{2}\left[e^{-jK_{pa}T_0} - 1\right] \sum_{n=-\infty}^{\infty} \frac{\delta(f + f_c + n f_0)}{K_{pa}T_0 - 2\pi n}$$