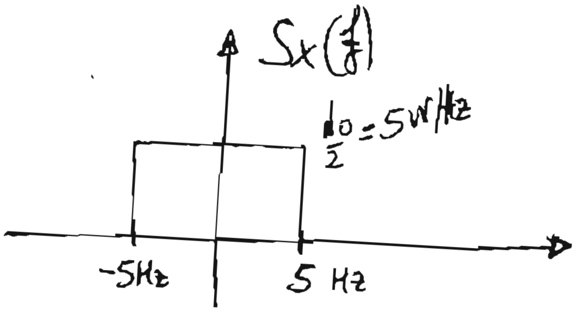
PROBLEMA 1.

①

(a) $\boxed{E[X(t)] = 0}$ $\qquad N_0 = 10 W/Hz$, estacionario



$P_X = E[X^2(t)] = var[X(t)] = 50\ W$

$S_X(f) = 5 \cdot \Pi\left(\frac{f}{10}\right)$ W/Hz

$R_X(\tau) \rightleftharpoons S_X(f)$

$$\boxed{R_X(\tau) = 50 \operatorname{sinc}(10\tau)}$$

$$\boxed{E[Y(t)] = E[X(t)\cos(100\pi t)] = \cos(100\pi t)\, E[X(t)] = 0}$$

$$\boxed{E[Z(t)] = E[Y(t) + W(t)] = E[Y(t)] + E[W(t)] = 0}$$

W(t) Ruido blanco, Gaussiano con $E[W(t)] = 0$, $N_0 = 1 W/Hz$, estacionario

$P_W = E[W^2(t)] = var[W(t)] = \infty$ $\qquad \boxed{S_W(f) = \frac{N_0}{2} = \frac{1}{2}\ W/Hz}$

$$\boxed{R_W(\tau) = \frac{1}{2}\,\delta(\tau)}$$

$$\boxed{E[S(t)]} = E[Z(t) * h(t)] = E\left[\int Z(\tau)\, h(t-\tau)\, d\tau\right] = \int E[Z(\tau)]\, h(t-\tau)\, d\tau =$$

$$= E[Z(t)] * h(t) = \boxed{0}$$

Todas las señales son estacionarias con respecto a la media, ya que es nula y por tanto no depende del instante de tiempo t.

(b) $\boxed{R_Y(t_1,t_2)} = E[Y(t_1)\cdot Y(t_2)] =$

$$= E[X(t_1)\cos(100\pi t_1)\, X(t_2)\cos(100\pi t_2)]$$

$$= E[X(t_1)X(t_2)]\cos(100\pi t_1)\cos(100\pi t_2)$$

$$= R_X(t_1-t_2)\left[\frac{1}{2}\cos[100\pi(t_1-t_2)] + \frac{1}{2}\cos[100\pi(t_1+t_2)]\right]$$

$$= \frac{1}{2}\cdot 50\,\text{sinc}[10(t_1-t_2)]\left(\cos[100\pi(t_1-t_2)] + \cos[100\pi(t_1+t_2)]\right)$$

$$\boxed{= 25\,\text{sinc}[10(t_1-t_2)]\left[\cos[100\pi(t_1-t_2)] + \cos[100\pi(t_1+t_2)]\right]}$$

Debido al término $\cos[100\pi(t_1+t_2)]$, la autocorrelación no sólo depende de la diferencia de tiempos $\tau = t_1 - t_2$, sino también de la suma, por lo que, no puede ser estacionaria en sentido amplio.

(c) $\boxed{R_Z(t_1,t_2)} = E[Z(t_1)Z(t_2)] = E[(Y(t_1)+W(t_1))(Y(t_2)+W(t_2))]$

$$= E[Y(t_1)Y(t_2) + Y(t_1)W(t_2) + W(t_1)Y(t_2) + W(t_1)W(t_2)]$$

$$= E[Y(t_1)Y(t_2)] + \cancel{E[Y(t_1)]E[W(t_2)]} + \cancel{E[W(t_1)]E[Y(t_2)]} + E[W(t_1)W(t_2)]$$

$$= R_Y(t_1,t_2) + R_W(t_1,t_2)$$

$$\boxed{= 25\,\text{sinc}[10(t_1-t_2)]\left[\cos[100\pi(t_1-t_2)] + \cos[100\pi(t_1+t_2)]\right] + \frac{1}{2}\delta(t_1-t_2)}$$

$$SNR(t) = \frac{\text{Pot. Señal}(t)}{\text{Pot. Ruido}(t)} = \frac{\text{Var. Señal}(t)}{\text{Var. Ruido}(t)}$$ (Supuesto media cero).

PROBLEMA 1 (CONT.)

Señal ⇒ $R_Z^{\text{señal}}(t_1,t_2) = R_Y(t_1,t_2)$

Ruido ⇒ $R_Z^{\text{ruido}}(t_1,t_2) = R_W(t_1,t_2)$

Var. señal $(t) = R_Y(t,t)$

Var. ruido $(t) = R_W(t,t)$

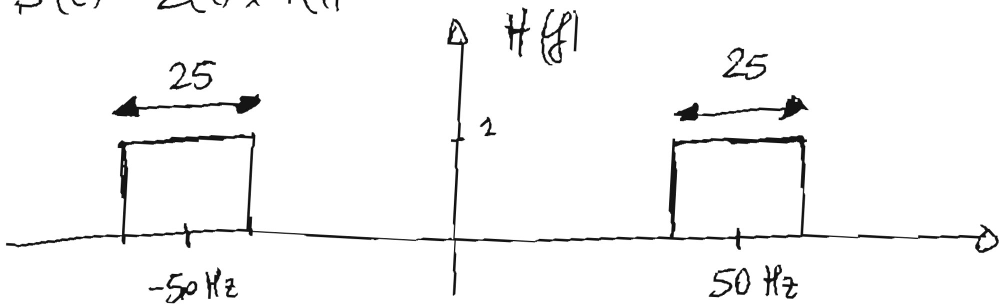
$$\boxed{R_Y(t,t) = 25\,\text{sinc}[10(t-t)]\left[\cos[100\pi(t-t)] + \cos[100\pi(t+t)]\right]}$$

$$\boxed{= 25\left[1 + \cos[200\pi t]\right] = \text{Var}[Y(t)]}$$

$$R_W(t,t) = \frac{1}{2}\delta(t-t) = \infty = \text{Var}[W(t)]$$

$$SNR(t) = \frac{R_Y(t,t)}{R_W(t,t)} = \frac{25[1+\cos[200\pi t]]}{\infty} = 0$$

(d) $S'(t) = Z(t) * h(t)$



$$S(t) = [Y(t) + W(t)] * h(t) = \underbrace{Y(t)*h(t)}_{\text{señal filtrada}} + \underbrace{W(t)*h(t)}_{\text{ruido filtrado.}}$$

La componente de señal no es estacionaria, pero la componente de ruido sí. No podemos determinar la densidad espectral de potencia para $Y(t)$, ya que no es estacionaria.

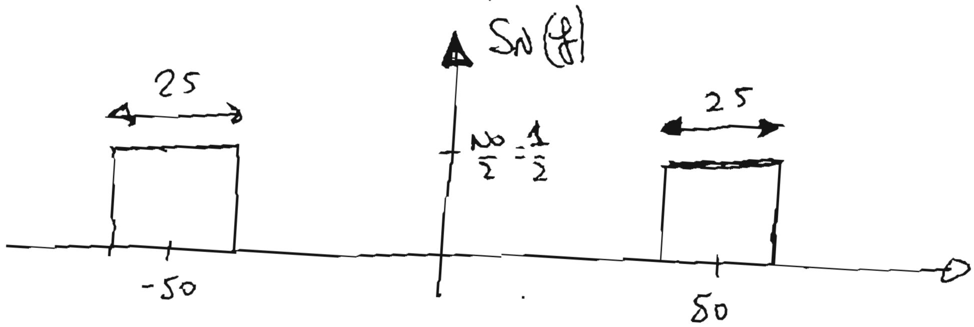
$Y(t) = X(t)\cdot\cos(100\pi t)$, $X(t)$ era una señal paso bajo con frecuencia máxima 5 Hz, $Y(t)$ será entonces una señal DSB

La portadora $\cos[100\pi t]$ a frecuencia $f_c = 50$ Hz. Dunque no tengo representación espectral sabemos que ocupará tras una señal DSB: $[f_c - W, f_c + W]$ con $f_c = 50$ Hz y $W = 5$ Hz en este caso, entonces ocupa la banda $[45, 55]$ por lo que pasa sin ser modificada por el filtro $h(t)$: 4

$$S(t) = Y(t) + W(t) * h(t).$$

Si llamamos $N(t) = W(t) * h(t)$, componente filtrada del ruido, entonces:

$$S_N(f) = S_W(f) \cdot |H(f)|^2$$



$$R_N(\tau) = R_N(t_1 - t_2) \rightleftharpoons S_N(f).$$

$$S_N(f) = \frac{1}{2}\Pi\left(\frac{f-50}{25}\right) + \frac{1}{2}\Pi\left(\frac{f+50}{25}\right)$$

$$\boxed{R_N(\tau) = \frac{1}{2} \cdot 25\,\mathrm{sinc}(25\tau)\exp(j2\pi 50\tau) + \frac{1}{2}25\,\mathrm{sinc}(25\tau)\exp(-j2\pi 50\tau)}$$

$$= 25\,\mathrm{sinc}(25\tau)\left[\frac{1}{2}e^{j100\pi\tau} + \frac{1}{2}e^{-j100\pi\tau}\right]$$

$$\boxed{= 25\,\mathrm{sinc}(25\tau)\cos(100\pi\tau)}$$

PROBLEMA 1 (CONT.)

5

$$S(t) = Y(t) + N(t)$$

$$E[Y(t)] = 0$$

$$R_Y(t_1,t_2) = 25\,\text{sinc}[(t_1-t_2)10]\Big[\cos[100\pi(t_1-t_2)] + \cos[100\pi(t_1+t_2)]\Big]$$

$$R_N(t_1,t_2) = 25\,\text{sinc}[25(t_1-t_2)]\cos[100\pi(t_1-t_2)]$$

$$E[N(t)] = E[w(t) * h(t)] = h(t) * E[w(t)] = 0.$$

$$\boxed{R_S(t_1,t_2)} = E[S(t_1)S(t_2)] = E\Big[[Y(t_1)+N(t_1)][Y(t_2)+N(t_2)]\Big]$$

$$= E\Big[Y(t_1)Y(t_2) + Y(t_1)N(t_1) + N(t_1)Y(t_2) + N(t_1)N(t_2)\Big]$$

$$= R_Y(t_1,t_2) + \cancel{E[Y(t_1)]E[N(t_1)]} + \cancel{E[N(t_1)]E[Y(t_2)]} + R_N(t_1,t_2)$$

$$= 25\cos[100\pi(t_1-t_2)]\Big(\text{sinc}(10(t_1-t_2)) + \text{sinc}(25(t_1-t_2))\Big) + 25\cos[100\pi(t_1+t_2)]\,\text{sinc}[10(t_1-t_2)]$$

$$\text{SNR}(t) = \frac{P_{Y(t)}}{P_N} = \frac{\text{var}[Y(t)]}{\text{var}[N(t)]} \quad \underline{\text{medias cero.}}$$

$$\text{var}[Y(t)] = R_Y(t,t) = 25[1+\cos[200\pi t]] \quad \text{calculado apartado (c)}$$

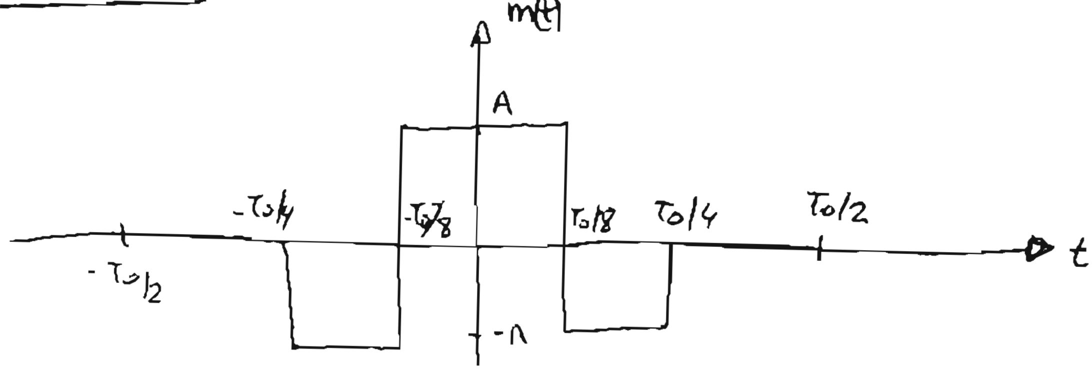
$$\text{var}[N(t)] = R_N(t,t) = 25\,\text{sinc}(25(t-t))\cos(100\pi(t-t)) = 25\ W.$$

$$\boxed{\text{SNR}(t) = \frac{25(1+\cos(200\pi t))}{25} = 1+\cos(200\pi t)}$$

Como $Y(t)$ no es estacionaria, la potencia de $Y(t)$ cambia con el tiempo y por tanto la SNR también. En $t=0$.

$$\boxed{\text{SNR}(0) = 1+\cos(200\pi t) = 1+1 = 2}$$

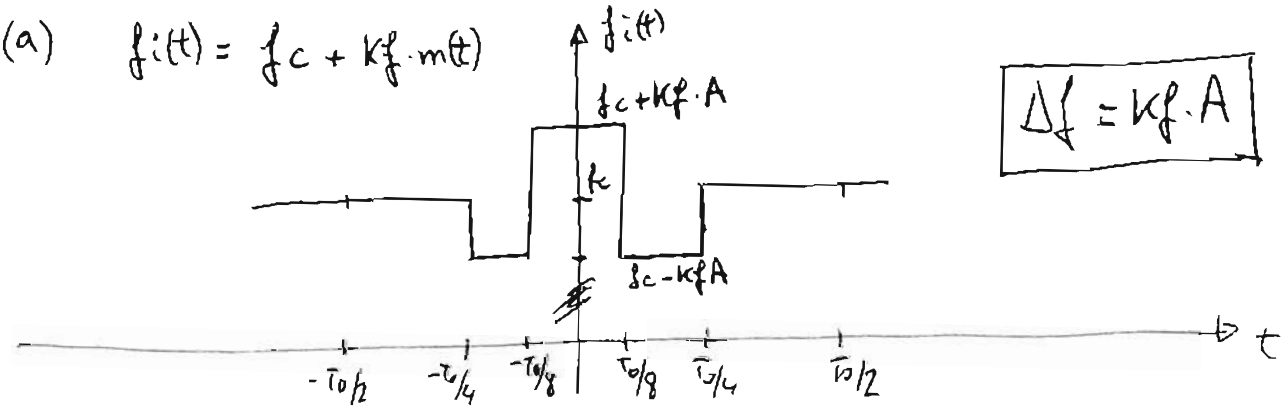
PROBLEMA 2.



$c(t) = A_c \cos(2\pi f_c t)$ $\qquad$ K_f : sensibilidad en frecuencia.

(a) $f_i(t) = f_c + K_f \cdot m(t)$

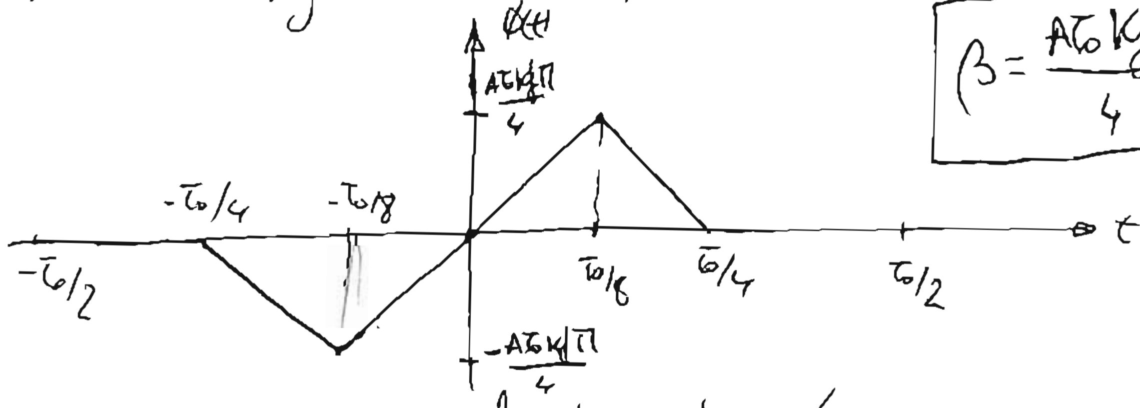
$$\boxed{\Delta f = K_f \cdot A}$$



(b)

$\phi(t) = 2\pi K_f \int m(t)\, dt$ $\qquad$ $\phi(0) = 0$

$$\boxed{\beta = \frac{A T_o K_f \pi}{4}}$$



Entre $-T_o/8$ y $T_o/8$ es lineal y además $\phi(0) = 0$

En $T_o/8$: área entre $[0, T_o/8]$ $\Rightarrow$ $A \cdot \frac{T_o}{8} \cdot 2\pi K_f = \frac{A T_o K_f \pi}{4}$

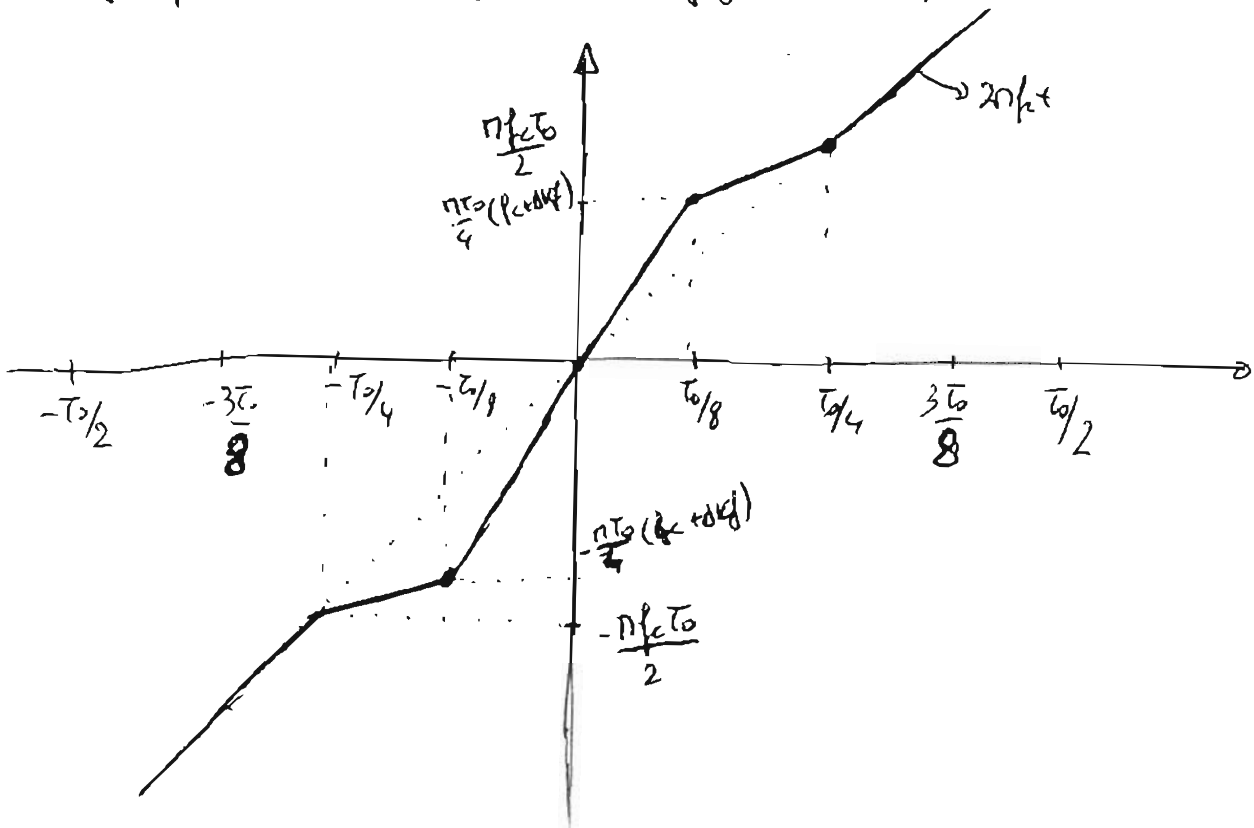
En $-T_o/8$: área entre $[-T_o/8, 0]$ $\Rightarrow$ $-\frac{A T_o K_f \pi}{4}$

De $T_o/8$ a $T_o/4$ se suma área : $-\frac{A T_o K_f \pi}{4}$, con lo que $\phi(T_o/4) = 0$

De $-T_o/4$ a $T_o/8$ se suma área $\frac{A T_o K_f \pi}{4}$, con lo que $\phi(-T_o/4) = 0$

(c) $\theta_i(t) = 2\pi f_c t + 2\pi K_f \int m(t)dt = 2\pi f_c t + \phi(t) = 2\pi \int f_i(t)dt$

Hay que sumar $2\pi f_c t$ a la figura de $\phi(t)$.



$$\theta_i(t)\Big|_{t=T_0/4} = \theta_i\left(\frac{T_0}{4}\right) = 2\pi f_c \frac{T_0}{4} = \frac{\pi f_c T_0}{2}$$

$$\theta_i\left(\frac{T_0}{8}\right) = 2\pi f_c \frac{T_0}{8} + \phi\left(\frac{T_0}{8}\right) = \frac{\pi f_c T_0}{4} + \frac{\pi A K_f T_0}{4} = \frac{\pi T_0}{4}(f_c + \Delta K_f)$$

(d) $\tilde{s}(t) = A_c \exp(j\phi(t)) = \sum_{n=-\infty}^{\infty} c_n \exp(j 2\pi n f_0 t) \qquad f_0 = \frac{1}{T_0}$

Además usamos que $\alpha = \Delta K_f T_0$.

Seleccionamos el intervalo de longitud T_0: $\left[-T_0/4, \frac{3T_0}{4}\right]$

En ese intervalo:

$$\phi(t) = \begin{cases} -2\pi A K_f \left(t + \frac{T_0}{4}\right) & -\frac{T_0}{4} < t < -\frac{T_0}{8} \\ 2\pi \Delta K_f t & |t| \leq \frac{T_0}{8} \\ -2\pi A K_f \left(t - \frac{T_0}{4}\right) & \frac{T_0}{8} < t < \frac{T_0}{4} \\ 0 & \frac{T_0}{4} < t < \frac{3T_0}{4} \end{cases}$$

PROBLEMA 2 (CONT.)

$$c_n = \frac{1}{T_0}\int_{-T_0/4}^{3T_0/4} \exp(j\phi(t))\exp(-j2\pi n f_0 t)dt =$$

$$= \frac{1}{T_0}\int_{-T_0/4}^{-T_0/8} \exp\left(-j2\pi\Delta k f\left(t+\frac{T_0}{4}\right)\right)\exp(-j2\pi n f_0 t)dt$$

$$+ \frac{1}{T_0}\int_{-T_0/8}^{T_0/8} \exp(j2\pi\Delta k f t)\exp(-j2\pi n f_0 t)dt$$

$$+ \frac{1}{T_0}\int_{T_0/8}^{T_0/4} \exp\left(-j2\pi\Delta k f\left(t-\frac{T_0}{4}\right)\right)\exp(-j2\pi n f_0 t)dt$$

$$+ \frac{1}{T_0}\int_{T_0/4}^{3T_0/4} \exp(j\cdot 0)\exp(-j2\pi n f_0 t)dt$$

$$= \frac{1}{T_0}\int_{-T_0/4}^{-T_0/8} \exp[-j2\pi(\Delta k f + n f_0)t]\exp\left(-j\frac{\pi\alpha}{2}\right)dt$$

$$+ \frac{1}{T_0}\int_{-T_0/8}^{T_0/8} \exp[j2\pi(\Delta k f - n f_0)t]dt$$

$$+ \frac{1}{T_0}\int_{T_0/8}^{T_0/4} \exp[-j2\pi(\Delta k f + n f_0)t]\exp\left(j\frac{\pi\alpha}{2}\right)dt + \frac{1}{T_0}\int_{T_0/4}^{\frac{3T_0}{4}} \exp(-j2\pi n f_0 t)dt$$

$$= \frac{1}{T_0}\exp\left(-j\frac{\pi\alpha}{2}\right)\left.\frac{\exp[-j2\pi(\Delta k f + n f_0)t]}{-j2\pi(\Delta k f + n f_0)}\right|_{-T_0/4}^{-T_0/8}$$

$$+ \frac{1}{T_0}\left.\frac{\exp[j2\pi(\Delta k f - n f_0)t]}{j2\pi(\Delta k f - n f_0)}\right|_{-T_0/8}^{T_0/8}$$

$$+ \frac{1}{T_0}\exp\left(j\frac{\pi\alpha}{2}\right)\left.\frac{\exp[-j2\pi(\Delta k f + n f_0)t]}{-j2\pi(\Delta k f + n f_0)}\right|_{T_0/8}^{T_0/4}$$

$$+ \frac{1}{T_0}\left.\frac{\exp(-j2\pi n f_0 t)}{-j2\pi n f_0}\right|_{T_0/4}^{3T_0/4}$$

(4)

$$= \frac{\exp\left(-j\frac{\pi\alpha}{2}\right)\exp\left(j\frac{\pi\alpha}{2}\right)\exp\left(j\frac{\pi n}{2}\right)}{j2\pi(\Delta kf+nf_0)T_0} - \frac{\exp\left(-j\frac{\pi\alpha}{2}\right)\exp\left(j\frac{\pi\alpha}{4}\right)\exp\left(j\frac{\pi n}{4}\right)}{j2\pi(\Delta kf+nf_0)T_0}$$

$$+ \frac{\exp\left(j\frac{\pi\alpha}{4}\right)\exp\left(-j\frac{\pi n}{4}\right)}{j2\pi(\Delta kf-nf_0)T_0} - \frac{\exp\left(-j\frac{\pi\alpha}{4}\right)\exp\left(j\frac{\pi n}{4}\right)}{j2\pi(\Delta kf-nf_0)T_0}$$

$$+ \frac{\exp\left(j\frac{\pi\alpha}{2}\right)\exp\left(-j\frac{\pi\alpha}{4}\right)\exp\left(-j\frac{\pi n}{4}\right)}{j2\pi(\Delta kf+nf_0)T_0} - \frac{\cancel{\exp\left(j\frac{\pi\alpha}{2}\right)}\cancel{\exp\left(-j\frac{\pi\alpha}{2}\right)}\exp\left(-j\frac{\pi n}{2}\right)}{j2\pi(\Delta kf+nf_0)T_0}$$

$$+ \frac{\exp\left(-j\frac{\pi n}{2}\right)}{j2\pi n} - \frac{\exp\left(-j\frac{3\pi n}{2}\right)}{j2\pi n}$$

$$= \frac{\exp\left(j\frac{\pi n}{2}\right) - \exp\left(j\frac{\pi(n-\alpha)}{4}\right) + \exp\left(-j\frac{\pi(n-\alpha)}{4}\right) - \exp\left(-j\frac{\pi n}{2}\right)}{j2\pi(\alpha+n)}$$

$$+ \frac{\exp\left(j\frac{\pi(\alpha-n)}{4}\right) - \exp\left(-j\frac{\pi(\alpha-n)}{4}\right)}{j2\pi(\alpha-n)} + \frac{\exp(-j\pi n)\left[\exp\left(j\frac{\pi n}{2}\right)-\exp\left(-j\frac{\pi n}{2}\right)\right]}{j2\pi n}$$

$$= \frac{\operatorname{sen}\left(\frac{\pi n}{2}\right) + \operatorname{sen}\left[\frac{\pi(\alpha-n)}{4}\right]}{\pi(\alpha+n)} + \frac{\operatorname{sen}\left[\frac{\pi(\alpha-n)}{4}\right]}{\pi(\alpha-n)} + (-1)^n \frac{\operatorname{sen}\left(\frac{\pi n}{2}\right)}{\pi n}$$

Usando ahora la fórmula $\operatorname{sen} x + \operatorname{sen} y = 2\operatorname{sen}\left(\frac{x+y}{2}\right)\cos\left(\frac{y-x}{2}\right)$

$$= \frac{2\operatorname{sen}\left[\frac{\pi(\alpha+n)}{8}\right]\cos\left[\frac{\pi(\alpha-3n)}{8}\right]}{\pi(\alpha+n)} + \frac{1}{4}\operatorname{sinc}\left(\frac{\alpha-n}{4}\right) + \frac{(-1)^n}{2}\operatorname{sinc}\left(\frac{n}{2}\right)$$

$$\boxed{c_n = \frac{1}{4}\cos\left[\frac{\pi(\alpha-3n)}{8}\right]\operatorname{sinc}\left(\frac{\alpha+n}{8}\right) + \frac{1}{4}\operatorname{sinc}\left(\frac{\alpha-n}{4}\right) + \frac{(-1)^n}{2}\operatorname{sinc}\left(\frac{n}{2}\right)}$$

PROBLEMA 2 (CONT.)

(e)

$$s(t) = \mathrm{Re}\left\{ \tilde{s}(t) \exp(j 2\pi f_c t) \right\}$$

$$\tilde{s}(t) = A_c \sum_{n=-\infty}^{\infty} c_n \exp(j 2\pi n f_0 t)$$

$$= \frac{A_c}{2} \sum_{n=-\infty}^{\infty} \left[\frac{1}{2} \cos\left[\frac{\pi(\alpha-3n)}{8}\right] \mathrm{sinc}\left(\frac{\alpha+n}{8}\right) + \frac{1}{2} \mathrm{sinc}\left(\frac{\alpha-n}{4}\right) + (-1)^n \mathrm{sinc}\left(\frac{n}{2}\right) \right] \exp(j 2\pi n f_0 t)$$

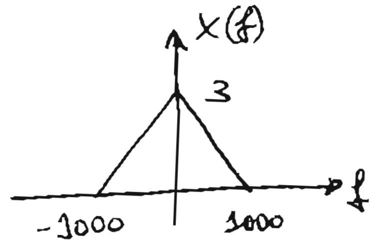
$$\boxed{s(t) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} \left(\frac{1}{2} \cos\left[\frac{\pi(\alpha-3n)}{8}\right] \mathrm{sinc}\left(\frac{\alpha+n}{8}\right) + \frac{1}{2} \mathrm{sinc}\left(\frac{\alpha-n}{4}\right) + (-1)^n \mathrm{sinc}\left(\frac{n}{2}\right) \right) \cos(2\pi f_c t + 2\pi n f_0 t)}$$

(f)

$$\boxed{S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} \left(\frac{1}{2} \cos\left[\frac{\pi(\alpha-3n)}{8}\right] \mathrm{sinc}\left(\frac{\alpha+n}{8}\right) + \frac{1}{2} \mathrm{sinc}\left(\frac{\alpha-n}{4}\right) + (-1)^n \mathrm{sinc}\left(\frac{n}{2}\right) \right) \left[\delta(f - f_c - n f_0) + \delta(f + f_c + n f_0) \right]}$$

PROBLEMA 3:

(a)



$$X(f) = 3\,\Lambda\left(\frac{f}{1000}\right)$$

Sabemos que

$$\text{sinc}^2(2Wt) \rightleftharpoons \frac{1}{2W}\,\Lambda\left(\frac{f}{2W}\right)$$

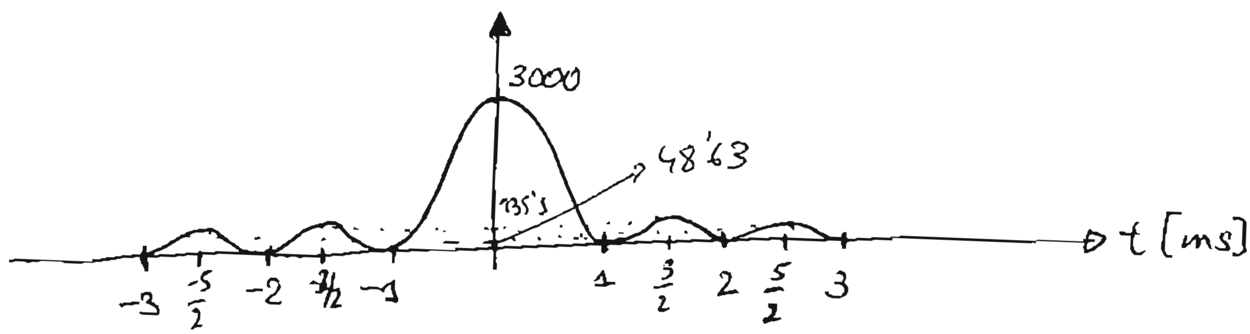
por lo que

$$2W\,\text{sinc}^2(2Wt) \rightleftharpoons \Lambda\left(\frac{f}{2W}\right) \quad \text{y} \quad W\,\text{sinc}^2(Wt) \rightleftharpoons \Lambda\left(\frac{f}{W}\right)$$

$$x(t) = 3000\,\text{sinc}^2(1000\,t) \qquad W = 100 \text{ Hz}$$

tenemos que $x(0) = 3000$

ceros en $t = \frac{n}{1000}$ con $n \neq 0$, es decir cada milisegundo.



$$x\left(\frac{3}{2}10^{-3}\right) = \frac{3000\,\text{sen}^2(\pi\cdot \cancel{1000}\cdot\frac{3}{2}\cdot\cancel{10^{-3}})}{[\pi\cdot\cancel{1000}\cdot\frac{3}{2}\cdot\cancel{10^{-3}}]^2} = \frac{3000\cdot 4}{\pi^2\cdot 9} = \frac{12000}{9\pi^2} = \frac{4000}{3\pi^2} = 135'1$$

$$x\left(\frac{5}{2}10^{-3}\right) = \frac{3000\,\text{sen}^2(\pi\cdot\cancel{1000}\cdot\frac{5}{2}\cdot\cancel{10^{-3}})}{[\pi\cdot\cancel{1000}\cdot\frac{5}{2}\cdot\cancel{10^{-3}}]^2} = \frac{3000\cdot 4}{\pi^2\cdot 25} = \frac{480}{\pi^2} = 48'63$$

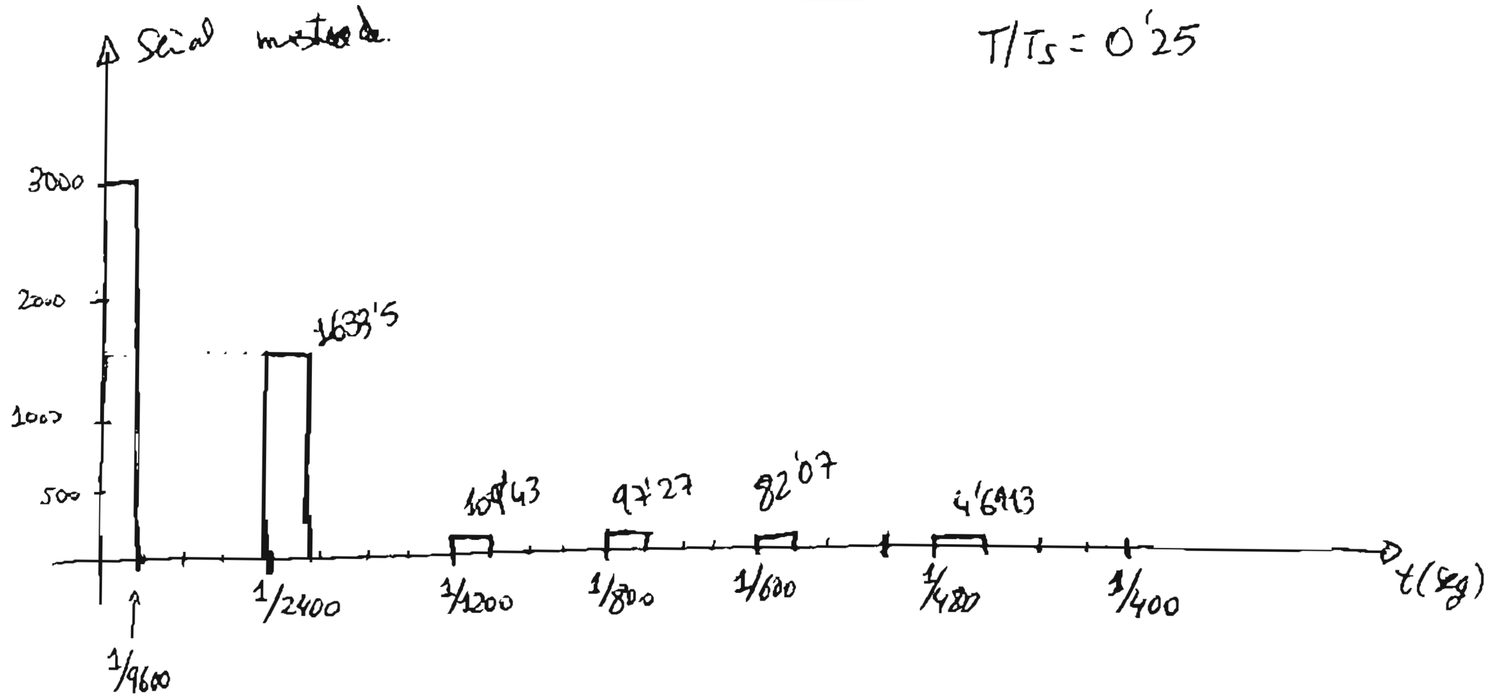
(b) $f_{Nyquist} = 2W = 2 \cdot 1000\,Hz = 2000\,Hz.$

$$f_s = 1'2\, f_{Nyquist} = 1'2 \cdot 2000 = 2400\,Hz$$

(c)

n	nT_s	$x(nT_s)$
0	0	3000
1	$\frac{1}{2400}$	$\frac{3000\, sen^2(\pi \cdot 1000/2400)}{\pi^2 (1000/2400)^2} = \frac{17280\, sen^2(\frac{\pi 5}{12})}{\pi^2} = 1633'5$
2	$\frac{1}{1200}$	$\frac{3000\, sen^2(\pi\, 1000/1200)}{\pi^2 (1000/1200)^2} = \frac{4320\, sen^2(\frac{5\pi}{6})}{\pi^2} = 109'43$
3	$\frac{1}{800}$	$\frac{3000\, sen^2(\pi\, 1000/800)}{\pi^2 (1000/800)^2} = \frac{1920\, sen^2(5\pi/4)}{\pi^2} = 97'27$
4	$\frac{1}{600}$	$\frac{3000\, sen^2(\pi\, 1000/600)}{\pi^2 (1000/600)^2} = \frac{1080\, sen^2(5\pi/3)}{\pi^2} = 82'07$
5	$\frac{1}{480}$	$\frac{3000\, sen^2(\pi\, 1000/480)}{\pi^2 (1000/480)^2} = \frac{3456\, sen^2(\frac{25\pi}{12})}{5\pi^2} = 4'6913$

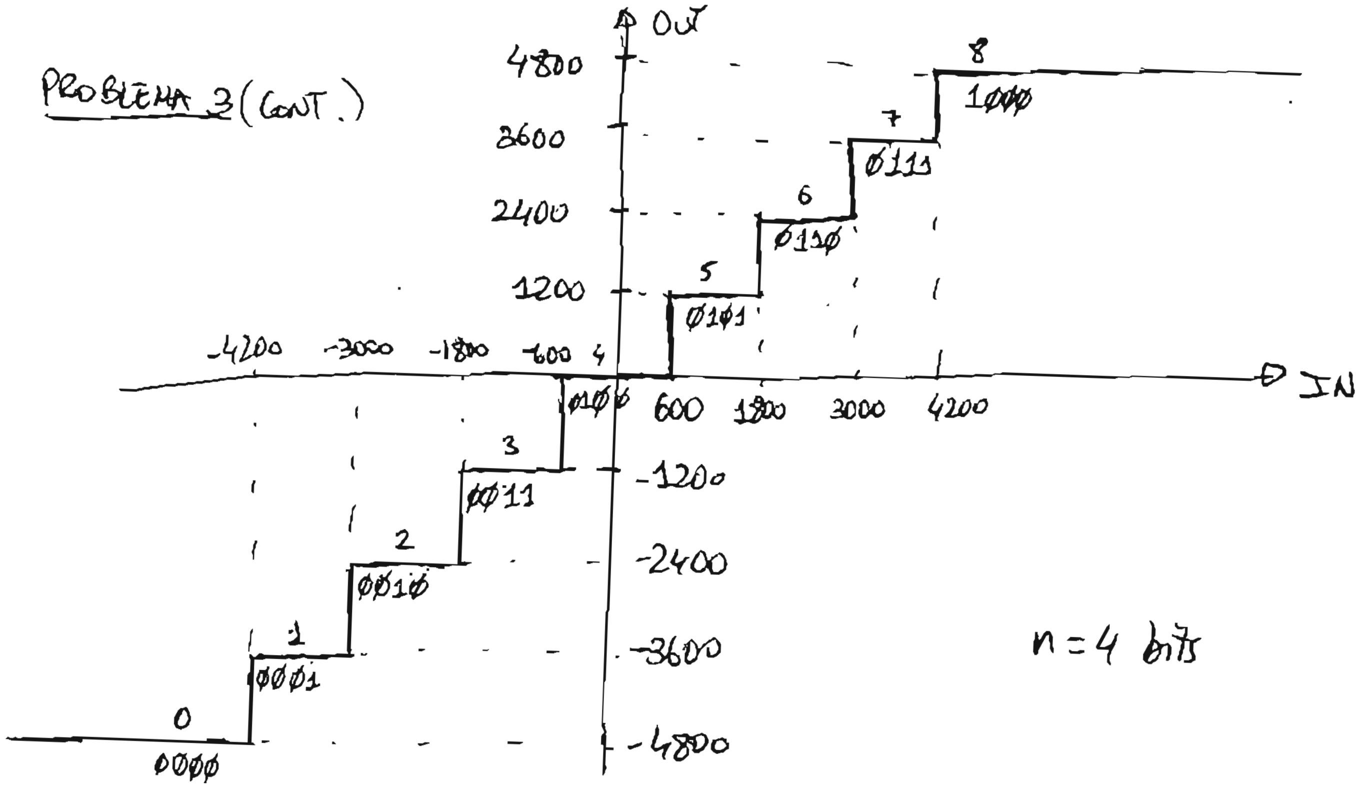
$T/T_s = 0'25$



(d) $\boxed{A_{máx} = 1'8 \cdot \max_t (x(t)) = 1'8 \cdot 3000 = 5400}$ $\quad L = 9$

$$\boxed{\delta = \frac{2A_{max}}{L} = \frac{2 \cdot 5400}{9} = 1200}$$

PROBLEMA 3 (CONT.)



$n=4$ bits

$x(nT_s)$	$\hat{x}(nT_s)$ cuantificado
3000	2400 (6)
1633'5	1200 (5)
109'43	0 (4)
97'27	0 (4)
82'07	0 (4)
4'6913	0 (4)

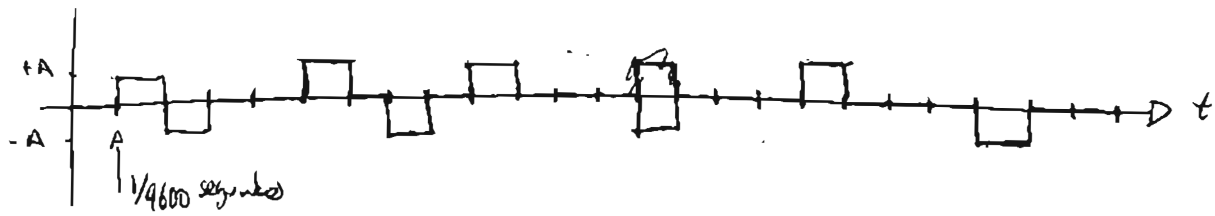
(e) $$E[x^2(nT_s)] \simeq \frac{1}{6}\left[3000^2 + 1633'5^2 + 109'43^2 + 97'27^2 + 82'07^2 + 4'6913^2\right] = 1'949\cdot 10^6$$

$$E[(x(nT_s)-\hat{x}(nT_s))^2] \simeq \frac{1}{6}\left[(3000-2400)^2 + (1633'5-1200)^2 + 109'43^2 + 97'27^2 + 82'07^2 + 4'6913^2\right]$$

$$= 9'602\cdot 10^4$$

$$SQNR = \frac{E[x^2(nT_s)]}{E[(x(nT_s)-\hat{x}(nT_s))^2]} = \frac{1'949\cdot 10^6}{9{,}602\cdot 10^4} = 20'30 \quad (13{,}08\,dB)$$

(f) ∅11∅ ∅1∅1 ∅1∅∅ ∅1∅∅ ∅1∅∅ ∅1∅∅



(g) $\mu = 125$ para valores normalizados:

$$|V_2| = \frac{\ln(1+\mu|V_1|)}{\ln(1+\mu)} = \frac{\ln(1+125|V_1|)}{\ln(126)}$$

$|V_1| = \frac{x(nT_s)}{A_{max}}$ ~~y~~ $|V_2| = \frac{y(nT_s)}{A_{max}}$ ($x(nT_s)$ siempre positivo)

$$\boxed{y(nT_s) = A_{max} \cdot \frac{\ln\left(1 + 125\,\frac{x(nT_s)}{A_{max}}\right)}{\ln(126)} = \frac{5400\ln\left(1+ \frac{5\,x(nT_s)}{216}\right)}{\ln(126)}}$$

$x(nT_s)$	$y(nT_s)$ (comp)	$\hat{y}(nT_s)$ (cuantificada)	[El expansor será el inverso] $\hat{x}(nT_s)$ (exp)
3000	4750'8	4800 (8)	3137,2
1633'5	4085'2	3600 (7)	1042,6
109'43	1409'3	1200 (5)	83,34
97'27	1316'6	1200 (5)	83,34
82'07	1188'7	1200 (5)	83,34
4'6913	115'11	0 (4)	0

Para el expansor:

$$\hat{y}(nT_s) = \frac{5400\ln\left(1+\frac{5\hat{x}(nT_s)}{216}\right)}{\ln(126)}$$ despejamos $\hat{x}(nT_s)$

$$\ln\left(1+\frac{5\hat{x}(nT_s)}{216}\right) = \frac{\ln(126)\,\hat{y}(nT_s)}{5400} \Rightarrow 1+\frac{5\hat{x}(nT_s)}{216} = \exp\left(\frac{\ln(126)\,\hat{y}(nT_s)}{5400}\right)$$

$$\boxed{\hat{x}(nT_s) = \frac{216\left(\exp\left(\frac{\ln(126)\,\hat{y}(nT_s)}{5400}\right)-1\right)}{5}}$$

PROBLEMA 3 (CONT.)

$E[\hat{x}^2(nT_s)] \simeq 1'949 \cdot 10^6$ (ya calculado en apartado (c))

$E[[x(nT_s) - \hat{x}(nT_s)]^2] = \frac{1}{6}\left[(3000-3137,2)^2 + (1638'5 - 1042,6)^2 + (109'43 - 83'34)^2 + \right.$

$\left. + (97'27 - 83'34)^2 + (82'07 - 83'34)^2 + 4'6913^2\right] = 6,149 \cdot 10^4$

$SQNR = \frac{E[x^2(nT_s)]}{E[(x(nT_s) - \hat{x}(nT_s))^2]} = \frac{1'949 \cdot 10^6}{6,149 \cdot 10^4} = 31,70$ (15 dB)

$\frac{SQNR(\text{no unif})}{SQNR(\text{unif})} = \frac{31'70}{20'30} = 1'5616 \Rightarrow$ 56'16% de mejora.

Señal Bipolar:

1000 0111 0101 0101 0101 0100