Ingeniería de las Ondas I
E.T.S.I. Telecomunicación
Universidad de Valladolid

TEORÍA ONDULATORIA DEL SONIDO: ecuaciones

$$\frac{d}{dt}\iiint_V \rho dV = -\iint_S \rho \mathbf{v}\cdot\mathbf{n}dS \quad (1)$$

$$\iint_S \mathbf{A}\cdot\mathbf{n}dS = \iiint_V \nabla\cdot\mathbf{A}dV \quad (2)$$

$$\iiint_V \left[\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v})\right]dV = 0 \quad (3)$$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0 \quad (4)$$

$$\frac{d}{dt}\iiint_{V*} \rho\mathbf{v}dV = \iint_{S*}\mathbf{f}_s dS + \iiint_{V*}\mathbf{f}_B dV \quad (5)$$

$$\mathbf{f}_s = -\mathbf{n}p \quad (6)$$

$$\iint_{S*}\mathbf{f}_s dA = -\iint_{S*} p\mathbf{n} = -\iiint_{V*}\nabla p dV \quad (7)$$

$$\frac{d}{dt}\mathbf{v}\left(x_p(t), y_p(t), z_p(t), t\right) = \frac{\partial\mathbf{v}}{\partial t} + \frac{\partial\mathbf{v}}{\partial x_p}\frac{dx_p}{dt} + \frac{\partial\mathbf{v}}{\partial y_p}\frac{dy_p}{dt} + \frac{\partial\mathbf{v}}{\partial z_p}\frac{dz_p}{dt} = \quad (8)$$

$$= \frac{\partial\mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = \frac{D\mathbf{v}}{Dt}$$

$$\frac{d}{dt}\iiint_{V*}\rho\mathbf{v}dV = \iiint_{V*}\rho\frac{D\mathbf{v}}{Dt}dV \quad (9)$$

$$\iiint_{V*}\left(\rho\frac{D\mathbf{v}}{Dt} + \nabla p\right)dV = 0 \quad (10)$$

$$\rho\frac{D\mathbf{v}}{Dt} = -\nabla p \quad (11)$$

$$p = p(\rho) \quad (12)$$

$$p = K\rho^\gamma \quad (13)$$

$$\frac{Ds}{Dt} = 0 \quad (14)$$

$$Tds = du + pd\rho^{-1} \quad (15)$$

$$p = p(\rho, s) \quad (16)$$

$$\rho T\frac{Ds}{Dt} = \kappa\nabla^2 T \quad (17)$$

$$p = p_0 + p' \quad \rho = \rho_0 + \rho' \quad \mathbf{v} = \mathbf{v}_0 + \mathbf{v}' \quad (18)$$

$$\frac{\partial}{\partial t}(\rho_0 + \rho') + \nabla\cdot\left[(\rho_0 + \rho')\mathbf{v}'\right] = 0 \quad (19)$$

$$(\rho_0 + \rho')\left(\frac{\partial}{\partial t} + \mathbf{v}'\cdot\nabla\right)\mathbf{v}' = -\nabla(p_0 + p') \quad (20)$$

$$p_0 + p' = p(\rho_0 + \rho', s_0) \quad (21)$$

$$p' = \left(\frac{\partial p}{\partial\rho}\right)_0\rho' + \frac{1}{2}\left(\frac{\partial^2 p}{\partial\rho^2}\right)_0(\rho')^2 + \ldots \quad (22)$$

$$\frac{\partial\rho'}{\partial t} + \rho_0\nabla\cdot\mathbf{v}' = 0 \quad (23)$$

$$\rho_0\frac{\partial\mathbf{v}'}{\partial t} = -\nabla p' \quad (24)$$

$$p' = c^2\rho' \quad c^2 = \left(\frac{\partial p}{\partial\rho}\right)_0 \quad (25)$$

$$T' = \left(\frac{\beta T}{\rho c_p}\right)_0 p' \quad (26)$$

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0 \quad (27)$$

$$\frac{\partial}{\partial t}(\nabla\times\mathbf{v}) = 0 \quad (28)$$

$$\mathbf{v} = \nabla\Phi \quad p = -\rho_0\frac{\partial\Phi}{\partial t} \quad (29)$$

$$\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = 0 \quad (30)$$

$$\frac{\partial p}{\partial t} + \rho_0 c^2\frac{\partial v}{\partial s} = 0 \quad \rho_0\frac{\partial v}{\partial t} = -\frac{\partial p}{\partial s} \quad (31)$$

$$\frac{\partial^2 p}{\partial s^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0 \quad (32)$$

$$\left(\frac{\partial p}{\partial s} - \frac{1}{c}\frac{\partial p}{\partial t}\right)\left(\frac{\partial p}{\partial s} + \frac{1}{c}\frac{\partial p}{\partial t}\right)p = 0 \quad (33)$$

$$-\frac{4}{c^2}\frac{\partial}{\partial\xi}\frac{\partial}{\partial\eta}p = 0 \quad (34)$$

$$p = f\left(t - c^{-1}s\right) + g\left(t + c^{-1}s\right) \quad (35)$$

$$\rho c\left(\frac{\partial}{\partial t} \pm c\frac{\partial}{\partial s}\right)v = \mp\left(\frac{\partial}{\partial t} \pm c\frac{\partial}{\partial s}\right)p \quad (36)$$

$$\frac{\partial}{\partial \eta}(\rho c v + p) = 0 \quad \frac{\partial}{\partial \xi}(\rho c v - p) = 0 \tag{37}$$

$$v = (\rho c)^{-1}\left[f\left(t - c^{-1}s\right) - g\left(t + c^{-1}s\right)\right] \tag{38}$$

$$f\left(t_2 - c^{-1}s\right) = f\left(t_1 - c^{-1}\left[s - (t_2 - t_1)c\right]\right) \tag{39}$$

$$p = f\left(t - c^{-1}\mathbf{n}\cdot\mathbf{x}\right) \tag{40}$$

$$\mathbf{v} = \frac{\mathbf{n}}{\rho c}p \quad \rho' = \frac{p}{c^2} \quad T' = \left(\frac{T\beta}{\rho c_p}\right)_0 p \tag{41}$$

$$p = p_{pk}\cos(wt - \phi) = p_{pk}\sin(wt - \phi') = \mathrm{Re}\left[\hat{p}e^{-iwt}\right] \tag{42}$$

$$\phi' = \phi - \frac{\pi}{2} \quad \hat{p} = p_{pk}e^{i\phi} \tag{43}$$

$$\sin\left(\alpha + \frac{\pi}{2}\right) = \cos\alpha \quad e^{i\alpha} = \cos\alpha + i\sin\alpha \tag{44}$$

$$f = \frac{w}{2\pi} \tag{45}$$

$$\left(p^2\right)_{av} = \frac{1}{T}\int_{t_0}^{t_0+T} p^2 dt = p_{rms}^2 \tag{46}$$

$$\cos^2\alpha = \frac{1}{2} + \frac{1}{2}\cos 2\alpha \tag{47}$$

$$\left(p^2\right)_{av} = \frac{1}{2}p_{pk}^2 = \frac{1}{2}\left|\hat{p}\right|^2 \tag{48}$$

$$X = \mathrm{Re}\,\hat{X}e^{-iwt} \quad Y = \mathrm{Re}\,\hat{Y}e^{-iwt} \tag{49}$$

$$(XY)_{av} = \frac{1}{2}\mathrm{Re}\,\hat{X}\hat{Y}^* \tag{50}$$

$$\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha - \beta) + \frac{1}{2}\cos(\alpha + \beta) \tag{51}$$

$$\left|\hat{X}\right|\cdot\left|\hat{Y}\right|\cos\left(\phi_y - \phi_x\right) = \mathrm{Re}\left(\left|\hat{X}\right|\cdot\left|\hat{Y}\right|e^{\pm i\left(\phi_y - \phi_x\right)}\right) \tag{52}$$

$$\mathrm{Re}\left[\left(-iw\hat{\rho} + \rho_0\nabla\cdot\hat{\mathbf{v}}\right)e^{-iwt}\right] = 0 \tag{53}$$

$$-iw\hat{p} + \rho c^2\nabla\cdot\hat{\mathbf{v}} = 0 \quad -iw\rho\hat{\mathbf{v}} = -\nabla\hat{p} \tag{54}$$

$$\nabla^2\hat{p} + k^2\hat{p} = 0 \tag{55}$$

$$f\left(t - c^{-1}s\right) = p = p_{pk}\cos\left(w\left(t - c^{-1}s\right) - \phi_0\right) = p_{pk}\cos\left(wt - ks - \phi_0\right) = \tag{56}$$

$$= p_{pk}\cos\left(wt - \mathbf{k}\cdot\mathbf{x} - \phi_0\right) = \mathrm{Re}\left[p_{pk}e^{i\phi_0}e^{i\mathbf{k}\cdot\mathbf{x}}e^{-iwt}\right]$$

$$\mathbf{k} = \frac{w}{c}\mathbf{n} = k\mathbf{n} \tag{57}$$

$$\hat{p}(\mathbf{x}) = p_{pk}e^{i\phi_0}e^{i\mathbf{k}\cdot\mathbf{x}} \tag{58}$$

$$\lambda f = c \tag{59}$$

$$c^2 = \frac{\partial p}{\partial \rho} = \frac{\partial}{\partial \rho}\left(K\rho^\gamma\right) = \gamma K\rho^{\gamma-1} = \frac{\gamma p}{\rho} \tag{60}$$

$$p = \rho RT \tag{61}$$

$$c = (\gamma RT)^{1/2} \tag{62}$$

$$R = \frac{R_0}{M} \tag{63}$$

$$c = 331 + 0.6T_c \tag{64}$$

$$c_{wet} = [1 + 0.16h]c_{dry} \tag{65}$$

$$\frac{T'}{T_0} = \frac{\gamma - 1}{\gamma}\frac{p'}{p_0} \quad p = p_0, T = T_0 \tag{66}$$

$$c = \left(\frac{K_s}{\rho}\right)^{1/2} \tag{67}$$

$$K_s = \rho\frac{\partial}{\partial \rho}p(\rho, s) \tag{68}$$

$$\frac{K_s - K_T}{K_s} = \frac{T\beta^2 K_T}{\rho c_p} = \frac{T\beta^2 c^2}{\gamma c_p} = \frac{\gamma - 1}{\gamma} \tag{69}$$

$$c = 1447 + 4.0\Delta T + \left(1.6\times10^{-6}\right)p \tag{70}$$

$$c = 1490 + 3.6\Delta T + \left(1.6\times10^{-6}\right)p + 1.3\Delta S \tag{71}$$

$$\rho = 999.7 + 0.048\times10^{-5}p - 0.088\Delta T - 0.007(\Delta T)^2 \tag{72}$$

$$\beta = \left(8.8 + 0.022\times10^{-5}p + 1.4\Delta T\right)\times10^{-5} \tag{73}$$

$$c_p = 4192 - 0.40\times10^{-5}p - 1.6\Delta T \tag{74}$$

$$K_T = \left(20.9 + 0.0058\times10^{-5}p + 0.10\Delta T\right)\times10^8 \tag{75}$$

$$\frac{\beta T}{\rho c_p} = \left(6.0\times10^{-9}\right)\left(1 + \frac{\Delta T}{6} + 0.0024\times10^{-5}p\right) \tag{76}$$

$$\frac{\gamma - 1}{\gamma} = 0.0011\left(1 + \frac{\Delta T}{6} + 0.0024\times10^{-5}p\right)^2 \tag{77}$$

$$\rho = 1027 + 0.043\times10^{-5}p - 0.16\Delta T - 0.004(\Delta T)^2 + 0.75\Delta S \tag{78}$$

$$\beta = \left(16.3 + 0.019\times10^{-5}p + 0.81\Delta T + 0.2\Delta S\right)\times10^{-5} \tag{79}$$

$$c_p = 3988 - 0.23\times10^{-5}p + 0.54\Delta T - 5.4\Delta S \tag{80}$$

$$K_T = \left(22.6 + 0.0062\times10^{-5}p + 0.10\Delta T + 0.051\Delta S\right)\times10^8 \tag{81}$$

$$\frac{\beta T}{\rho c_p} = \left(1.1\times10^{-8}\right)\left(1 + \frac{\Delta T}{20} + 0.0012\times10^{-5}p + 0.012\Delta S\right) \tag{82}$$

$$\frac{\gamma - 1}{\gamma} = 0.0041\left(1 + \frac{\Delta T}{20} + 0.0012\times10^{-5}p + 0.012\Delta S\right)^2 \tag{83}$$

$$\frac{\partial^2\rho'}{\partial t^2} - \nabla^2 p = 0 \tag{84}$$

$$\rho_0 c_p\frac{\partial}{\partial t}\left(\rho' - \frac{p}{c^2}\right) = \kappa\nabla^2\left(\rho' - \frac{p}{c_T^2}\right) \tag{85}$$

$$\frac{\partial}{\partial t}\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)p = \left(\frac{\kappa}{\rho c_p}\right)\nabla^2\left(\nabla^2 - \frac{1}{c_T^2}\frac{\partial^2}{\partial t^2}\right)p \tag{86}$$

$$p = \mathrm{Re}\,Ae^{-iwt}e^{iks} \tag{87}$$

$$\frac{k^2 - \left(w/c\right)^2}{k^2 - \left(w/c_T\right)^2} = \frac{\kappa}{\rho c_p}\frac{k^2}{iw} \tag{88}$$

$$w_{TC} = \frac{\rho c_p c^2}{\kappa} = 2\pi f_{TC} \tag{89}$$

$$\rho c_p c^2 = \frac{\gamma^2}{\gamma-1} R p_0 = 1.4\times10^8\ {}^{W}\!/_{(m\cdot s\cdot K)}$$ presion de 1 atm.

Se llega por medio de: (90)

$c^2 = \frac{\gamma p_0}{\rho_0}$ (Hipoteis de Laplace adiabatica, gas ideal)

$c_p = \frac{\gamma}{\gamma-1} R$ (gas ideal)

$$\mathbf{v}\cdot\left(\rho_0 \frac{\partial \mathbf{v}}{\partial t}\right) = -\mathbf{v}\cdot\nabla p = -\nabla\cdot(\mathbf{v}p) + p\nabla\cdot\mathbf{v} = = -\nabla\cdot(p\mathbf{v}) - p\rho_0^{-1}\frac{\partial \rho'}{\partial t} \quad (91)$$

$$\frac{\partial w}{\partial t} + \nabla\cdot\mathbf{I} = 0 \quad (92)$$

$$w = \frac{1}{2}\rho_0 v^2 + \frac{1}{2}\frac{p^2}{\rho_0 c^2} \quad \mathbf{I} = p\mathbf{v} \quad (93)$$

$$\frac{d}{dt}\iiint_v w dV + \iint_S \mathbf{I}\cdot\mathbf{n} dA = 0 \quad (94)$$

$$\frac{\partial E}{\partial t} + \nabla\cdot(E\mathbf{v} + \rho\mathbf{v}) = 0 \quad (95)$$

$$\frac{d}{dt}\iiint_v E dV + \iint_S E\mathbf{v}\cdot\mathbf{n} dA + \iint \rho\mathbf{v}\cdot\mathbf{n} dA = 0 \quad (96)$$

$$E = \tfrac{1}{2}\rho v^2 + \rho U_p(\rho, s) \quad U_p = \int_{1/\rho}^{1/\rho_0} p d\frac{1}{\rho} \quad (97)$$

$$\rho U_p \approx \frac{p_0}{\rho_0}(\rho-\rho_0) + \frac{1}{2}\frac{c^2}{\rho_0}(\rho-\rho_0)^2 \quad (98)$$

$$E \approx \frac{1}{2}\rho_0 v^2 + \left[\frac{p_0}{\rho_0}(\rho-\rho_0)\right] + \frac{1}{2}\frac{(p-p_0)^2}{\rho_0 c^2} \quad (99)$$

$$(E+p)\mathbf{v} \approx \left[\frac{p_0}{\rho_0}\rho\mathbf{v}\right] + (p-p_0)\mathbf{v} \quad (100)$$

$$\frac{\partial}{\partial t}\left[\frac{1}{2}\rho_0 v^2 + \frac{1}{2}\frac{(p-p_0)^2}{\rho_0 c^2}\right] + \nabla\cdot\left[(p-p_0)\mathbf{v}\right] = 0 \quad (101)$$

$$\frac{1}{2}\rho v^2 = \frac{1}{2}\frac{p^2}{\rho c^2} = \frac{w}{2} \qquad \mathbf{I} = \frac{\mathbf{n}p^2}{\rho c} = c\mathbf{n}w \quad (102)$$

$$w_{av} = \frac{1}{4}\rho\hat{\mathbf{v}}\hat{\mathbf{v}}^* + \frac{1}{4}\frac{|\hat{p}|^2}{\rho c^2} \quad (103)$$

$$\mathbf{I}_{av} = \mathrm{Re}\frac{1}{2}\hat{p}^*\hat{\mathbf{v}} \quad (104)$$

$$\nabla\cdot\mathbf{I}_{av} = 0 \quad (105)$$

$$\iint_S \mathbf{I}_{av}\cdot\mathbf{n} dS = 0 \quad (106)$$

$$P_{av} = \iint_S \mathbf{I}_{av}\cdot\mathbf{n}_{out} dS \quad (107)$$

$$\iint_S \mathbf{I}_{av}\cdot\mathbf{n} dS = \iint_{S_2}\mathbf{I}_{av}\cdot\mathbf{n}_{out} dS_2 - \iint_{S_1}\mathbf{I}_{av}\cdot\mathbf{n}_{out} dS_1 = 0 \quad (108)$$

$$P_{av} = \sum_i P_{av,i} = \sum_i \iint_{S_i}\mathbf{I}_{av}\cdot\mathbf{n}_{out} dS_i \quad (109)$$

$$I_{r,av} = \frac{P_{av}}{4\pi r^2} \quad (110)$$

$$\frac{\partial}{\partial x_i} p(r,t) = \frac{\partial p}{\partial r}\frac{\partial r}{\partial x_i} = \frac{x_i}{r}\frac{\partial p}{\partial r} \quad (111)$$

$$\frac{\partial^2}{\partial x_i^2} p(r,t) = \frac{1}{r}\frac{\partial p}{\partial r} + \frac{x_i^2}{r}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial p}{\partial r}\right) \quad (112)$$

$$\nabla^2 p(r,t) = \frac{3}{r}\frac{\partial p}{\partial r} + r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial p}{\partial r}\right) = \frac{\partial^2 p}{\partial r^2} + \frac{2}{r}\frac{\partial p}{\partial r} = \frac{1}{r}\frac{\partial^2}{\partial r^2} rp \quad (113)$$

$$\frac{1}{r}\frac{\partial^2}{\partial r^2} rp - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0 \quad (114)$$

$$p(r,t) = r^{-1} f\left(t - c^{-1}r\right) + r^{-1} g\left(t + c^{-1}r\right) \quad (115)$$

$$v_r = -\frac{1}{\rho}\frac{\partial}{\partial r}\frac{F\left(t-c^{-1}r\right)}{r} \quad p = \frac{\partial}{\partial t}\frac{F\left(t-c^{-1}r\right)}{r} \quad (116)$$

$$v_r = \frac{p}{\rho c} + \frac{F\left(t-c^{-1}r\right)}{\rho r^2} \quad (117)$$

$$I_r = \frac{p^2}{\rho c} + \frac{\partial}{\partial t}\left[\frac{F^2\left(t-c^{-1}r\right)}{2\rho r^3}\right] \quad (118)$$

$$I_{r,av} = \frac{\left(p^2\right)_{av}}{\rho c} \quad (119)$$

$$p = |A| r^{-1}\cos\left(wt - kr - \phi_A\right) = r^{-1}\mathrm{Re}\left[Ae^{-iwt}e^{ikr}\right] \quad (120)$$

$$\rho c v_r = |A| r^{-1}\cos\left(wt-kr-\phi_A\right) + |A| k^{-1} r^{-2}\sin\left(wt-kr-\phi_A\right) = = r^{-1}\mathrm{Re}\left[\left(1+\frac{i}{kr}\right)Ae^{-iwt}e^{ikr}\right] \quad (121)$$

$$I_{r,av} = \frac{|A|^2}{2\rho c r^2} \quad (122)$$

$$\frac{1}{2}\frac{\left(p^2\right)_{av}}{\rho c^2} = \frac{|A|^2}{4\rho c^2 r^2} = \frac{I_{r,av}}{2c} \quad (123)$$

$$\tfrac{1}{2}\rho\left(v_r^2\right)_{av} = \frac{|A|^2}{4\rho c^2 r^2}\left[1 + \frac{1}{(kr)^2}\right] \quad (124)$$

$$p \approx \frac{1}{r} f\left(t - c^{-1}r, \theta, \phi\right) \quad \mathbf{v} = \frac{p\mathbf{e}_r}{\rho c} \quad (125)$$

$$\mathbf{I}_{av} = \frac{J(\theta,\phi)}{r^2}\mathbf{e}_r \quad J(\theta,\phi) = \frac{1}{\rho c T}\int_{t_0}^{t_0+T} f^2(t,\theta,\phi) dt \quad (126)$$

$$P_{av} = \iint_S \mathbf{I}_{av}\mathbf{n}_{out} dS = \int_0^{2\pi}\int_0^{\pi} J(\theta,\phi)\sin\theta d\theta d\phi \quad (127)$$