PROBLEMA 1.

$M(t)$ con ancho de banda W, estacionario, Gaussiana, media nula.

Señal AM $\quad X(t) = A_c[1 + K_a M(t)] \cos(2\pi f_c t + \Theta)$

$\Theta \sim U(0, 2\pi)$ independiente de $M(t)$.

(a) $\quad K_a = \frac{2}{3} \qquad \sigma_M^2 = \frac{1}{4} \quad , \sigma_M = \frac{1}{2}$

Se produce sobremodulación cuando se invierte la fase de la portadora

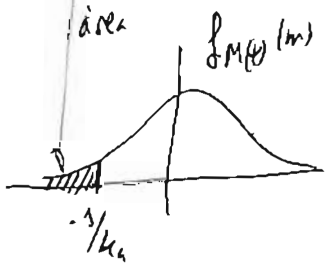
$$C(t) = A_c \cdot \cos(2\pi f_c t + \Theta)$$

es decir cuando $1 + K_a M(t)$ tome valores negativos

$$p(1 + K_a M(t) < 0) = p(K_a M(t) < -1) = p\left(M(t) < -\frac{1}{K_a}\right)$$

$M(t)$ es Gaussiana con media cero y varianza σ_M^2:

$$f_{M(t)}(m) = \frac{1}{\sqrt{2\pi}\,\sigma_M} \exp\left(-\frac{m^2}{2\sigma_M^2}\right)$$



$$p(\text{sobremodulación}) = \int_{-\infty}^{-\frac{1}{K_a}} f_{M(t)}(m)\,dm = \frac{1}{\sqrt{2\pi}\,\sigma_M} \int_{-\infty}^{-\frac{1}{K_a}} \exp\left(-\frac{m^2}{2\sigma_M^2}\right) dm$$

Sabemos que $\quad \operatorname{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^{\infty} \exp(-z^2)\,dz$

Hacemos cambio de variable $\quad z = -\frac{m}{\sqrt{2}\,\sigma_M} \quad , \; dz = -\frac{dm}{\sqrt{2}\,\sigma_M}$

$$p(\text{sobremodulación}) = \frac{1}{\sqrt{\pi}} \int_{\frac{1}{\sqrt{2} K_a \sigma_M}}^{\infty} \exp(-z^2)\,dz = \frac{1}{2} \operatorname{erfc}\left(\frac{1}{\sqrt{2} \cdot K_a \cdot \sigma_M}\right)$$

$$u = \frac{1}{\sqrt{2} \cdot K_a \cdot \sigma_M} = \frac{1}{\sqrt{2} \cdot 2/3 \cdot \frac{1}{2}} = \frac{3}{\sqrt{2}} = 2'12$$

, podemos usar aproximación

$$\boxed{P(\text{sobremodulación}) = \frac{1}{2}\frac{\exp(-u^2)}{\sqrt{\pi}\cdot u}} = \frac{1}{2}\frac{\exp(-2'12)}{\sqrt{\pi}\cdot 2'12} = \boxed{0'0035}$$

exacto usando tablas: 0'0013.

(b) $V(t) = A\cdot X(t) + BX^2(t) = A\cdot A_c[1+K_a M(t)]\cos(2\pi f_c t + \Theta)$

$$+ B A_c^2 [1+K_a M(t)]^2 \cos^2(2\pi f_c t + \Theta)$$

$$= A\cdot A_c[1+K_a M(t)]\cos(2\pi f_c t+\Theta) + B\cdot A_c^2[1+2K_a M(t)+K_a^2 M^2(t)]\left[\frac{\cos(4\pi f_c t+2\Theta)+1}{2}\right]$$

$$= \frac{BA_c^2}{2}[1+2K_a M(t)+K_a^2 M^2(t)] + A\cdot A_c[1+K_a M(t)]\cos(2\pi f_c t+\Theta)$$

$$+ \frac{BA_c^2}{2}[1+2K_a M(t)+K_a^2 M^2(t)]\cos(4\pi f_c t + 2\Theta)$$

(c) $E[M(t)] = 0$ $\quad M(t)$ y Θ independientes

$$E[\cos(2\pi f_c t+\Theta)] = \int_{-\infty}^{\infty} f_\Theta(\theta)\cos(2\pi f_c t+\theta)d\theta = \int_0^{2\pi}\frac{1}{2\pi}\cos(2\pi f_c t+\theta)d\theta$$

$$= \frac{1}{2\pi}\left[\operatorname{sen}(2\pi f_c t+\theta)\right]_0^{2\pi} = 0.$$

$$E[\cos(4\pi f_c t+2\Theta)] = \int_0^{2\pi}\frac{1}{2\pi}\cos(4\pi f_c t+2\theta)d\theta = \frac{1}{2\pi}\left[\frac{\operatorname{sen}(4\pi f_c t+2\theta)}{2}\right]_0^{2\pi} = 0.$$

$$E[M^2(t)] = \operatorname{var}[M(t)] = R_M(0).$$

$$E[V(t)] = \frac{BA_c^2}{2}[1+0+K_a^2 R_M(0)] + A\cdot A_c[1+0]\cdot 0 +$$

$$\frac{BA_c^2}{2}[1+0+K_a^2 R_M(0)]\cdot 0 = \boxed{\frac{BA_c^2}{2}[1+K_a^2 R_M(0)]}$$

$$\boxed{E[V(t)] = \frac{B\cdot A_c^2}{2} + \frac{BA_c^2K_a^2}{2}R_M[0]}$$

No depende del tiempo, estacionario con respecto a la media.

PROBLEMA 1 (CONTINUACIÓN)

(d) Todos los términos con frecuencia superior a 3W son eliminados por el filtro. Entonces:

$$\boxed{Y(t) = \frac{BA_c^2}{2} + BA_c^2 K_a M(t) + \frac{BA_c^2 K_a^2}{2} M^2(t)}$$

(e)
$$\boxed{E[Y(t)]} = \frac{BA_c^2}{2} + B\cdot A_c^2\cdot K_a \cdot \cancel{E[M(t)]}^{\,0} + \frac{BA_c^2 K_a^2}{2} \underbrace{E[M^2(t)]}_{R_M(0)}$$

$$\boxed{E[Y(t)] = E[V(t)] = \frac{BA_c^2}{2} + \frac{BA_c^2 K_a^2}{2} R_M(0)}$$

No cambia la media y sigue siendo estacionaria al no depender del tiempo.

$$R_Y(\tau) = E[Y(t)\,Y(t+\tau)] = \frac{B^2A_c^4}{4} E\Big[\big(1+2K_a M(t) + K_a^2 M^2(t)\big)\big(1+2K_a M(t+\tau) + K_a^2 M^2(t+\tau)\big)\Big]$$

$$= \frac{B^2A_c^4}{4} + \frac{B^2A_c^4}{2} K_a \cancel{E[M(t+\tau)]}^{\,0} + \frac{B^2A_c^4}{4} K_a^2 \underbrace{E[M^2(t+\tau)]}_{R_M(0)} + \frac{B^2A_c^4}{2} K_a \cancel{E[M(t)]}^{\,0} +$$

$$+ B^2A_c^4 K_a^2 \underbrace{E[M(t)M(t+\tau)]}_{R_M(\tau)} + \frac{B^2A_c^4}{2} K_a^3 \underbrace{E[M(t)\overbrace{M^2(t+\tau)}^{Z(t+\tau)}]}_{R_{M,Z}(\tau)} + \frac{B^2A_c^4}{4} K_a^2 \underbrace{E[M^2(t)]}_{R_M(0)}$$

$$+ \frac{B^2A_c^4}{2} K_a^3 \underbrace{E[\overbrace{M^2(t)}^{Z(t)} M(t+\tau)]}_{R_{Z,M}(\tau) = R_{M,Z}(-\tau)} + \frac{B^2A_c^4}{4} K_a^4 \underbrace{E[\overbrace{M^2(t)}^{Z(t)}\overbrace{M^2(t+\tau)}^{Z(t+\tau)}]}_{R_Z(\tau)}$$

$$R_Y(\tau) = \frac{B^2A_c^4}{4} + \frac{B^2A_c^4}{2} R_M(0) + B^2A_c^4 K_a^2 R_M(\tau) + \frac{B^2A_c^4}{2} K_a^3\big(R_{M,Z}(\tau) + R_{M,Z}(-\tau)\big) + \frac{B^2A_c^4}{4} K_a^4 R_Z(\tau)$$

$$\boxed{R_Y(\tau) = B^2A_c^4\left[\frac{1}{4} + \frac{R_M(0)}{2} + K_a^2 R_M(\tau) + \frac{K_a^3}{3} R_{M,Z}(\tau) + \frac{K_a^3}{3} R_{M,Z}(-\tau) + \frac{K_a^4}{4} R_Z(\tau)\right]}$$

Estacionaria en sentido amplio pues ni $E[Y(t)]$ ni $R_Y(\tau)$ dependen de t.

PROBLEMA 2.

$m(t)$ con $W = 15\,KHz$ y $P = 10\,W$
normalizada tal que $\max_t |m(t)| = 1$

para AM al 90%, $SNR_o^{AM} = 30\,dB \equiv 1000$

(a) $\mu = 0.9 = K_a \cdot \max_t |m(t)| = K_a \cdot 1 = K_a \Rightarrow \boxed{K_a = 0.9}$

Como $SNR_o = 30\,dB$ estamos por encima del umbral, por lo que

$$SNR_o^{AM} = \frac{A_c^2 K_a^2 P}{2W N_0} \Rightarrow \frac{A_c^2}{N_0} = \frac{2 \cdot SNR_o^{AM} \cdot W}{P \cdot K_a^2}$$

La relación portadora ruido

$$\rho = CNR = \frac{A_c^2}{4WN_0} = \frac{A_c^2}{N_0} \cdot \frac{1}{4W} = \frac{\cancel{2} \cdot SNR_o^{AM} \cdot \cancel{W}}{{}_2\cancel{4}\cancel{W} P K_a^2} = \frac{SNR_o^{AM}}{2 \cdot P \cdot K_a^2}$$

$$\boxed{\rho = \frac{1000}{2 \cdot 10 \cdot 0'9^2} = 61'73 \equiv 17'9\,dB}$$

La probabilidad de que el ruido esté por encima de la señal:

$$\boxed{p(R \geq A_c) = \exp(-\rho) = \exp(-61'72) = 1'55 \cdot 10^{-27}}$$

Con lo que la suposición de estar por encima del umbral fue correcta.

(b) La potencia de señal de entrada en AM:

$$P_{S_I}^{AM} = \frac{A_c^2}{2}(1 + K_a^2 P) = \frac{\cancel{2}\, SNR_o^{AM} \cdot W \cdot N_0}{\cancel{2}\, P \cdot K_a^2}(1 + K_a^2 P) = SNR_o^{AM}\, W N_0 \frac{1 + K_a^2 P}{K_a^2 P}$$

$$P_{S_I}^{AM} = 1000 \cdot 15 \cdot 10^3 \frac{1 + 0'9^2 \cdot 10}{0'9^2 \cdot 10} \cdot N_0 = 1'69 \cdot 10^7 N_0$$

Para DSB

$$P_{S_I}^{DSB} = P_{S_I}^{AM} = 1'69 \cdot 10^7 N_0 = \frac{A_c^2 P}{2} \Rightarrow A_c^2 = \frac{2 P_{S_I}^{AM}}{P} = 3'37 \cdot 10^6 N_0$$

Entonces $\frac{A_c^2}{N_0} = 3'37 \cdot 10^6$

$$\boxed{SNR_o^{DSB} = \frac{A_c^2 \cdot P}{2WN_o} = 3'37\cdot 10^6 \cdot \frac{10}{2\cdot 15\cdot 10^3} = 1123'5 \equiv 30'5\,dB}$$

(c) Para SSB:

$$P_{S_I}^{SSB} = P_{S_I}^{AM} = 1'69\cdot 10^7 N_o = \frac{A_c^2 P}{4} \Rightarrow \frac{A_c^2}{N_o} = \frac{4\cdot 1'69\cdot 10^7}{10} = 6'74\cdot 10^6$$

$$\boxed{SNR_o^{SSB} = \frac{A_c^2 P}{4N_oW} = 6'67\cdot 10^6 \frac{10}{4\cdot 15\cdot 10^3} = 1123'5 \equiv 30'5\,dB}$$

(d) En FM igualamos potencia de señal recibida.

$$P_{S_I}^{FM} = P_{S_I}^{AM} = 1'69\cdot 10^7 N_o = \frac{A_c^2}{2} \Rightarrow \frac{A_c^2}{N_o} = 2\cdot 1'69\cdot 10^7 = 3'37\cdot 10^7$$

$$SNR_o^{FM} = \frac{3A_c^2\cdot K_f^2\cdot P}{2\cdot W^3\cdot N_o} = SNR_o^{AM} \Rightarrow K_f = \sqrt{\frac{2\cdot SNR_o^{AM}\cdot W^3}{3\left(\frac{A_c^2}{N_o}\right)\cdot P}}$$

$$K_f = \sqrt{\frac{2\cdot 1000\cdot 15000^3}{3\cdot 3'37\cdot 10^7\cdot 10}} = 2584\ Hz/V$$

$$\boxed{\Delta f = K_f\cdot \max_t |m(t)| = 2584\cdot 1 = 2584\ Hz}$$

Para que esté funcionando por encima del umbral

$$P_{S_I}^{FM} > 20 B_T N_o \Rightarrow \frac{A_c^2}{2} > 20 B_T N_o \Rightarrow \frac{A_c^2}{N_o} > 40 B_T$$

Calculamos el ancho de banda de transmisión:

$$B_T = 2W + 2\Delta f = 2\cdot 15000 + 2\cdot 2584 = 35168\ Hz$$

Entonces $\frac{A_c^2}{N_o} > 40\cdot 35168 = 1'41\cdot 10^6$

Como $\frac{A_c^2}{N_o} = 3'37\cdot 10^7$ estamos por encima del umbral

PROBLEMA 2 (CONTINUACION)

(e)

$$SNR_o^{FM} = \frac{3 \cdot A_c^2 \cdot K_f^2 \cdot P}{2 \cdot W^3 \cdot N_o} = 3'37 \cdot 10^7 \cdot \frac{3 \cdot 75000^2 \cdot 10}{2 \cdot 15000^3} = 8'43 \cdot 10^5$$

$$\boxed{SNR_o^{FM} = 59'26\,dB}$$

Volvemos a comprobar si

$$\frac{A_c^2}{N_o} > 40 B_T$$

El nuevo ancho de banda: $B_T = 2W + 2\Delta f = 2 \cdot 15000 + 2 \cdot 75000 = 180KHz$

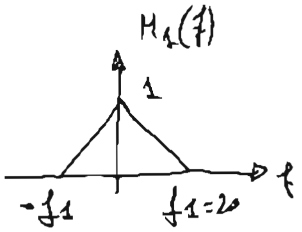
Para estar por encima del umbral:

$$\frac{A_c^2}{N_o} > 40 \cdot 180 \cdot 10^3 = 7'2 \cdot 10^6$$

Como $\frac{A_c^2}{N_o} = 3'37 \cdot 10^7$, estamos por encima del umbral.

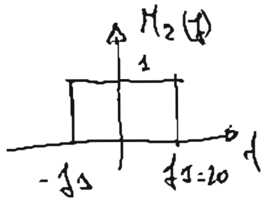
PROBLEMA 3

(a) $M_1(f) = \Lambda\left(\frac{f}{f_1}\right)$



$\boxed{W_1 = f_1 = 20\ Hz}$

$M_2(f) = \Pi\left(\frac{f}{2f_1}\right)$



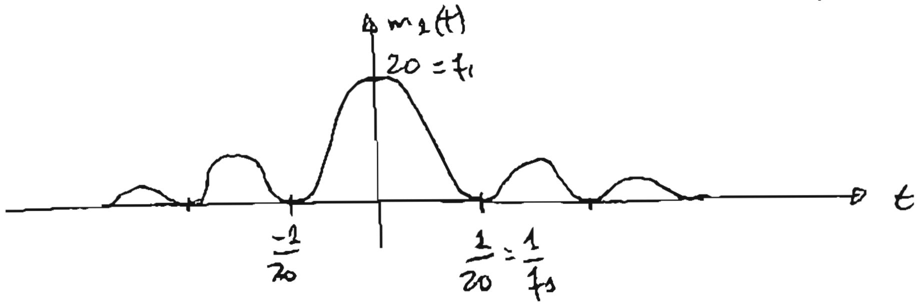
$\boxed{W_2 = f_1 = 20 Hz}$

(b) Sabemos de la tabla de transformadas de Fourier inmediatas:

$$\Lambda\left(\frac{t}{T}\right) \rightleftharpoons T\ \text{sinc}^2(fT)$$

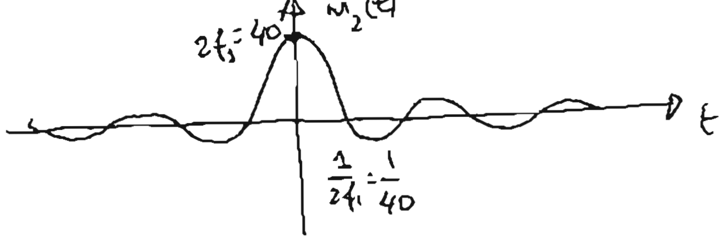
Aplicando la propiedad de dualidad cambiando tiempos y frecuencias:

$$m_1(t) = f_1\ \text{sinc}^2(f_1 \cdot t) \rightleftharpoons M_1(f) = \Lambda\left(\frac{f}{f_1}\right)$$



Para $m_2(t)$: $\quad W\ \text{sinc}(WT) \rightleftharpoons \Pi\left(\frac{f}{W}\right)$ $\quad$ entonces:

$$m_2(t) = 2f_1\ \text{sinc}(2f_1 t) \rightleftharpoons M_2(f) = \Pi\left(\frac{f}{2f_1}\right)$$



(c) $s(t) = m_1(t)\cos(2\pi f_c t) + m_2(t)\ \text{sen}(2\pi f_c t) =$

$= f_1\ \text{sinc}^2(f_1 t)\cos(2\pi f_c t) + 2f_1\ \text{sinc}(2f_1 t)\ \text{sen}(2\pi f_c t) =$

$$= \cancel{f_1}\,\frac{\operatorname{sen}(\pi f_1 t)\,\operatorname{sen}(\pi f_1 t)\cos(2\pi f_c t)}{\pi^2 f_1^{\cancel{2}} t^2} + \cancel{2}\,\cancel{f_1}\,\frac{\operatorname{sen}(2\pi f_1 t)\cos(2\pi f_c t)}{\cancel{2}\pi \cancel{f_1} t}$$

$$= \frac{1}{\pi^2 f_1 t^2}\left(\frac{1}{2} - \frac{1}{2}\cos(2\pi f_1 t)\right)\cos(2\pi f_c t) + \frac{1}{\pi t}\operatorname{sen}(2\pi f_1 t)\operatorname{sen}(2\pi f_c t)$$

sabemos que $\operatorname{sen} a \operatorname{sen} b = \frac{1}{2}\cos(a-b) - \frac{1}{2}\cos(a+b)$

$$\boxed{s(t)} = \frac{1}{2\pi^2 f_1 t^2}\cos(2\pi f_c t) - \frac{1}{4\pi^2 f_1 t^2}\cos(2\pi(f_c+f_1)t) - \frac{1}{4\pi^2 f_1 t^2}\cos[2\pi(f_c-f_1)t]$$

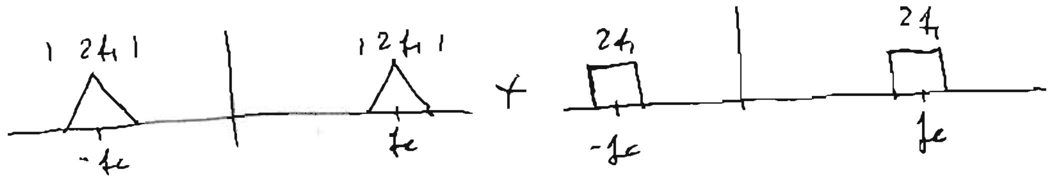
$$+ \frac{1}{2\pi t}\cos[2\pi(f_c-f_1)t] - \frac{1}{2\pi t}\cos[2\pi(f_c+f_1)t]$$

$$= \frac{1}{2\pi t}\left[\frac{1}{\pi f_1 t}\cos(2\pi f_c t) + \left(1 - \frac{1}{2\pi f_1 t}\right)\cos(2\pi(f_c-f_1)t) - \left(1+\frac{1}{2\pi f_1 t}\right)\cos(2\pi(f_c+f_1)t)\right]$$

$$\boxed{= \frac{1}{2\pi t}\left[\frac{1}{20\pi t}\cos(1970\pi t) + \left(1 - \frac{1}{40\pi t}\right)\cos(1930\pi t) - \left(1+\frac{1}{40\pi t}\right)\cos(2010\pi t)\right]}$$

(d)

$$s(t) = \underbrace{m_1(t)\cos(2\pi f_c t)}_{DSB_1} + \underbrace{m_2(t)\operatorname{sen}(2\pi f_c t)}_{DSB_2}$$



$$\left.\begin{array}{l}\boxed{f_{min} = f_c - f_1 = 965\ Hz} \\ \boxed{f_{max} = f_c + f_1 = 1005\ Hz}\end{array}\right\} \boxed{B_W = 2f_1 = 40\ Hz}$$

(e) Si la señal se considera banda base, el teorema de Nyquist dice que la tasa debe ser al menos el doble de la frecuencia f_{max} $\Rightarrow$ $\boxed{f_s = 2 f_{max} = 2\cdot 1005 = 2010\ Hz}$

PROBLEMA 3 (CONTINUACION)

$$\boxed{T_s = \frac{1}{2010} = 4'98 \cdot 10^{-4} \text{ segundos}}$$

$$s(0) = f_1 = 20$$

$$s(T_s) = -17'46$$

$$s(2 \cdot T_s) = 14'84$$

$$s(3T_s) = -12'18$$

(f) Primero hay que determinar r con la expresión:

$$r \leq \frac{f_c + f_1}{2f_1} \leq r+1$$

$$r \leq \frac{985+20}{2 \cdot 20} = \frac{1005}{40} = 25'125 < r+1 \Rightarrow \boxed{r = 25}$$

El nuevo ancho de banda:

$$B_w' = \frac{f_c + f_1}{r} = \frac{1005}{25} = 40'2 \text{ Hz}$$

Con lo cual el teorema de Nyquist:

$$\boxed{f_s' = 2 \cdot B_w' = 80'4 \text{ Hz}} \Rightarrow T_s' = 0'0124 \text{ segundos}$$

$$s(0) = 20$$

$$s(T_s') = 25'46$$

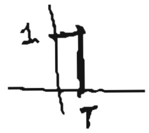
$$s(2 \cdot T_s') = -8'19$$

$$s(3 \cdot T_s') = 8'57$$

(g) Señal PAM $\quad K_a = 0'03$

$$\frac{T}{T_s'} = 0'25 \qquad s(t) = A_c \sum_{n=-\infty}^{\infty} \left[1 + K_a m(nT_s')\right] g(t - nT_s')$$

$T = 0'25 \cdot T_s' = 0'031$

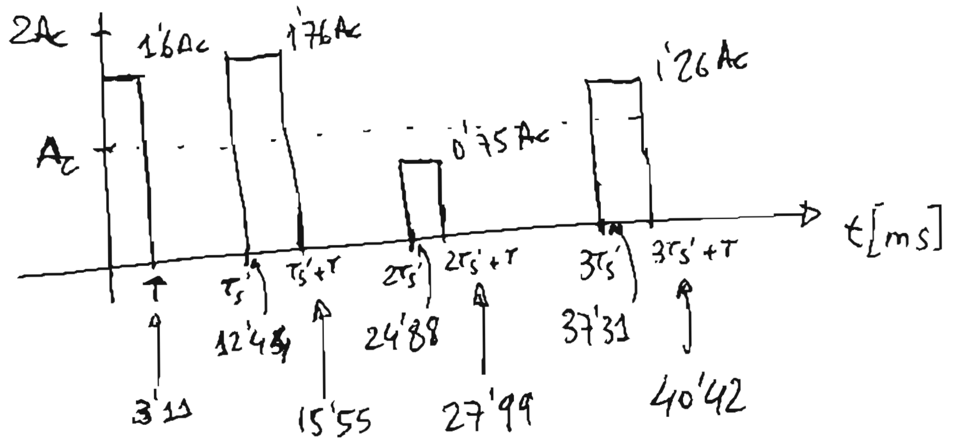
$g(t)$ suponemos 

$$1 + K_a m(0) = 1 + 0'03 \cdot 20 = 1'6$$

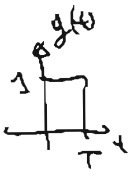
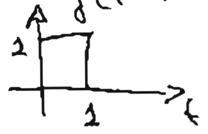
$$1 + K_a m(T_s') = 1 + 0'03 \cdot 25'46 = 1'76$$

$$1 + K_a m(2T_s') = 1 + 0'03 \cdot (-8'19) = 0'75$$

$$1 + K_a m(3T_s') = 1 + 0'03 \cdot 8'57 = 1'26$$



(h) $K_d = 0'035 \qquad s(t) = A_c \sum_{n=-\infty}^{\infty} g\left(\frac{2T(t - nT_s')}{T_s'(1 + K_d m(nT_s'))}\right)$

$g(t)$ suponemos  $\Rightarrow$ $g(T-t)$ $\Rightarrow$ 

Duraciones:

$$\frac{T_s'}{2}\left(1 + K_d\, m(nT_s')\right) \Rightarrow \begin{cases} \frac{12'44\,ms}{2}(1 + 0'035 \cdot 20) = 10'57\,ms \\ 6'22\,ms\,(1 + 0'035 \cdot 25'46) = 11'76\,ms \\ 6'22\,ms\,(1 + 0'035(-8'19)) = 4'44\,ms \\ 6'22\,ms\,(1 + 0'035 \cdot 8'57) = 8'08\,ms \end{cases}$$