Noise estimation and diffusion signal reconstruction: 
From cradle to parallel imaging

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Abstract

Noise is known to be one of the main sources of quality deterioration in magnetic resonance (MR) data. Not only visual inspection but also processing techniques such as segmentation, registration or tensor estimation in diffusion tensor MRI (DT-MRI) will be affected or biased due to the presence of noise. In High Angular Resolution Diffusion Imaging (HARDI) for instance, to achieve a speedup in the acquisition time the temporal averaging is reduced; as a consequence, the strength of the noise is increased proportionally to the square root of the speedup. Noise in the k-space in MR data is usually assumed to be a zero-mean uncorrelated complex Gaussian process in each scanner coil, with equal variance in both the real and imaginary parts. As a result, sometimes it is easy to derive the distribution of the magnitude data (as in single coil systems) and other not-so easy, as in multiple coil parallel reconstruction.

Many filtering methods have been proposed either to remove the noise or to estimate the signal out of the noise. Different approaches may be found in the literature: (1) general filtering methods (anisotropic diffusion filtering, total variation methods) and (2) methods based on the statistical properties of the signal and noise (LMMSE, ML...). The talk will concentrate on discussing the nature of noise and means to address the challenges associated with it.

1 MRI Noise models

Noise in the k-space in Magnetic Resonance (MR) data from each coil is assumed to be a zero-mean uncorrelated Gaussian process with equal variance in both the real and imaginary parts. As a result, in single coil systems magnitude data in the spatial domain is modeled using a Rician distribution Drumheller (1993); Gudbjartsson and Patz (1995). In the same way, the composite signal in coils systems with multiple channels may be modeled as non-central Chi distributed Constantinides et al. (1997) if no subsampling of the k-space is assumed. The acquisition rate can be increased with parallel MRI (pMRI) techniques via subsampled acquisitions of the k-space data. In these cases, reconstruction methods have to be used in order to suppress the aliasing and underlying artifacts created by the subsampling. Dominant among these are SENSE Pruessmann et al. (1999) and GRAPPA Griswold et al. (2002), reviews of which can be found in Hoge et al. (2005); Larkman and Nunes (2007). From a statistical point of view, such a reconstruction will affect the stationarity of the noise in the reconstructed data, i.e. the spatial distribution of the noise across the image Thünberg and Zetterberg (2007). As a result, the variance of noise may vary for different image locations. Moreover it may also vary from one coil to another. However, under the assumption of a nearly homogeneous variance Dietrich et al. (2008), the data may be considered to follow a general non-central Chi distribution Aja-Fernández et al. (2010); Aja-Fernández et al. (2010a). This distribution reduces to a Rician if SENSE is used.

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2 Noise estimation

Noise in MR data, either from multiple or single coil acquisitions, is known to affect the visual quality of the MR images and different processing techniques, such as segmentation, registration or tensor estimation in Diffusion Tensor MRI (DT-MRI) Aja-Fernández et al. (2008b). Accordingly, the estimated noise power gives a measure of the quality of the data. This estimation can be used to measure the Signal-to-Noise Ratio (SNR) and as an input parameter in MR processing algorithms. Many filtering methods to improve SNR in MRI need an estimated value for $\sigma^2_n$: as the conventional approach McGibney and Smith (1993), maximum likelihood based methods Jiang and Yang (2003); Sijbers et al. (1998a,c), expectation maximization formulations with Rician noise assumptions DeVore et al. (2000), Linear Minimum Mean Square Error (LMMSE) based schemes Aja-Fernández et al. (2007, 2008a,b); Tristán-Vega and Aja-Fernández (2008) and unbiased non-local mean schemes Aja-Fernández and Krissian (2008); Coupé et al. (2008); Wiest-Daesslé et al. (2008). New techniques for DTI tensor estimation Fillard et al. (2007); Landman et al. (2007), segmentation methods based on the Rician distribution and fiber orientation estimators Clarke et al. (2008) also depend upon an estimated $\sigma^2_n$ value.

Noise estimation in MR is usually done over the (composite) magnitude image, since it is the usual output of the scanning process. However, if data in the complex spatial domain are available, the estimation may be easier done in that domain. Noise estimation is carried out assuming that the noise is uncorrelated and with identical variance in each pixel and it will be reduced to the estimation of the variance in a well-known Gaussian problem over one of the components.

If no complex data are available, noise estimation is carried out over the composite magnitude image, assuming again that the noise is uncorrelated and with identical variance in each pixel. Methods performing such estimation from magnitude data may roughly be divided into two groups: (i) approaches estimating the noise variance using a single magnitude image and (ii) approaches using multiple images. In this paper we will focus on the former. Noise estimation using a single image is usually based on background intensities, where the true signal amplitude should vanish and the Rayleigh or central Chi assumptions hold. Some of these techniques require a previous background segmentation. See overview in Table 2. Notation in Table 1.

Main references

- Rician noise estimation: Nowak (1999); Sijbers et al. (1998a,b,c) (background based) Aja-Fernández et al. (2009a); Brummer et al. (1993); Chang et al. (2005); Sijbers et al. (2006, 2007) (maximization of argument), Aja-Fernández et al. (2008a, 2009a); Sijbers et al. (2006) (mode based).
- Non-central $\chi$: Aja-Fernández et al. (2009a); Constantinides et al. (1997); Dietrich et al. (2008); Koay and Basser (2006)
$E\{X\}$: Expectation of random variable $X$

$\sigma^2_X$: Variance of random variable $X$

$\langle M(x) \rangle$: (Global) Sample mean of image $M(x)$

$\langle M(x) \rangle_x$: Local sample mean of image $M(x)$

$\text{Var}(M(x))_x$: Sample local variance of $M(x)$

$M(x_B)$: Background area of image $M(x)$

$M(x_R)$: $M(x)$ in the region $R$

$\text{mode}\{I(x)\}$: Mode of the distribution of $I(x)$

$\hat{a}$: Estimator of parameter $a$

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Table 1: Notation

3 Filtering and signal estimation

Since the amount of data to acquire in DTI studies is usually large, there is a tradeoff between acquisition time and quality that leads to a poorer Signal to Noise Ratio (SNR) associated to the Diffusion Weighted Images (DWI) used to infer fiber orientations when compared to conventional Magnetic Resonance Imaging (MRI). This is especially the case with modern High Angular Resolution Diffusion Imaging (HARDI) techniques, where very strong gradients have to be applied in order to improve the angular contrast. This produces a strong attenuation which worsens the SNR. Moreover, the Rician nature of the noise in DWI prevents the use of conventional Gaussian-based filtering techniques.

Main references

- Anisotropic Diffusion Schemes: Gerig et al. (1992); Krissian and Aja-Fernández (2009); Perona and Malik (1990); Weickert et al. (1998)
- Rician Based: McGibney and Smith (1993) (Conventional approach), Basu et al. (2006); Koay and Basser (2006),
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<th>Assump.</th>
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Table 2: Survey of noise estimators for single and multiple coil MR data. Note that estimators in boxes 1 and 4 require background segmentation. From Aja-Fernández et al. (2009a)
4 Noise and tensor estimation

Despite the known weakness of the single tensor approach in certain cases, as fiber crossing or kissing, Diffusion Tensor Imaging (DTI) is still widely used, mainly due to its simplicity and straight visual interpretation. Least Squares (LS) techniques have become the \textit{de facto} standard to estimate the Diffusion Tensor (DT) in DTI Salvador et al. (2005); Tristán-Vega et al. (2009). Although other approaches are possible, LS are fast and robust, and they show optimal properties when Weighted Least Squares (WLS) is used. Recent studies show that when single coil acquisition is considered, the estimation is nearly unbiased, so WLS is in this case the Best Linear Unbiased Estimator (BLUE) Salvador et al. (2005). When multiple receiver coils for simultaneous acquisition are considered, the bias may be an important source of error; the larger the number of receiving coils the more critical. As stated in Tristán-Vega et al. (2009), the variance in the estimation may be reduced by increasing the number of gradient directions, but the error bias remains. Results in Salvador et al. (2005); Tristán-Vega et al. (2009) provide a theoretical framework to properly estimate the DT from single- and multiple-coil systems.

Main references


5 Image quality assessment methods

Full-reference methods for quality assessment are those in which a signal is compared to a \textit{ground truth} image, i.e. a \textit{golden standard}. Within these methods, the most frequently used are those error based methods, as the Mean Squared Error (MSE) Eskicioglu and Fisher (1995); Tang and Cahill (1992). The limitations of such methods have been widely reported in the literature (see Girod (1993) for example). Consequently, some additional variations of the MSE have also been used in order to better deal with the features of the Human Visual System Eskicioglu and Fisher (1995); Tang and Cahill (1992). In Miyahara et al. (1998) a new index is proposed, namely, the objective Picture Quality Scale (PQS), basically intended to measure the degradation in coding and compression of images. It takes into account properties of visual perception of both global features and of disturbances. It turns out to be bounded, being the maximum value 5.797, obtained when an image is compared with itself. Experiments show that although it is a good measure when dealing with compression, it is not so good a measure for other sources of degradation Sheikh et al. (2006). Recently, some methods based on \textit{Natural Scene Statistics} have been reported Sheikh and Bovik (2006); Sheikh et al. (2005).

In Wang et al. (2004) Wang \textit{et al.} proposed a full-reference quality assessment method based on the structural similarity of two images, the so-called Structural Similarity (SSIM) index. The method is a modification of their Quality Index, originally proposed in Wang and Bovik (2002). As of today, this method has proved to be versatile and robust in many different environments Sheikh et al. (2006). In Aja-Fernández et al. (2006) a new method based on the structural information of the image and, specifically, on the statistics of the sample local variance; it is intended to penalize degradations in which the structural content is filtered out by accounting for the non-stationary content in the image. Additionally, it intends not to over-penalize noise as long as noise content does not obscure structure.

Main references


References


