PROBLEMA 1

(a) Determinando la ecuación de la recta:

$$f_A(a) = \begin{cases} -\frac{K}{b}(a-b) & 0 \le a \le b \\ 0 & \text{Resto} \end{cases} = -\frac{K}{b}(a-b)\cdot \Pi\left(\frac{a-\frac{b}{2}}{b}\right)$$

para que sea función de densidad tiene que ser siempre positiva y con área unidad, por lo que, mediante el área del triángulo,

$$\frac{K\cdot b}{2} = 1 \qquad \boxed{K = \frac{2}{b}}$$

entonces

$$\boxed{f_A(a) = -\frac{2}{b^2}(a-b)\,\Pi\left(\frac{a-b/2}{b}\right)}$$

(b) La función de distribución por la definición:

$$F_A(a) = \int_{-\infty}^{a} f_A(\alpha)\,d\alpha$$

puesto que $f_A(a)$ sólo esta definida en el intervalo $0 \le a \le b$

entonces $F_A(a) = 0 \quad a \le 0$ y $F_A(a) = 1 \quad a \ge b$

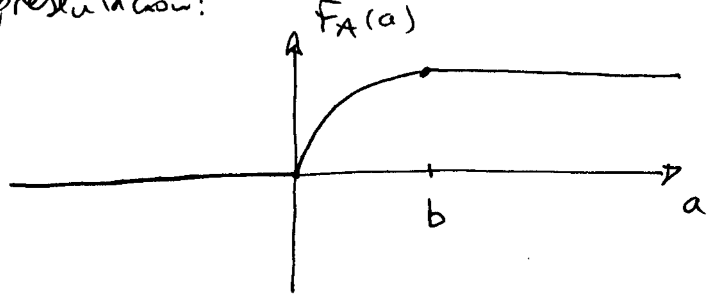
En el intervalo $0 \le a \le b$, se tiene

$$F_A(a) = \int_0^a -\frac{2}{b^2}(\alpha - b)\,d\alpha = \int_0^a \left(-\frac{2\alpha}{b^2} + \frac{2}{b}\right)d\alpha = -\frac{2}{b^2}\left[\frac{\alpha^2}{2}\right]_0^a + \frac{2}{b}[\alpha]_0^a$$

$$= -\frac{\cancel{2}}{b^2}\frac{a^2}{\cancel{2}} + \frac{2}{b}a = -\frac{a^2}{b^2} + \frac{2a}{b} = \frac{2ab-a^2}{b^2}$$

$$\boxed{F_A(a) =} \begin{cases} \frac{2ab-a^2}{b^2} & 0 \le a \le b \\ 0 & a \le 0 \\ 1 & a > b \end{cases} = \boxed{\frac{2ab-a^2}{b^2}\Pi\left(\frac{a-b/2}{b}\right) + u(a-b)}$$

Su representación:



en $0 \leq a \leq b$ es parabólica. Puesto que su derivada $f_A(a)$ es discontinua en $a=0$ $F_A(a)$ tiene un vértice en dicho punto. Además en $a=b$, $f_A(a)$ vale cero y es continua, por lo que $F_A(a)$ no tiene vértice en $a=b$ y la parábola alcanza un máximo (zona plana con pendiente cero).

(c) $$\boxed{E[A]} = \int_{-\infty}^{\infty} a\, f_A(a)\, da = \int_0^b a\left(-\frac{2}{b^2}(a-b)\right) da = \int_0^b \left(-\frac{2a^2}{b^2} + \frac{2a}{b}\right) da$$

$$= -\frac{2}{b^2}\left[\frac{a^3}{3}\right]_0^b + \frac{2}{b}\left[\frac{a^2}{2}\right]_0^b = -\frac{2}{\cancel{b^2}}\cdot\frac{b^{\cancel{3}}}{3} + \frac{\cancel{2}}{\cancel{b}}\cdot\frac{b^{\cancel{2}}}{\cancel{2}} = -\frac{2}{3}b + b = \boxed{\frac{b}{3}}$$

(d) $$\boxed{E[A^2]} = \int_{-\infty}^{\infty} a^2 f_A(a)\, da = \int_0^b a^2\left(-\frac{2}{b^2}(a-b)\right) da = \int_0^b \left(-\frac{2a^3}{b^2} + \frac{2a^2}{b}\right) da$$

$$= -\frac{2}{b^2}\left[\frac{a^4}{4}\right]_0^b + \frac{2}{b}\left[\frac{a^3}{3}\right]_0^b = -\frac{2}{b^2}\cdot\frac{b^4}{4} + \frac{2}{b}\cdot\frac{b^3}{3} = -\frac{b^2}{2} + \frac{2}{3}b^2 = \frac{-3+4}{6}b^2 = \boxed{\frac{b^2}{6}}$$

$$\boxed{\mathrm{var}[A] = E[A^2] - E^2[A] = \frac{b^2}{6} - \left(\frac{b}{3}\right)^2 = \frac{b^2}{6} - \frac{b^2}{9} = \frac{3-2}{18}b^2 = \frac{b^2}{18}}$$

(e) $X(t) = A\cdot\cos(2\pi f_0 t)$

$$\boxed{E[X(t)] = E[A\cdot\cos(2\pi f_0 t)] = E[A]\cdot\cos(2\pi f_0 t) = \frac{b}{3}\cos(2\pi f_0 t)}$$

$$\boxed{R_X(t,t+\tau)} = E\left[A\cos(2\pi f_0 t)\cdot A\cos(2\pi f_0(t+\tau))\right] = E[A^2]\cos(2\pi f_0 t)\cos(2\pi f_0(t+\tau))$$

$$= \frac{E[A^2]}{2}\left(\cos(2\pi f_0 \tau) + \cos(2\pi f_0(2t+\tau))\right) = \boxed{\frac{b^2}{12}\left[\cos(2\pi f_0 \tau) + \cos(2\pi f_0(2t+\tau))\right]}$$

No es estacionario ya que la media depende de t y la autocorrelación también depende de t.

PROBLEMA 1 (cont)

(f) $Y(t) = \cos(2\pi f_0 t + A)$

Si $b = 2\pi$ $\quad \boxed{f_A(a) = -\frac{2}{4\pi^2}(a-2\pi)\prod\left(\frac{a-\pi}{2\pi}\right) = -\frac{1}{2\pi^2}(a-2\pi)\prod\left(\frac{a-\pi}{2\pi}\right)}$

$$\boxed{E[Y(t)]} = E[\cos(2\pi f_0 t + A)] = \int_{-\infty}^{\infty} f_A(a)\cos(2\pi f_0 t + a)\,da = \int_0^{2\pi} -\frac{a-2\pi}{2\pi^2}\cos(2\pi f_0 t + a)\,da$$

$$= -\frac{1}{2\pi^2}\int_0^{2\pi} a\cos(2\pi f_0 t + a)\,da + \frac{1}{\pi}\int_0^{2\pi} \cancel{\cos(2\pi f_0 t + a)\,da}^{\,0} \quad \left|\begin{array}{ll} u = a & du = da \\ dv = \cos(2\pi f_0 t + a)\,da & v = \operatorname{sen}(2\pi f_0 t + a)\end{array}\right.$$

$$= -\frac{1}{2\pi^2}\left[a\operatorname{sen}(2\pi f_0 t + a)\right]_0^{2\pi} + \frac{1}{2\pi^2}\int_0^{2\pi} \cancel{\operatorname{sen}(2\pi f_0 t + a)\,da}^{\,0} = -\frac{1}{2\pi^2}\left[-0 + 2\pi\operatorname{sen}(2\pi f_0 t)\right]$$

$$\boxed{= -\frac{1}{\pi}\operatorname{sen}(2\pi f_0 t)}$$

$$\boxed{R_Y(t, t+\tau)} = E[Y(t)Y(t+\tau)] = E[\cos(2\pi f_0 t + A)\cos(2\pi f_0(t+\tau) + A)]$$

$$= \frac{1}{2}\cos(2\pi f_0\tau) + \frac{1}{2}E[\cos(2\pi f_0(2t+\tau) + 2A)] = \frac{1}{2}\cos(2\pi f_0\tau) + \frac{1}{2}\int_{-\infty}^{\infty} f_A(a)\cos(2\pi f_0(2t+\tau) + 2a)\,da$$

$$= \frac{1}{2}\cos(2\pi f_0\tau) + \frac{1}{2}\int_0^{2\pi} -\frac{a-2\pi}{2\pi^2}\cos(2\pi f_0(2t+\tau) + 2a)\,da =$$

$$= \frac{1}{2}\cos(2\pi f_0\tau) - \frac{1}{4\pi^2}\int_0^{2\pi} a\cos(2\pi f_0(2t+\tau) + 2a)\,da + \frac{1}{2\pi}\int_0^{2\pi} \cancel{\cos(2\pi f_0(2t+\tau) + 2a)\,da}^{\,0}$$

$$\left|\begin{array}{ll} u = a & du = da \\ dv = \cos(2\pi f_0(2t+\tau) + 2a)\,da & v = \frac{1}{2}\operatorname{sen}(2\pi f_0(2t+\tau) + 2a)\end{array}\right|$$

$$= \frac{1}{2}\cos(2\pi f_0\tau) - \frac{1}{4\pi^2}\left[\frac{a}{2}\operatorname{sen}(2\pi f_0(2t+\tau) + 2a)\right]_0^{2\pi} + \frac{1}{4\pi^2}\int_0^{2\pi} \cancel{\frac{1}{2}\operatorname{sen}(2\pi f_0(2t+\tau) + 2a)\,da}^{\,0}$$

$$= \frac{1}{2}\cos(2\pi f_0\tau) - \frac{1}{4\pi^2}\left[\frac{\cancel{2}\pi}{\cancel{2}}\operatorname{sen}(2\pi f_0(2t+\tau)) - 0\right] = \boxed{\frac{1}{2}\cos(2\pi f_0\tau) - \frac{1}{4\pi}\operatorname{sen}(2\pi f_0(2t+\tau))}$$

Tanto la media como la autocorrelación son función del tiempo t, por lo que la señal $Y(t)$ no puede ser estacionaria en sentido amplio

PROBLEMA 2

(a) $\theta_i(t) = 2\pi f_c t + K_p\, m(t)$

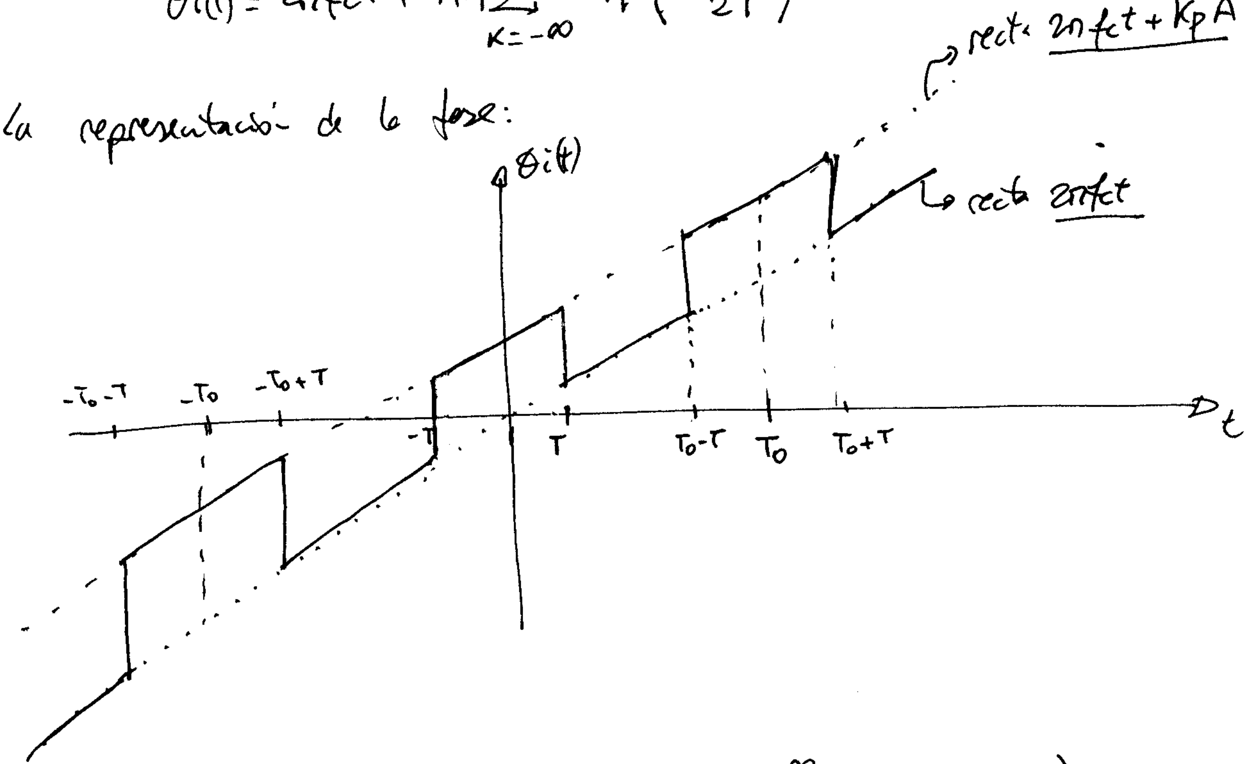
podemos usar para $m(t)$ la expresión:

$$m(t) = A \sum_{K=-\infty}^{\infty} \Pi\left(\frac{t-KT_0}{2T}\right)$$

entonces:

$$\theta_i(t) = 2\pi f_c t + A K_p \sum_{K=-\infty}^{\infty} \Pi\left(\frac{t-KT_0}{2T}\right)$$

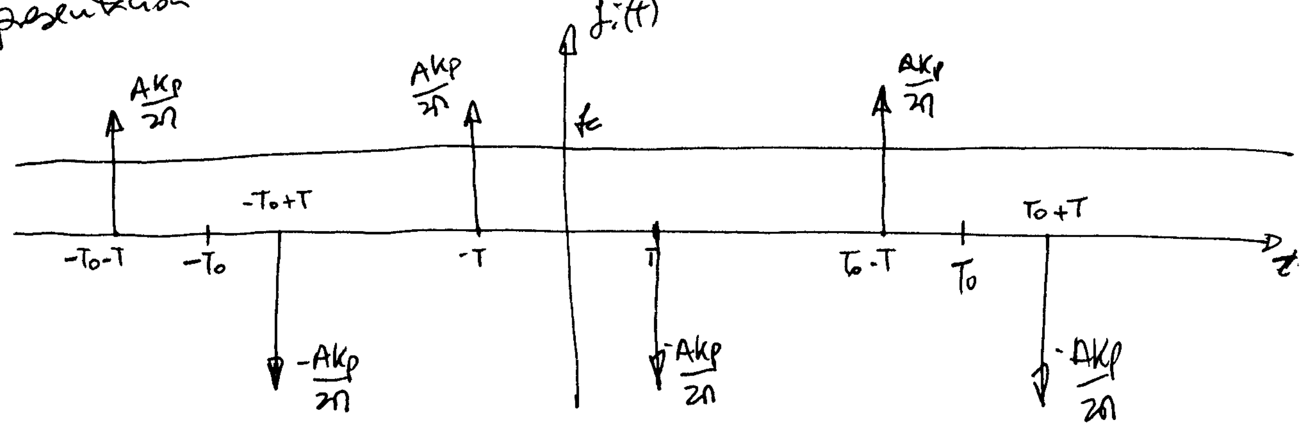
La representación de la fase:



(b) $$\boxed{f_i(t)} = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = f_c + \frac{A K_p}{2\pi}\frac{d}{dt}\sum_{K=-\infty}^{\infty} \Pi\left(\frac{t-KT_0}{2T}\right)$$

$$= f_c + \frac{AK_p}{2\pi}\sum_{K=-\infty}^{\infty}\frac{d}{dt}\Pi\left(\frac{t-KT_0}{2T}\right) = \boxed{f_c + \frac{AK_p}{2\pi}\sum_{K=-\infty}^{\infty}\left[\delta(t+T-KT_0) - \delta(t-T-KT_0)\right]}$$

La representación



(c) $\boxed{s(t) = A_c \cos(\theta_i(t)) = A_c \cos\left[2\pi f_c t + A k_p \sum_{k=-\infty}^{\infty} \Pi\left(\frac{t-kT_0}{2T}\right)\right]}$

$$\hat{s}(t) = A_c \,\text{sen}(\theta_i(t)) = A_c \,\text{sen}\left[2\pi f_c t + A k_p \sum_{k=-\infty}^{\infty} \Pi\left(\frac{t-kT_0}{2T}\right)\right]$$

$$s_+(t) = s(t) + j\hat{s}(t) = A_c \cos(\theta_i(t)) + j A_c \,\text{sen}(\theta_i(t)) = A_c \exp(j\theta_i(t))$$

$$= A_c \exp\left[j 2\pi f_c t + j A k_p \sum_{k=-\infty}^{\infty} \Pi\left(\frac{t-kT_0}{2T}\right)\right]$$

$$\boxed{\tilde{s}(t) = s_+(t)\exp(-j2\pi f_c t) = A_c \exp\left[j A k_p \sum_{k=-\infty}^{\infty} \Pi\left(\frac{t-kT_0}{2T}\right)\right]}$$

otra expresión interesante:

$$\tilde{s}(t) = A_c \prod_{k=-\infty}^{\infty} \exp\left[j A k_p \Pi\left(\frac{t-kT_0}{2T}\right)\right]$$

(d) La envolvente compleja $\tilde{s}(t)$ es periódica con periodo T_0, entonces se podrá poner como

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} c_n \exp\left(j\frac{2\pi n t}{T_0}\right)$$

siendo c_n los coeficientes complejos de la serie,

$$c_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \tilde{s}(t)\exp\left(-j\frac{2\pi n t}{T_0}\right)dt = \frac{1}{T_0}\int_{-T}^{T} A_c \exp(jAk_p)\exp\left(-j\frac{2\pi n t}{T_0}\right)dt$$

$$+ \frac{1}{T_0}\int_{-T_0/2}^{-T} A_c \exp\left(-j\frac{2\pi n t}{T_0}\right)dt + \frac{1}{T_0}\int_{T}^{T_0/2} A_c \exp\left(-j\frac{2\pi n t}{T_0}\right)dt$$

$$= \frac{A_c}{T_0}\exp(jAk_p)\left[\frac{\exp\left(-j\frac{2\pi n t}{T_0}\right)}{-j\frac{2\pi n}{T_0}}\right]_{-T}^{T} + \frac{A_c}{T_0}\left[\frac{\exp\left(-j\frac{2\pi n t}{T_0}\right)}{-j\frac{2\pi n}{T_0}}\right]_{-T_0/2}^{-T} + \frac{A_c}{T_0}\left[\frac{\exp\left(-j\frac{2\pi n t}{T_0}\right)}{-j\frac{2\pi n}{T_0}}\right]_{T}^{T_0/2}$$

PROBLEMA 2 (cont.)

$$= \frac{A_c}{T_0} \exp(jAK_p) \frac{\exp\left(j\frac{2\pi nT}{T_0}\right) - \exp\left(-j\frac{2\pi nT}{T_0}\right)}{j\frac{2\pi n}{T_0}} + \frac{A_c}{T_0} \frac{\exp(j\pi n) - \exp\left(j\frac{2\pi nT}{T_0}\right) + \exp\left(-j\frac{2\pi nT}{T_0}\right) - \exp(-j\pi n)}{j\frac{2\pi n}{T_0}}$$

$$= \frac{A_c}{\cancel{T_0}} \exp(jAK_p) \cdot \frac{\cancel{T_0}}{\pi n} \operatorname{sen}\left(\frac{2\pi nT}{T_0}\right) + \frac{A_c}{\cancel{T_0}} \cdot \frac{\cancel{T_0}}{\pi n}\left[\operatorname{sen}(\pi n) - \operatorname{sen}\left(\frac{2\pi nT}{T_0}\right)\right]$$

$$= \frac{A_c}{\pi n} \operatorname{sen}\left(\frac{2\pi nT}{T_0}\right)\left(\exp(jAK_p) - 1\right) + A_c \operatorname{sinc}(n)$$

$$= \frac{A_c}{\pi n} \cdot \frac{2T}{2T} \cdot \frac{T_0}{T_0} \operatorname{sen}\left(\frac{2\pi nT}{T_0}\right)\left(\exp(jAK_p) - 1\right) + A_c \operatorname{sinc}(n)$$

$$\boxed{c_n = \frac{2A_cT}{T_0}\left(\exp(jAK_p) - 1\right) \operatorname{sinc}\left(\frac{2nT}{T_0}\right) + A_c \operatorname{sinc}(n)}$$

$$\boxed{\tilde{s}(t)} = \sum_{n=-\infty}^{\infty} \frac{2A_cT}{T_0}\left[\exp(jAK_p) - 1\right] \operatorname{sinc}\left(\frac{2nT}{T_0}\right) \exp\left(j\frac{2\pi nt}{T_0}\right) + A_c \sum_{n=-\infty}^{\infty} \operatorname{sinc}(n) \exp\left(j\frac{2\pi nt}{T_0}\right)$$

$$= \boxed{\frac{2A_cT}{T_0}\left(\exp(jAK_p) - 1\right) \sum_{n=-\infty}^{\infty} \operatorname{sinc}\left(\frac{2nT}{T_0}\right) \exp\left(j\frac{2\pi nt}{T_0}\right) + A_c}$$

(e) $s(t) = \operatorname{Re}\{\tilde{s}(t) \exp(j2\pi f_c t)\}$

$$= \operatorname{Re}\{A_c \exp(j2\pi f_c t)\} + \frac{2A_cT}{T_0} \sum_{n=-\infty}^{\infty} \operatorname{sinc}\left(\frac{2nT}{T_0}\right) \operatorname{Re}\left[\left\{\exp\left(jAK_p + j\frac{2\pi nt}{T_0}\right) - \exp\left(j\frac{2\pi nt}{T_0}\right)\right\} \exp(j2\pi f_c t)\right]$$

$$= A_c \cos(2\pi f_c t) + \frac{2A_cT}{T_0} \sum_{n=-\infty}^{\infty} \operatorname{sinc}\left(\frac{2nT}{T_0}\right)\left[-\cos\left(2\pi f_c t + \frac{2\pi n t}{T_0}\right) + \cos\left(2\pi f_c t + \frac{2\pi nt}{T_0} + AK_p\right)\right]$$

$$= A_c \cos(2\pi f_c t) + \frac{2A_cT}{T_0} \sum_{n=-\infty}^{\infty} \operatorname{sinc}\left(\frac{2nT}{T_0}\right)\left[-\cos(2\pi(f_c + nf_0)t) + \cos(2\pi(f_c + nf_0)t)\cos(AK_p) - \operatorname{sen}(2\pi(f_c + nf_0)t)\operatorname{sen}(AK_p)\right]$$

$$= A_c \cos(2\pi f_c t) + \frac{2A_cT}{T_0}\left[\cos(AK_p) - 1\right] \sum_{n=-\infty}^{\infty} \operatorname{sinc}\left(\frac{2nT}{T_0}\right) \cos(2\pi(f_c + nf_0)t) - \frac{2A_cT}{T_0} \operatorname{sen}(AK_p) \sum_{n=-\infty}^{\infty} \operatorname{sinc}\left(\frac{2nT}{T_0}\right) \cdot \operatorname{sen}(2\pi(f_c + nf_0)t)$$

con $f_0 = \frac{1}{T_0}$.

(4)

$$S'(f) = \frac{A_c}{2}\left[\delta(f-f_c)+\delta(f+f_c)\right] + \frac{A_c T}{T_0}\left[-1+\cos(Ak_p)\right]\sum_{n=-\infty}^{\infty} \operatorname{sinc}\left(\frac{2nT}{T_0}\right)\left[\delta(f-f_c-nf_0)+\delta(f+f_c+nf_0)\right]$$

$$+j\frac{A_c T}{T_0}\sin(Ak_p)\sum_{n=-\infty}^{\infty} \operatorname{sinc}\left(\frac{2nT}{T_0}\right)\left[\delta(f-f_c-nf_0)-\delta(f+f_c+nf_0)\right]$$