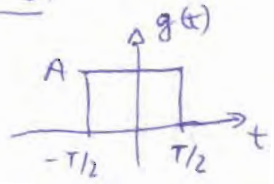
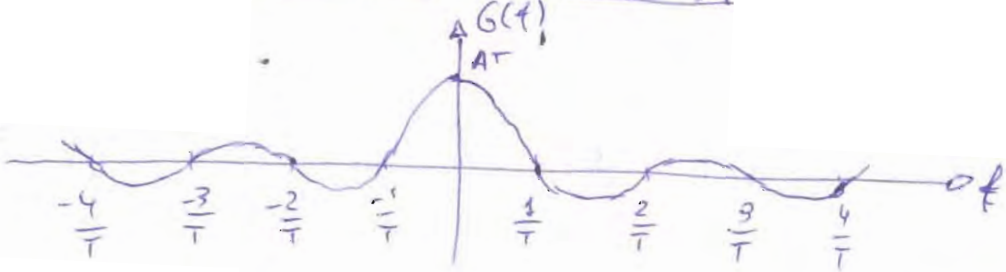
PROBLEMA 1.

a)

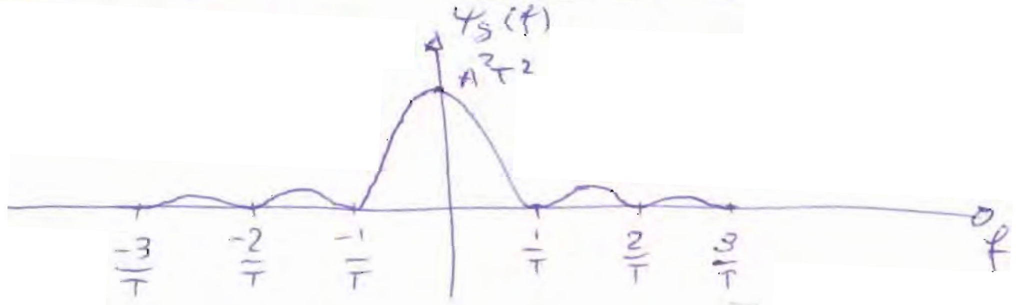


b) T. Inmediata $\boxed{G(f) = A \cdot T \cdot \text{sinc}(fT)}$



c) $\Psi_g(f) = |G(f)|^2 = A^2 T^2 |\text{sinc}^2(fT)| = \frac{A^2 T^2 |\text{sen}^2(\pi f T)|}{|\pi f T|^2} = \frac{A^2 \text{sen}^2(\pi f T)}{\pi^2 f^2}$

$\boxed{\Psi_g(f) = A \frac{1 - \cos(2\pi f T)}{2\pi^2 f^2} = A^2 T^2 \text{sinc}^2(fT)}$ función suave.



d) T. Inmediata considerando $f_c \gg \frac{1}{T}$ (ancho de banda de $g(t)$)

$$g(t)\cos(2\pi f_c t) \overset{T.H.}{\rightleftharpoons} g(t)\,\text{sen}(2\pi f_c t)$$

además sabemos que T. H. de $\hat{x}(t)$ es $-x(t)$ $\Rightarrow$

$$x(t) = g(t)\,\text{sen}(2\pi f_c t) \overset{T.H.}{\rightleftharpoons} -g(t)\cos(2\pi f_c t) = \hat{x}(t)$$

$$\boxed{\hat{x}(t) = -g(t)\,\text{sen}(2\pi f_c t) = -A \cdot \text{sen}(2\pi f_c t)\,\Pi\left(\frac{t}{T}\right)}$$

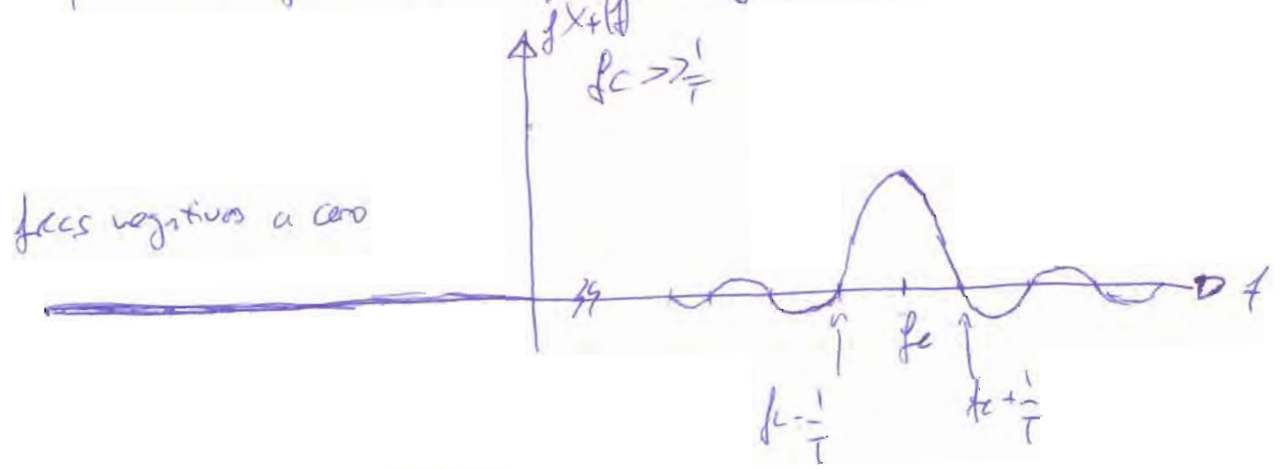
e) $\boxed{x_+(t)} = x(t) + j\,\hat{x}(t) = g(t)\,\text{sen}(2\pi f_c t) - j\,g(t)\cos(2\pi f_c t)$

$= -j\left[\cos(2\pi f_c t) + j\,\text{sen}(2\pi f_c t)\right] g(t) = -j\,g(t)\,e^{j2\pi f_c t}$

$\boxed{= -jA\,\Pi\left(\frac{t}{T}\right)\exp(j2\pi f_c t)}$

$\boxed{X_+(f) = -jA\cdot T\,\text{sinc}(fT) * \delta(f - f_c) = -jA\cdot T\,\text{sinc}((f-f_c)T)}$

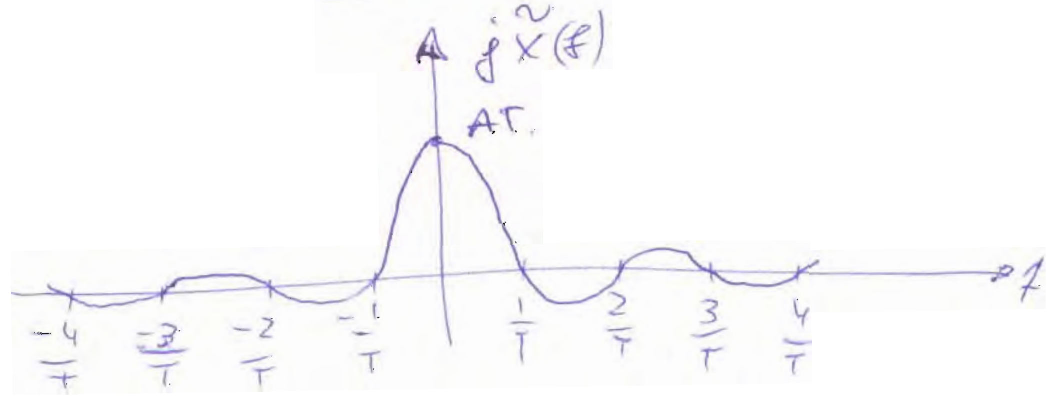
para dibujar cosas reales: $j\cdot X_+(f) = AT\,\text{sinc}((f-f_c)T)$



f) $\boxed{\tilde{x}(t)} = x_+(t)\,e^{-j2\pi f_c t} =$

$= -jA\,\Pi\left(\frac{t}{T}\right)e^{j2\pi f_c t}\cdot e^{-j2\pi f_c t} = \boxed{-jA\,\Pi\left(\frac{t}{T}\right)}$

$\boxed{\tilde{X}(f) = X_+(f+f_c) = -jA\cdot T\cdot\text{sinc}(fT)} \Rightarrow j\tilde{X}(f) = AT\,\text{sinc}(fT) = G(f)$



PROBLEMA 1 (CONT.)

g) $\tilde{x}(t) = x_c(t) + j\, x_s(t) \Rightarrow \begin{cases} x_c(t) = 0 \\ x_s(t) = -A\,\Pi\left(\frac{t}{T}\right) = -g(t) \end{cases}$

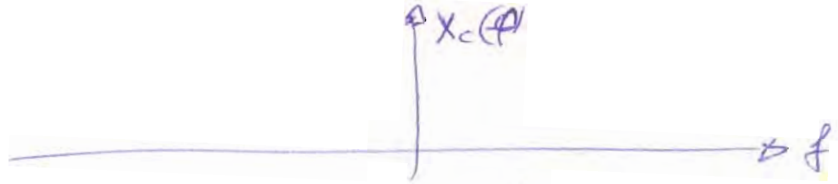
otra forma, usando la forma canonica:

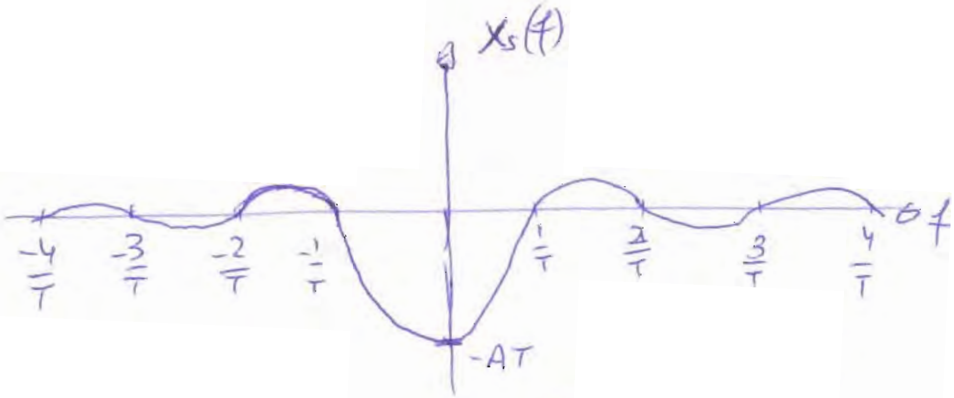
$$x(t) = x_c(t)\cos(2\pi f_c t) - x_s(t)\,\text{sen}(2\pi f_c t) = g(t)\,\text{sen}(2\pi f_c t) = A\,\Pi\left(\frac{t}{T}\right)\text{sen}(2\pi f_c t)$$

e identificando términos: $\boxed{\begin{cases} x_c(t) = 0 \\ x_s(t) = -g(t) = -A\,\Pi\left(\frac{t}{T}\right) \end{cases}}$

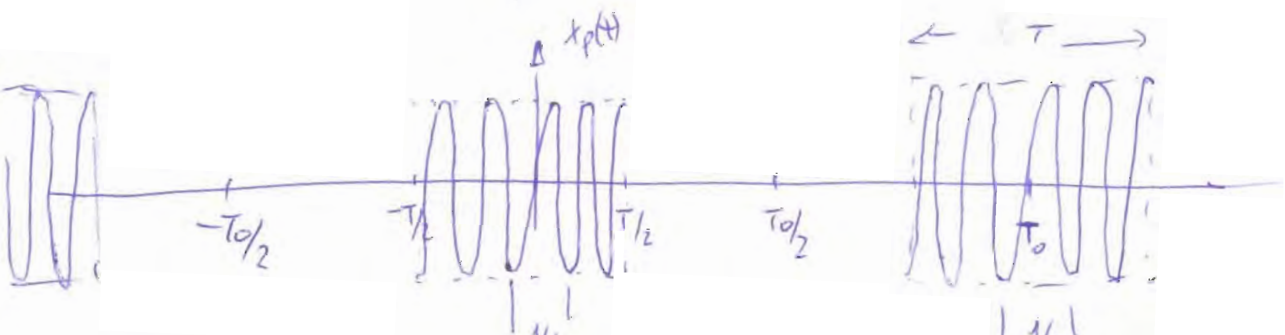
$X_c(f) = 0$

$X_s(f) = -G(f) = -AT\cdot \text{sinc}(fT)$





h)



i) $S_{x_p}(f) = \sum_{n=-\infty}^{\infty} |c_n|^2\, \delta\left(f - \frac{n}{T_0}\right)$ siendo c_n, los coeficientes de la serie de Fourier de $x_p(t)$.

$c_n = \frac{1}{T_0} X\left(\frac{n}{T_0}\right)$ siendo $X(f)$ la T.F. de la función generadora

que es $x(t) = g(t) \sin(2\pi f_c t)$.

$$X(f) = G(f) * \frac{1}{2j}\left[\delta(f - f_c) - \delta(f + f_c)\right]$$

$$= \frac{1}{2j} G(f - f_c) - \frac{1}{2j} G(f + f_c)$$

con $G(f) = AT \operatorname{sinc}(fT)$ entonces.

$$X(f) = \frac{AT}{2j}\left[\operatorname{sinc}[(f - f_c)T] - \operatorname{sinc}[(f + f_c)T]\right]$$

$$c_n = \frac{AT}{2jT_0}\left[\operatorname{sinc}\left(\left(\frac{n}{T_0} - f_c\right)T\right) - \operatorname{sinc}\left(\left(\frac{n}{T_0} + f_c\right)T\right)\right]$$

$$\boxed{S_{x_p}(f) = \sum_{n=-\infty}^{\infty} \frac{A^2T^2}{4T_0^2}\left(\operatorname{sinc}\left[\left(\frac{n}{T_0} - f_c\right)T\right] - \operatorname{sinc}\left(\left(\frac{n}{T_0} + f_c\right)T\right]\right)^2 \delta\left(f - \frac{n}{T_0}\right)}$$

PROBLEMA 2:

a) El desarrollo en serie de una función $g(x)$, en el punto x_0:

$$g(x) = g(x_0) + \frac{x-x_0}{1!} \left.\frac{dg(x)}{dx}\right|_{x=x_0} + \frac{(x-x_0)^2}{2!} \left.\frac{d^2g(x)}{dx}\right|_{x=x_0} + \dots$$

$$= \sum_{n=0}^{\infty} \frac{(x-x_0)^n}{n!} \left.\frac{d^n g(x)}{dx}\right|_{x=x_0}$$

Si el punto es $x_0=0$, entonces

$$g(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} \left.\frac{d^n g(x)}{dx}\right|_{x=0}$$

Luego hay que ir viendo lo que vale la función y sus derivadas en $x=0$

$$g(0) = \operatorname{atan}(0) = 0$$

$$\frac{dg(x)}{dx} = g'(x) = \frac{1}{1+x^2} \text{ (inmediata)} \qquad g'(0) = 1$$

$$\frac{d^2g(x)}{dx^2} = g''(x) = -(1+x^2)^{-2} \cdot 2x = \frac{-2x}{(1+x^2)^2} \qquad g''(0) = 0$$

$$\frac{d^3g(x)}{dx^3} = g'''(x) = \frac{-2(1+x^2)^2 + 2x \cdot 2(1+x^2)2x}{(1+x^2)^4} = \frac{-2(1+x^2)+8x^2}{(1+x^2)^3} =$$

$$= \frac{-2-2x^2+8x^2}{(1+x^2)^3} = \frac{6x^2-2}{(1+x^2)^3} \qquad g'''(0) = -2$$

$$\frac{d^4g(x)}{dx^4} = g^{iv}(x) = \frac{12x(1+x^2)^3 - (6x^2-2)3(1+x^2)^2 2x}{(1+x^2)^6} = \frac{12x(1+x^2) - 6x(6x^2-2)}{(1+x^2)^4}$$

$$= \frac{12x+12x^3-36x^3+12x}{(1+x^2)^4} = \frac{24x-24x^3}{(1+x^2)^4} \qquad g^{iv}(0) = 0$$

$$\boxed{g(x) \simeq \cancel{0} + \frac{x}{1!} \cdot 1 + \cancel{\frac{x^2}{2!} \cdot 0} + \frac{x^3}{3!}(-2) + \cancel{\frac{x^4}{4!} \cdot 0} + O(x^5) = x - \frac{x^3}{3} + O(x^5)}$$

b) $y(t) \simeq x(t) - \frac{x^3(t)}{3}$ con $x(t) = m(t) + c(t)$

$$y(t) \simeq m(t) + c(t) - \frac{1}{3}\left(m(t) + c(t)\right)^3 = m(t) + c(t) - \frac{m^3(t)}{3} - m^2(t)c(t) - m(t)c^2(t) - \frac{c^3(t)}{3}$$

Sabemos que $c(t) = A \cdot \cos(2\pi f_0 t)$

$$c^2(t) = A^2\cos^2(2\pi f_0 t) = \frac{A^2}{2} + \frac{A^2}{2}\cos(4\pi f_0 t)$$

$$c^3(t) = A^3\cos^3(2\pi f_0 t) = \left[\frac{A^3}{2} + \frac{A^3}{2}\cos(4\pi f_0 t)\right]\cos(2\pi f_0 t)$$

$$= \frac{A^3}{2}\cos(2\pi f_0 t) + \frac{A^3}{2}\cos(4\pi f_0 t)\cos(2\pi f_0 t) = \frac{A^3}{2}\cos(2\pi f_0 t) + \frac{A^3}{4}\cos(6\pi f_0 t) + \frac{A^3}{4}\cos(2\pi f_0 t)$$

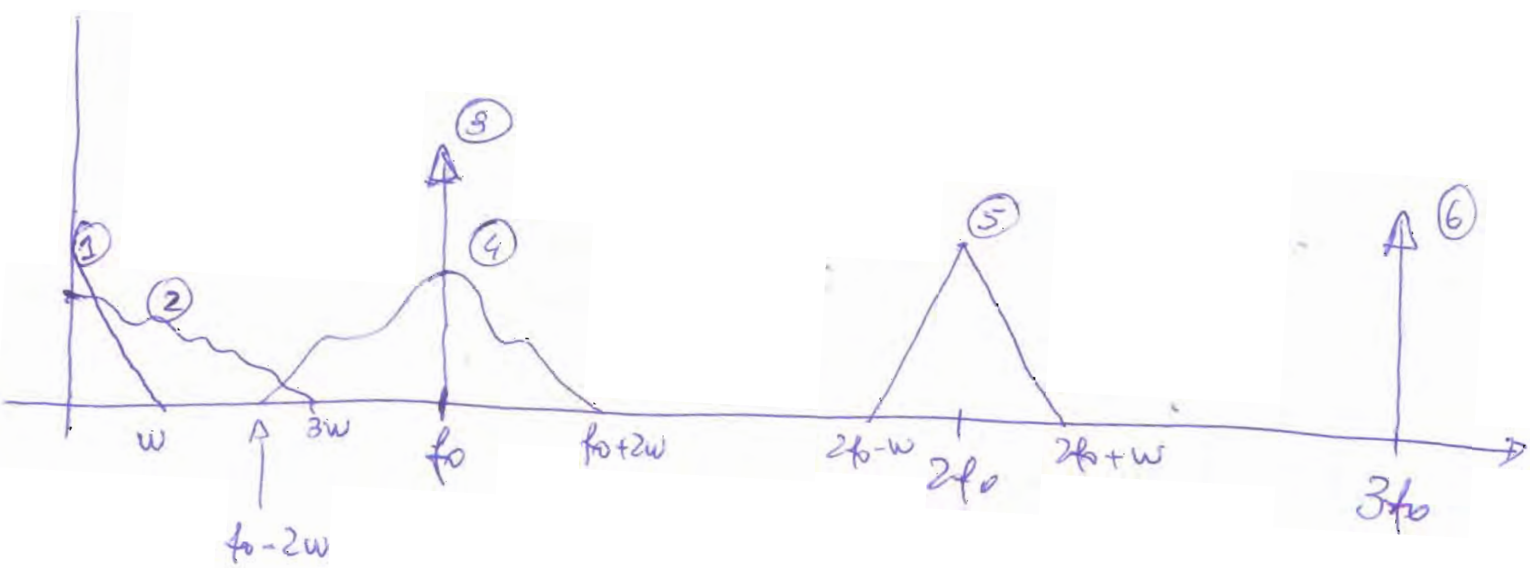
$$= \frac{3A^3}{4}\cos(2\pi f_0 t) + \frac{A^3}{4}\cos(6\pi f_0 t)$$

$$y(t) \simeq m(t) + c(t) - \frac{m^3(t)}{3} - m^2(t)c(t) - \frac{A^2}{2}m(t) - \frac{A^2}{2}m(t)\cos(4\pi f_0 t) - \frac{A^3}{4}\cos(2\pi f_0 t) - \frac{A^3}{12}\cos(6\pi f_0 t)$$

$$\simeq \left(1 - \frac{A^2}{2}\right)m(t) - \frac{m^3(t)}{3} + \left(A - \frac{A^3}{4}\right)\cos(2\pi f_0 t) - A\,m^2(t)\cos(2\pi f_0 t) - \frac{A^2}{2}m(t)\cos(4\pi f_0 t) - \frac{A^3}{12}\cos(6\pi f_0 t)$$

$$\boxed{y(t) \simeq \underbrace{\left(1 - \frac{A^2}{2}\right)m(t)}_{1} - \underbrace{\frac{m^3(t)}{3}}_{2} + \underbrace{A\left(1 - \frac{A^2}{4}\right)\cos(2\pi f_0 t)}_{3} - \underbrace{A\,m^2(t)\cos(2\pi f_0 t)}_{4} - \underbrace{\frac{A^2}{2}m(t)\cos(4\pi f_0 t)}_{5} - \underbrace{\frac{A^3}{12}\cos(6\pi f_0 t)}_{6}}$$

c) Dibujamos frecuencias positivas solamente



PROBLEMA 2 (CONT.)

Explicación términos

① $\left(1-\frac{A^2}{2}\right)m(t)$ moduladora original

② $-\frac{m^3(t)}{3}$ armónico 3 de la moduladora con ancho de banda 3W (distorsión)

③ $A\left(1-\frac{A^2}{4}\right)\cos(2\pi f_0 t)$ portadora a frecuencia f_0.

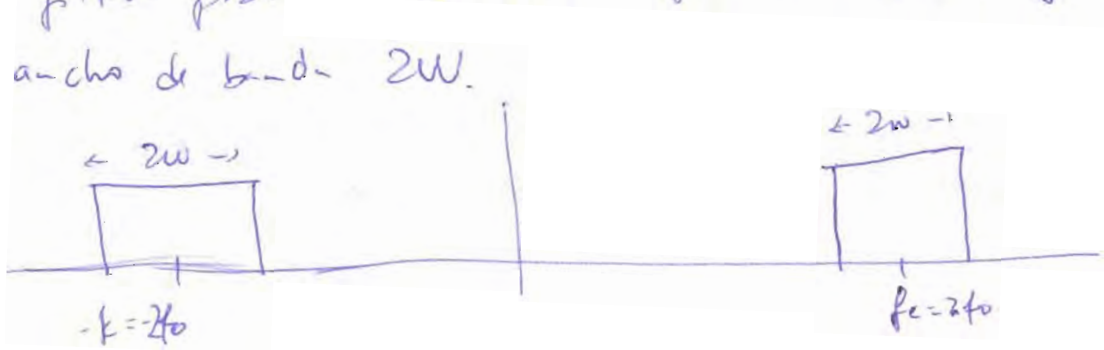
④ $-A\,m^2(t)\cos(2\pi f_0 t)$ señal doble banda lateral con moduladora $m^2(t)$, 2º armónico de $m(t)$ (Distorsión), a frecuencia f_0

⑤ $-\frac{A^2}{2}m(t)\cos(4\pi f_0 t)$ señal doble banda lateral con moduladora $m(t)$ y portadora $2f_0$. (INTERES)

⑥ $-\frac{A^3}{12}\cos(6\pi f_0 t)$ señal portadora a $3f_0$.

d) Según explicamos antes la señal deseada es el término ⑤ entonces $\boxed{s(t) = -\frac{A^2}{2}m(t)\cos(2\pi f_c t)}$, con $\boxed{f_c = 2f_0}$

El filtro paso banda tiene frecuencia central $f_c = 2f_0$ y ancho de banda 2W.



e) $\boxed{s(t) = -\frac{A^2}{2}m(t)\cos(2\pi f_c t)}$

Comprobamos condiciones en frecuencia y nos quedamos con la más estricta para que no haya solapamientos de otros componentes en la banda de $s(t)$

⑤ y ⑥ $3f_0 > 2f_0 + W \Rightarrow f_0 > W$

⑤ y ④ $2f_0 - W > f_0 + 2W \Rightarrow f_0 > 3W$ * La más restrictiva

⑤ y ② $2f_0 - W > 3W \Rightarrow f_0 > 2W$

⑤ y ③ $2f_0 - W > f_0 \Rightarrow f_0 > W$

⑤ y ① $2f_0 - W > W \Rightarrow f_0 > W$

Entonces $\boxed{f_0 > 3W}$

(f) Señal AM con índice μ:

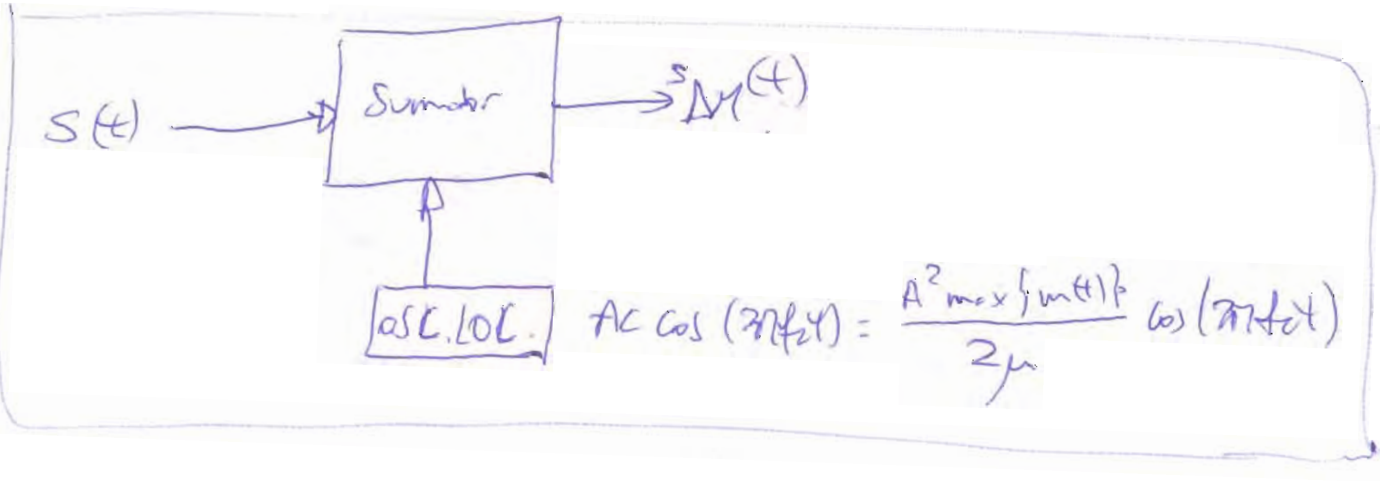
$$s_{AM}(t) = A_c\left[1 + \mu \frac{m(t)}{\max\{m(t)\}}\right]\cos(2\pi f_c t)$$

$$= A_c \cos(2\pi f_c t) + \underline{\frac{A_c\,\mu}{\max\{m(t)\}}\, m(t)\cos(2\pi f_c t)}$$

y tenemos $s(t) = \underline{-\frac{A^2}{2}\, m(t)\cos(2\pi f_c t)}$

igualando constantes $\frac{A^2}{2} = \frac{A_c\,\mu}{\max\{m(t)\}} \Rightarrow A_c = \frac{A^2 \max\{m(t)\}}{2\mu}$

Sumando una portadora con amplitud $A_2 = \frac{A^2 \max\{m(t)\}}{2\mu}$ tenemos AM con índice μ:



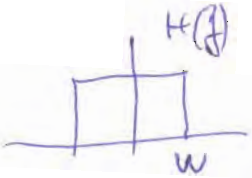
PROBLEMA 3

a) Cuando se transmite "1":

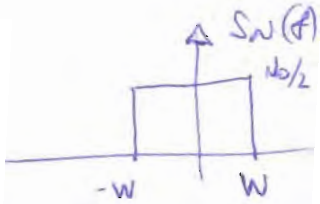
$$y_K = B + n(t_K)$$

donde $n(t_K)$ es una muestra del ruido $n(t)$, que a su vez es la versión filtrada del ruido $w(t)$

$$w(t) \longrightarrow \boxed{\text{Filtro Paso Bajo}} \longrightarrow n(t) \Rightarrow S_N(f) = S_w(f) \cdot |H(f)|^2$$



el filtro paso bajo, entonces:



$n(t_K)$ será una muestra de una variable Gaussiana, con media cero y varianza:

$$\sigma_N^2 = \int_{-W}^{W} N_0/2 \, df = N_0 \cdot W$$

Entonces $y_K = B + n(t_K)$ es una variable Gaussiana con media B y varianza $\sigma_N^2 = N_0 W$:

$$\boxed{f_{Y_K}(y_K/\text{"1"}) = \frac{1}{\sqrt{2\pi N_0 W}} \exp\left(-\frac{(y_K - B)^2}{2 N_0 W}\right)}$$

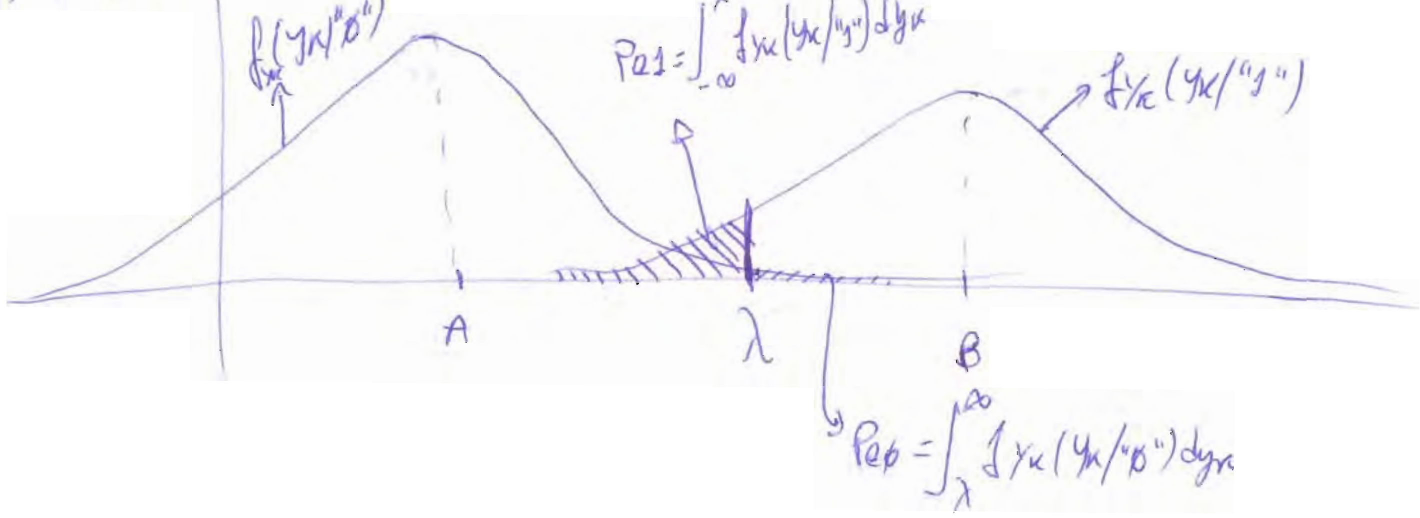
Cuando se transmite "0":

$$y_K = A + n(t_K)$$

Por las razones dichas, y_K es una variable Gaussiana con media A y varianza $\sigma_N^2 = N_0 W$

$$f_{Y_K}(y_K/\text{"0"}) = \frac{1}{\sqrt{2\pi N_0 W}} \exp\left(-\frac{(y_K - A)^2}{2 N_0 W}\right)$$

b) $A<\lambda<B$



c) $BER = P_e = P_1\, P_{e1} + P_\emptyset \cdot P_{e\emptyset}$ (teorema probabilidad total)

Se dice que $P_1 = p$, entonces $P_\emptyset = 1-p$

$BER = p\cdot P_{e1} + (1-p)\,P_{e\emptyset}$.

$$P_{e\emptyset} = \int_{\lambda}^{\infty} \frac{1}{\sqrt{2\pi N_0 W}} \exp\left(-\frac{(y_K-A)^2}{2N_0W}\right) dy_K \quad \left|\quad \begin{array}{l} \dfrac{y_K-A}{\sqrt{2N_0W}} = z \\ \dfrac{dy_K}{\sqrt{2N_0W}} = dz \end{array} \quad\right|\quad \begin{array}{l} y_K=\infty \Rightarrow z=\infty \\ y_K=\lambda \Rightarrow z=\dfrac{\lambda-A}{\sqrt{2N_0W}} \end{array}$$

$$P_{e\emptyset} = \frac{1}{\sqrt{\pi}} \int_{\frac{\lambda-A}{\sqrt{2N_0W}}}^{\infty} e^{-z^2}\,dz = \frac{1}{2}\,\mathrm{erfc}\left(\frac{\lambda-A}{\sqrt{2N_0W}}\right)$$

$$P_{e1} = \int_{-\infty}^{\lambda} \frac{1}{\sqrt{2\pi N_0 W}} \exp\left(-\frac{(y_K-B)^2}{2N_0W}\right) dy_K \quad \left|\quad \begin{array}{l} \dfrac{y_K-B}{\sqrt{2N_0W}} = -z \\ \dfrac{dy_K}{\sqrt{2N_0W}} = -dz \end{array} \quad\right|\quad \begin{array}{ll} y_K=\lambda & z=\dfrac{B-\lambda}{\sqrt{2N_0W}} \\ y_K=-\infty & z=\infty \end{array}$$

$$= -\frac{1}{\sqrt{\pi}} \int_{\infty}^{\frac{B-\lambda}{\sqrt{2N_0W}}} \exp(-z^2)\,dz = \frac{1}{\sqrt{\pi}} \int_{\frac{B-\lambda}{\sqrt{2N_0W}}}^{\infty} \exp(-z^2)\,dz = \frac{1}{2}\,\mathrm{erfc}\left(\frac{B-\lambda}{\sqrt{2N_0W}}\right)$$

$$\boxed{BER = p/2\ \mathrm{erfc}\left(\frac{B-\lambda}{\sqrt{2N_0W}}\right) + \frac{1-p}{2}\,\mathrm{erfc}\left(\frac{\lambda-A}{\sqrt{2N_0W}}\right)}$$

PROBLEMA 3 (CONT.)

d) Minimizamos $BER(\lambda)$ con respecto a λ:

$$\frac{d\,BER(\lambda)}{d\lambda} = \frac{p}{2}\left[+\frac{2}{\sqrt{\pi}}\exp\left(-\frac{(B-\lambda)^2}{2N_0W}\right)\cdot\left(+\frac{1}{\sqrt{2N_0W}}\right)\right] + \frac{1-p}{2}\left[-\frac{2}{\sqrt{\pi}}\exp\left(-\frac{(\lambda-A)^2}{2N_0W}\right)\cdot\left(\frac{1}{\sqrt{2N_0W}}\right)\right]$$

$$= \frac{p}{\sqrt{\pi\cdot 2N_0W}}\exp\left(-\frac{(B-\lambda)^2}{2N_0W}\right) - \frac{1-p}{\sqrt{\pi\cdot 2N_0W}}\exp\left(-\frac{(\lambda-A)^2}{2N_0W}\right) = 0$$

$$p\exp\left(-\frac{(B-\lambda)^2}{2N_0W}\right) = (1-p)\exp\left(-\frac{(\lambda-A)^2}{2N_0W}\right)$$

$$\frac{p}{1-p} = \exp\left(\frac{(B-\lambda)^2-(\lambda-A)^2}{2N_0W}\right)$$

$$\ln\frac{p}{1-p} = \frac{B^2+\cancel{\lambda^2}-2B\lambda-\cancel{\lambda^2}-A^2+2A\lambda}{2N_0W} = \frac{B^2-A^2}{2N_0W} - \lambda\frac{B-A}{N_0W}$$

$$\lambda\cdot\frac{B-A}{N_0W} = \ln\frac{1-p}{p} + \frac{B^2-A^2}{2N_0W} \qquad \Big| \qquad B^2-A^2 = (B-A)(B+A)$$

$$\boxed{\lambda^* = \frac{N_0W}{B-A}\ln\frac{1-p}{p} + \frac{B+A}{2}}$$

e) Si $p=0.5$, $1-p=0.5$ y $p=1-p \Rightarrow \ln\frac{1-p}{p} = \ln 1 = 0$

Entonces el umbral óptimo $\lambda^* = \frac{B+A}{2}$ (la media de niveles, como era de esperar)

$$\left.\begin{aligned} B-\lambda &= B-\frac{B+A}{2} = \frac{2B-B-A}{2} = \frac{B-A}{2} \\ \lambda-A &= \frac{B+A}{2}-A = \frac{B+A-2A}{2} = \frac{B-A}{2} \end{aligned}\right\} B-\lambda = \lambda-A$$

$$\boxed{P_e^* = \frac{1}{2}\,\mathrm{erfc}\left(\frac{B-A}{2\sqrt{2N_0W}}\right)}$$